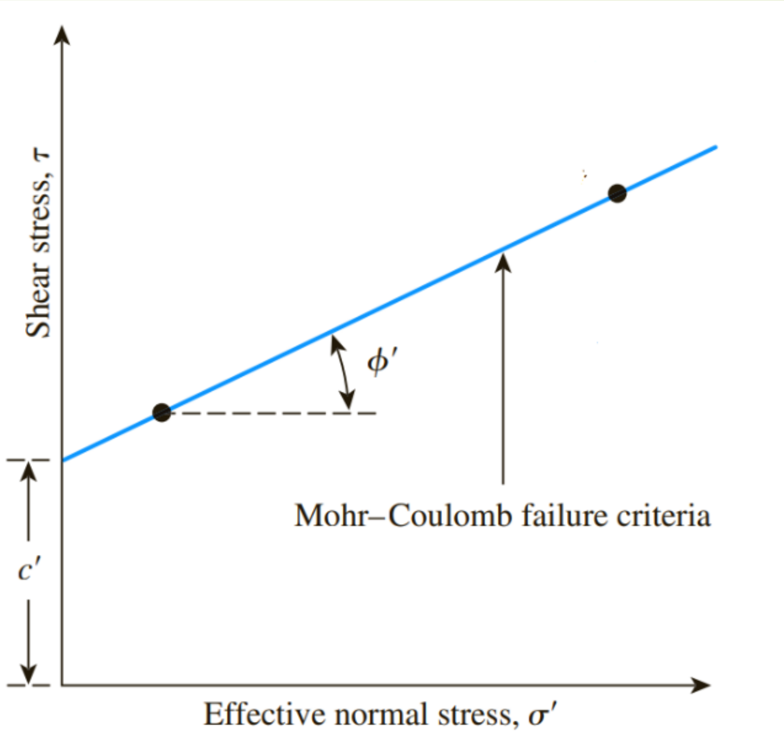
DAY 07

PANA ACADEMY

2.3 Shear strength of soil and stability of slopes: Concept of shear strength, principal planes and principal stresses; Mohr-Coulomb theory of shear strength; calculation of normal and shear stresses at different planes; relation of principle stress at failure condition; types of shear tests; stability of slopes. (ACiE0203)



Shear strength of soil (τ)

Cohesion (C):

Angle of internal friction:

 # Mohr-Coulomb theory of shear strength

$$\tau_f = c + \tan\phi$$

Limitations of Mohr-Column Theory:

- It neglects the effect of intermediate principal stress.
- It approximates the curved failure envelop by a straight line.
- For some clayey soil, there is no fixed relationship between the normal and shear stresses on the plane of failure. The theory cannot be used for such soils.

 # Principle plane and Principle stresses

If a plane is inclined at angle θ to major principle plane,

Normal stress (σ) is given by, $\sigma = \frac{\sigma_1 + \sigma_3}{2} + \frac{\sigma_1 - \sigma_3}{2}\cos 2\theta$

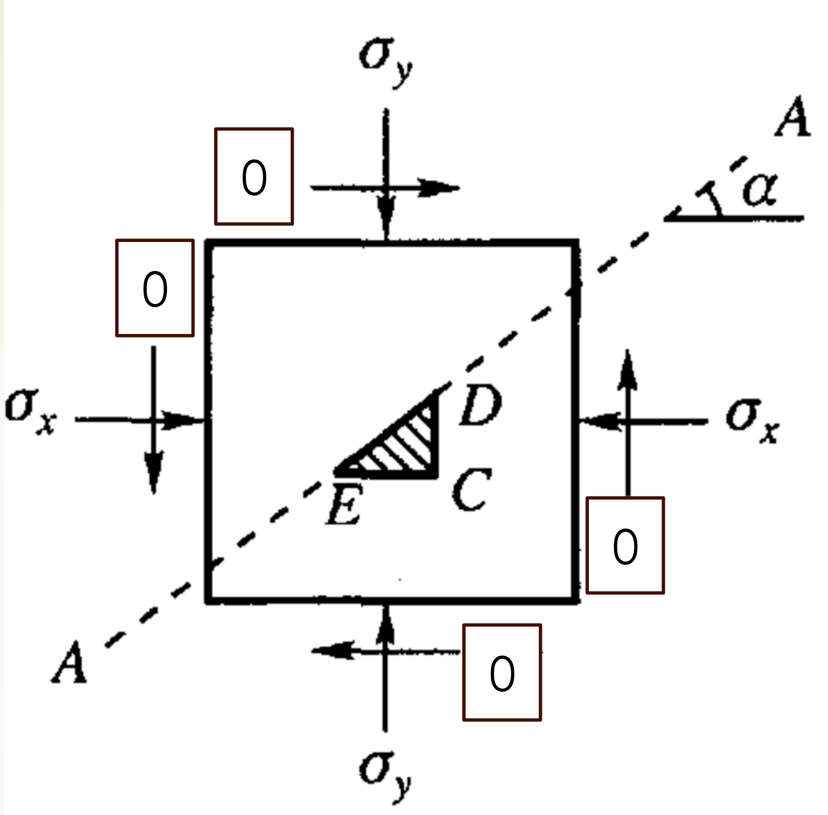
Shear stress (τ) = $\frac{\sigma_1 - \sigma_3}{2}\sin 2\theta$

Maximum shear stress (τ_{max}) = $\frac{\sigma_1 - \sigma_3}{2}$, occurs at 45^0 to principle plane

Where, σ_1 = major principle stress

σ_3 = minor principle stress

Θ = 45 + Ø/2

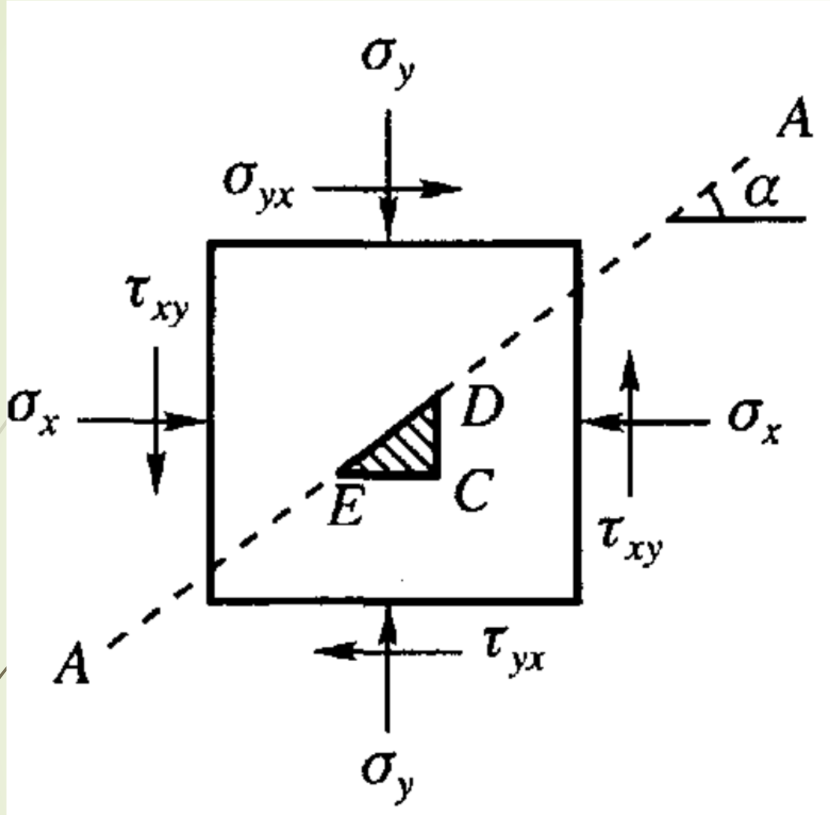


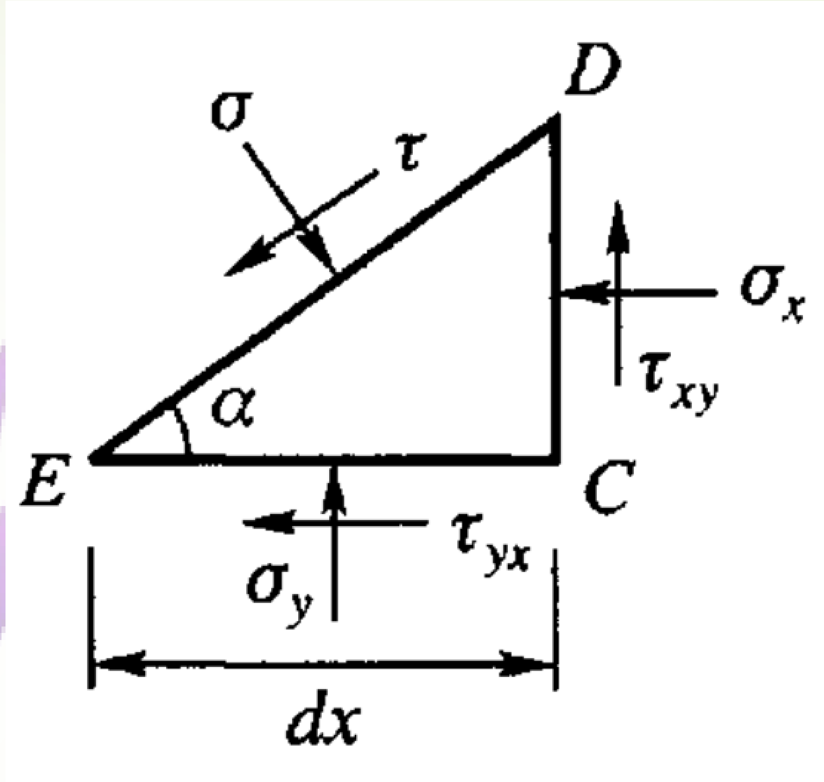
Mohr-Column failure criterion is given by the following relation. This gives the **relation between principal stresses**, cohesion and angle of friction at **failure condition.** $\mathrm{Tan}^2\left(45 + \frac{\phi}{2}\right) = N_\phi$ is called flow ratio.

$$\sigma_1' = \sigma_3' \tan^2\left(45^\circ + \frac{\phi'}{2}\right) + 2c' \tan\left(45^\circ + \frac{\phi'}{2}\right)$$

Failure plane makes $45 + \frac{\phi}{2}$ with major principal plane.

Calculation of normal and shear stresses at different planes





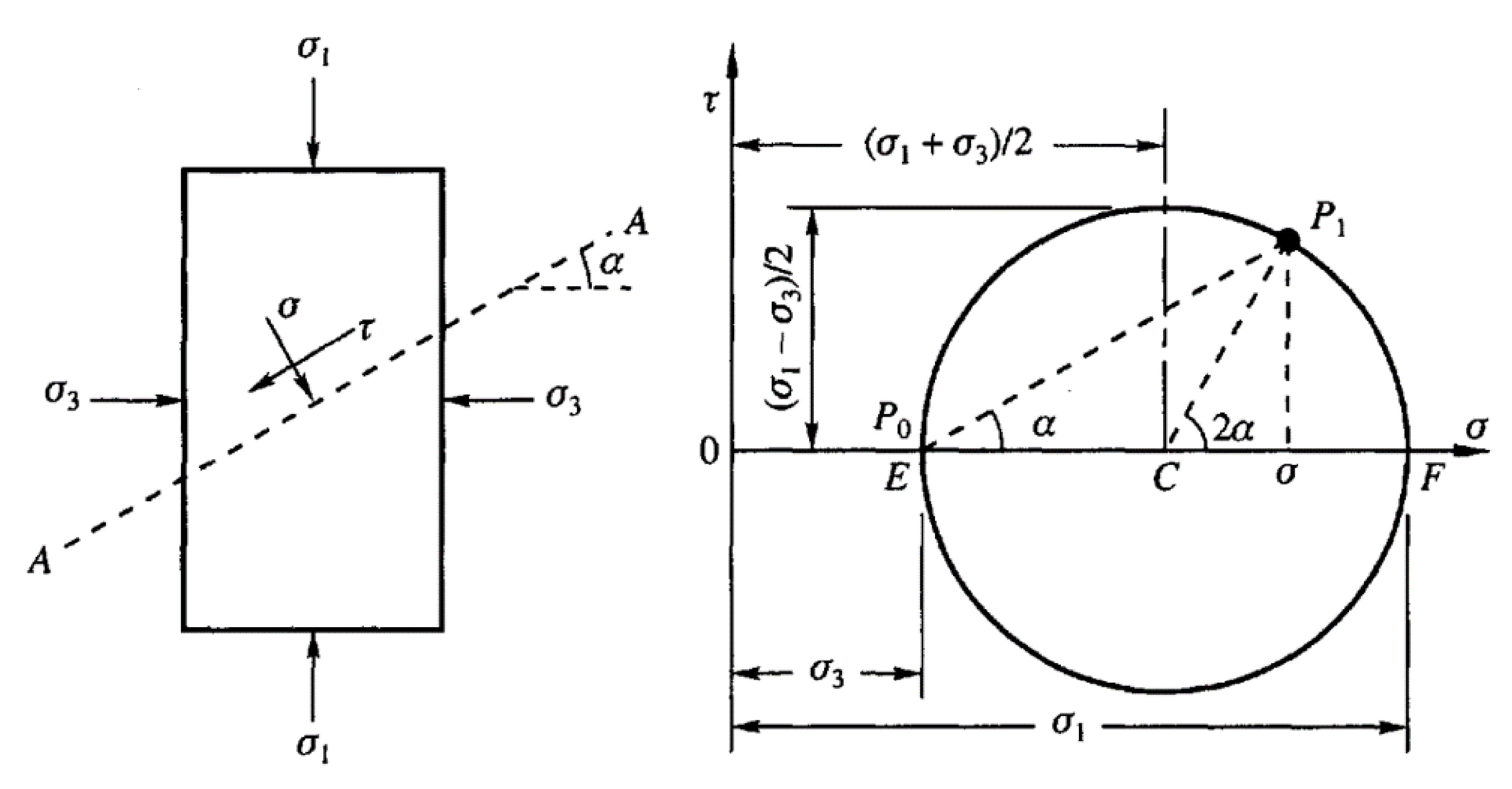
$$\sigma = \frac{\sigma_y + \sigma_x}{2} + \frac{\sigma_y - \sigma_x}{2}\cos 2\alpha + \tau_{xy}\sin 2\alpha$$

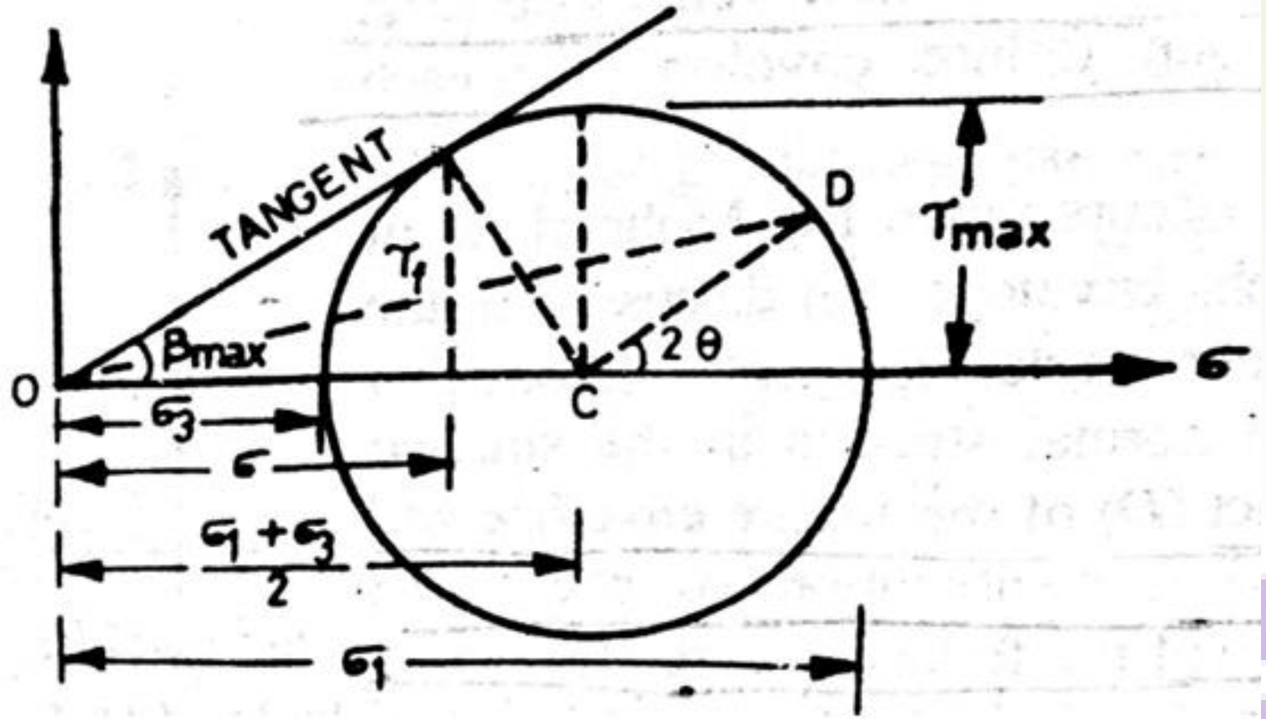
$$\tau = \frac{\sigma_y - \sigma_x}{2}\sin 2\alpha - \tau_{xy}\cos 2\alpha$$

Orientation of principal plane:

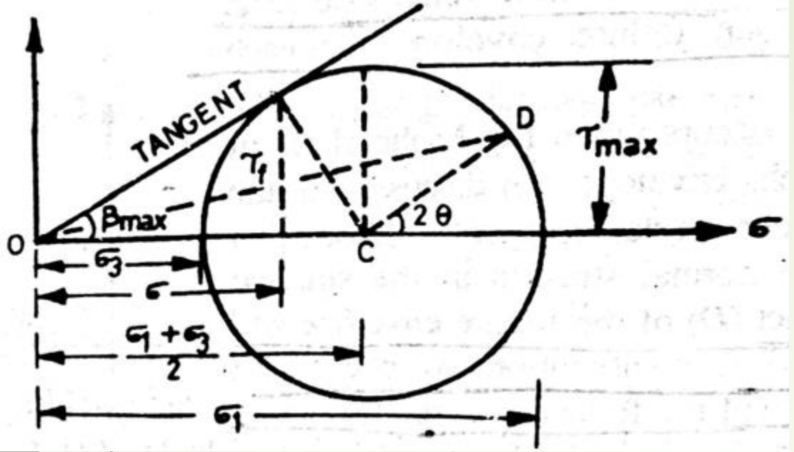
$$\tan 2\alpha = \frac{2\tau_{xy}}{\sigma_y - \sigma_x}$$

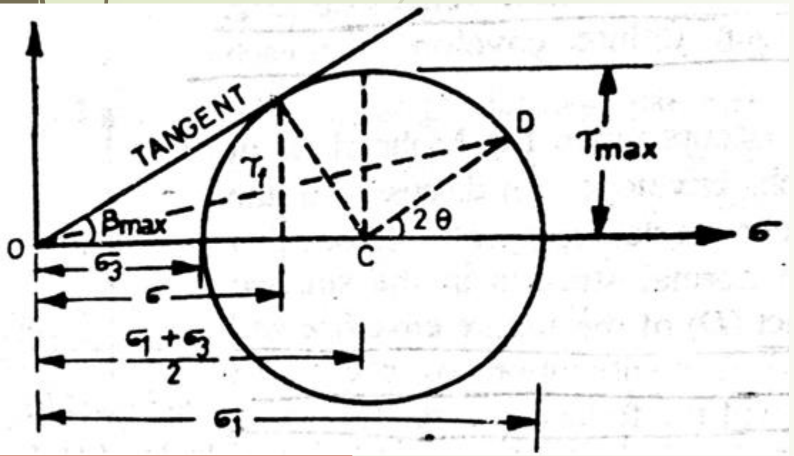
Mohr's Circle of Stress

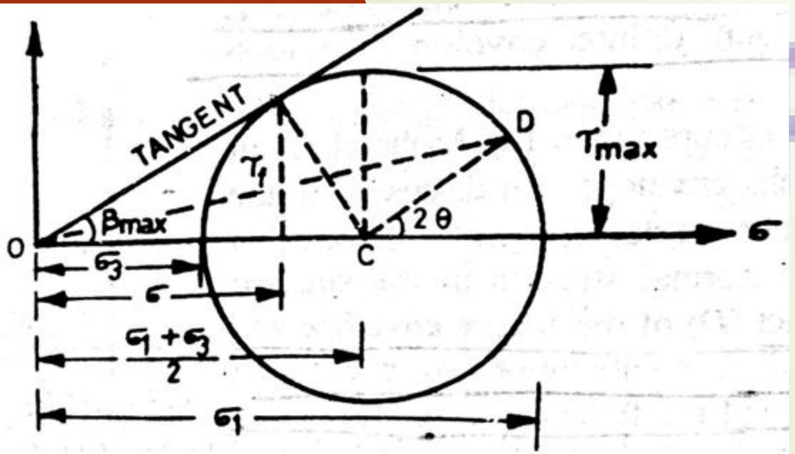




- The maximum shear stress τ_{max} is numerically equal to $\frac{\sigma_1 - \sigma_3}{2}$ (i.e., radius of Mohr's circle) and it occurs in a plane inclined at 45^0 to the principal planes.







- The point D on the Mohr circle represents the stresses (σ, τ) on a plane making an angle θ with the major principal plane.
  The resultant stress on that plane is equal to $\sqrt{\sigma^2 + \tau^2}$ and its angle of obliquity with the normal of the plane is equal to angle β, given by:

$$\beta = \tan^{-1}(\tau/\sigma)$$

- The maximum angle of obliquity β_{max} is obtained by drawing a tangent to the circle form origin O.

$$\beta_{max} = \sin^{-1}\frac{\frac{\sigma_1 - \sigma_3}{2}}{\frac{\sigma_1 + \sigma_3}{2}} = \sin^{-1}\frac{\sigma_1 - \sigma_3}{\sigma_1 + \sigma_3}$$

- The shear stress τ_f on the plane of the maximum obliquity is less than the maximum shear stress τ_{max}.
- Shear stress on the plane right angle to each other are numerically equal but are of opposite sign.

There is no need to be rigid about sign convention for plotting the shear stresses in Mohr's circle. These can be plotted either upward or downward. Although the sign convention is required for locating the orientation of the planes, the numerical results are not affected.

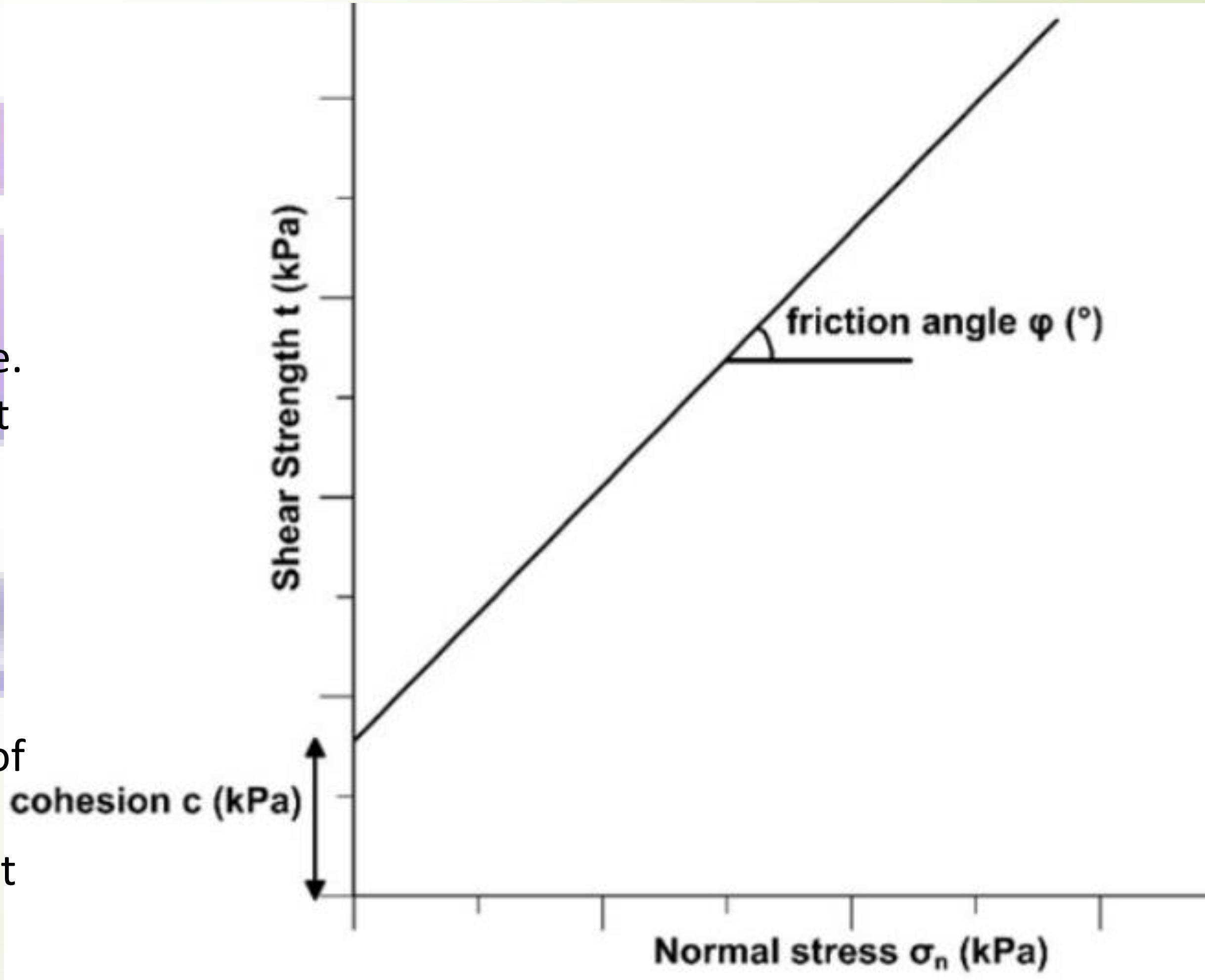
Types of shear tests

i. Direct shear test

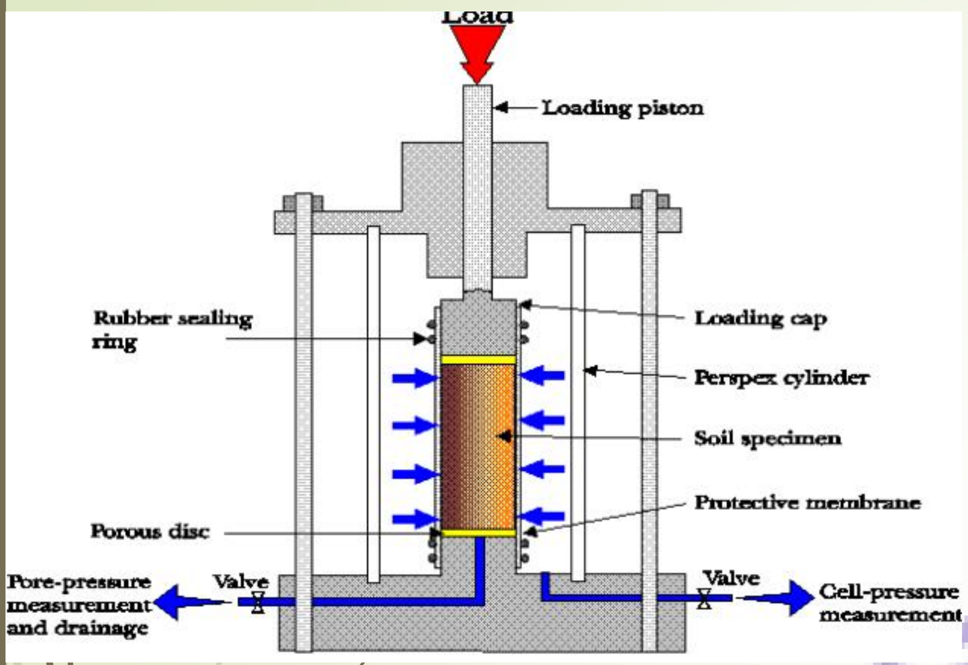
This test is normally applicable for **cohesion less soil** (coarse grained soil). The test is conducted on a device called "shear box apparatus" of size $60 \times 60 \times 50\ mm$ whereas the size of soil specimen- is $60 \times 60 \times 25\ \mathrm{mm}$. , it is ideally suited for drained test on cohesionless soils

Disadvantages of direct shear test are:

- Drainage condition during the test cannot be controlled.
- The area of failure plane is not constant during shear test.
- Soil is sheared on a predefined horizontal plane. This forced plane is not necessarily the weakest plane. This is the most important limitation of the Direct shear test.
- The distribution of normal and shear stress along the predetermined failure plane is very complex.
- The effect of lateral restraint by the side walls of the shear box is likely to affect the results.
- The measurement of pore water pressure is not possible.



ii. Triaxial compression test



S.N	Drainage condition during the two stages of triaxial testing		Type of test
	First stage (Application of cell pressure (σ_3) only)	Second Stage (Application of additional axial stress (σ_d) at constant cell pressure	
1	Drainage Allowed i.e., consolidated	Drainage allowed i.e., drained	Consolidated Drained (CD or Drained Test) [S Test]
2	Drainage Allowed	Drainage not allowed	Consolidated Undrained (CU test) [R test]
3	Drainage not allowed	Drainage not allowed	Unconsolidated undrained test (UU test) or [Q test]

Advantages of Triaxial test:

- The **stress distribution is uniform on the shear plane** during the progress of testing.
- The soil specimen fails in **shear on the weakest shear plane**.
- Complete **control of the drainage** is possible.
- The pore water pressure can be measured accurately during the test.
- The possibility to vary the cell pressure is possible in this test to **simulate the field** conditions for the sample.

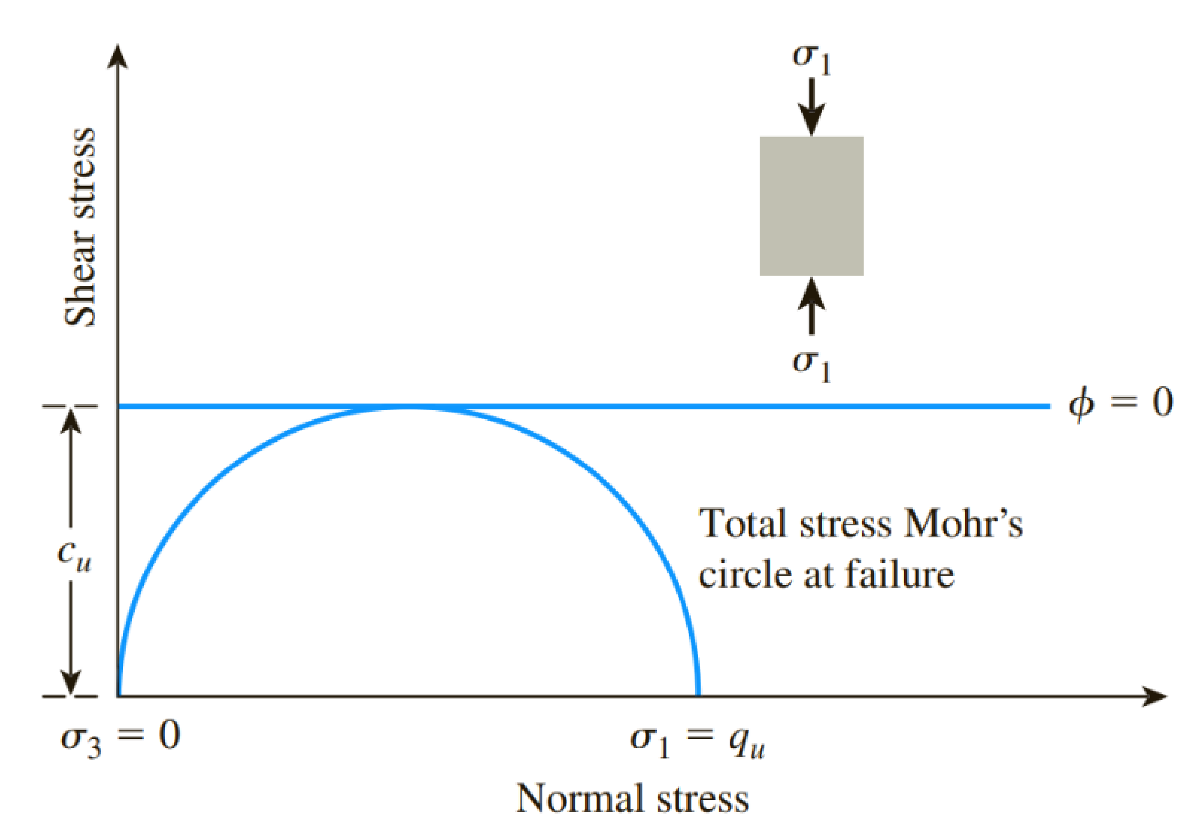
Disadvantages of Triaxial test:

- The apparatus is elaborate, costly and bulky.
- The drained test takes more time.
- It is not possible to determine the **cross-sectional area of the specimen at large strains**, as the assumption that the specimen remains cylindrical does not hold good.
- The consolidation of the specimen is isotropic which is contrary to the field where it is anisotropic.

iii. Unconfined compression test

$$A = \frac{A_0}{1 - \varepsilon}$$

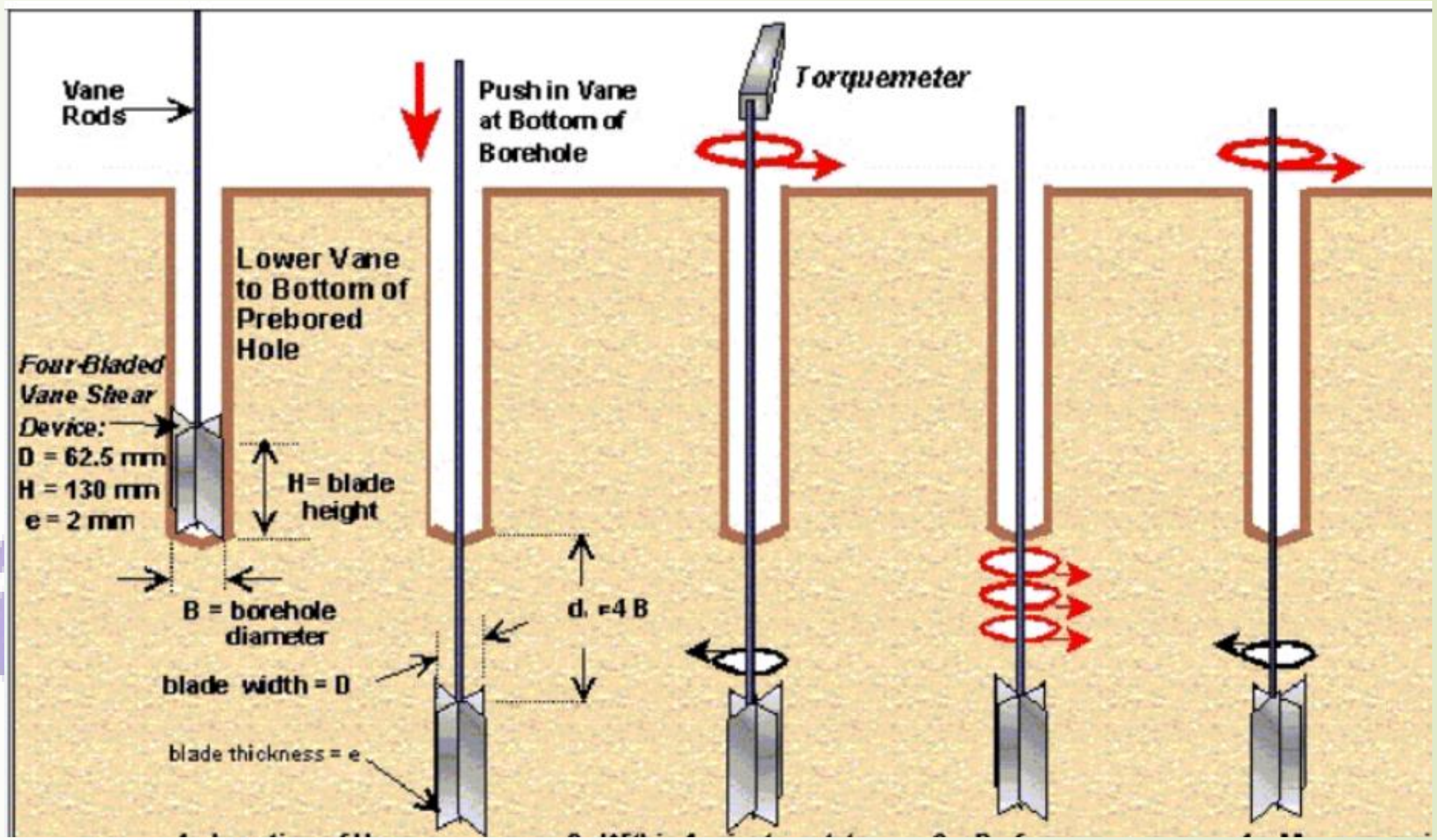
Cohesion intercept is equal to the radius of Mohr's circle. Its advantages are that it is convenient, simple, quick and is ideally suited for measuring unconsolidated undrained shear strength of **saturated clays.** Whereas, its disadvantages are that it **cannot be conducted on fissured clays** and it can be misleading one for soils for which the angle of shearing resistance is not zero.



iv. Vane shear test

$$\tau_f = \frac{T}{\pi D^2\left(\frac{H}{2} + \frac{D}{6}\right)}$$

If top of the vane is above the soil surface and the depth of the vane inside the soil surface is H_1 then,



$$\tau_f = \frac{T}{\pi D^2\left(\frac{H_1}{2} + \frac{D}{12}\right)}$$

Specially used for **soft sensitive clays**, for which sampling is difficult.

43. At a depth of 6 m below the ground surface at a site, a vane share test gave a torque value of 6040 N-cm. The vane was 10 cm high and 7 cm across the blades. Estimate the shear strength of the soil and choose the correct answer from the following options.
    a. 64 kN/m^2
    b. 69 kN/m^2
    c. 72 kN/m^2
    d. 62 kN/m^2
44. For a sandy soil, the angle of internal friction is 30^0. If the major principal stress is 50 kN/m^2, then the corresponding minor principal stress will be
    a. 12.2
    b. 16.66
    c. 20.8
    d. 27.2
45. The effective stress strength parameters of soil are c'=10 kPa and $\varphi=30^0$, the shear strength on a plane within the saturated soil mass at a point where total normal stress is 300 kPa and pore water pressure is 150 kPa, will be
    a. 90.5 kPa
    b. 96.6 kPa
    c. 101.5 kPa
    d. 105.5 kPa

46. The expansion of soil due to shear at constant value of pressure is called;
    a. Apparent Cohesion b. True Cohesion c. Dilatancy d. Consistency

47. The useful method of finding the shear strength of very plastic cohesive soil is by means of:
    a. Cone test b. Penetration Test c. Vane shear test d. Torsional shear test

T = 6040 N cm = 60.40 × 10^{-3} kN m, D = 7 cm = 0.070 m, h = 10 cm = 0.1 m

Vane Shear test at depth of 6 m below the ground surface at a site so we use the two-way shearing formula for shear strength of soil

$$S = \frac{T}{\pi D^2(\frac{H}{2}+\frac{D}{6})} = \frac{60.40\times 10^{-3}}{\pi(0.07)^2(\frac{0.1}{2}+\frac{0.07}{6})}$$

S = 63.6 kN/m²

For a sandy soil

C = 0

Now,

$$\sigma_1 = \sigma_3 \tan^2\left(45^\circ + \frac{\phi}{2}\right) + 2c\tan\left(45^\circ + \frac{\phi}{2}\right)$$

$$50 = \sigma_3 \tan^2\left(45^\circ + \frac{30}{2}\right) + 0$$

50 = σ_3 $\tan^2$ (60°)

$$\frac{50}{\tan^2(60^\circ)} = \sigma_3$$

= 16.66 kN/m².

Given:

c' = 10 kPa

φ = 30°

σ = 300 kPa

u = 150 kPa

Calculations:

$\bar{\sigma} = \sigma - u = 300 - 150 = 150$ kPa

S = 10 + (150) x tan30°

= 96.6 KPa

BCDCB BDB

48. Pick up the correct statement:
    a. Failure plane carries maximum shear stress.
    b. Failure plane doesn't carries maximum shear stress.
    c. Failure plane carries shear stress equal to maximum shear stress only on special condition.
    d. None of these
49. For testing saturated clay for shear strength, the test recommended, is:
    a. Direct shear test
    b. Triaxial compression test
    c. Unconfined compression test
    d. All of the above
50. A clay is subjected to pressure in excess to its present over-burden, is said to be:
    a. Pre-compressed
    b. Pre-consolidated
    c. Over-Consolidated
    d. All of the above
51. Which of the following is responsible for shear resistance of soil?
    a. Intergranular friction
    b. Cohesion and adhesion between the soil particles
    c. Both (a) and (b)
    d. None of the above
52. In an undrained plastic clay, the shear strength is due to
    a. Internal friction
    b. Cohesion
    c. Inter-granular friction
    d. None of the above
53. The angle made by the failure envelope with horizontal in the Mohr's Circle gives
    a. Cohesion
    b. Angle of internal friction
    c. Angle of wall friction
    d. Surcharge Angle
54. The angle between two planes on which the shearing stress is zero is:
    a. Zero
    b. 30^0
    c. 45^0
    d. 90^0
55. The shape of plot between shear and normal stresses according to Mohr's theory is
    a. Straight Line
    b. Curve
    c. Elliptical
    d. All

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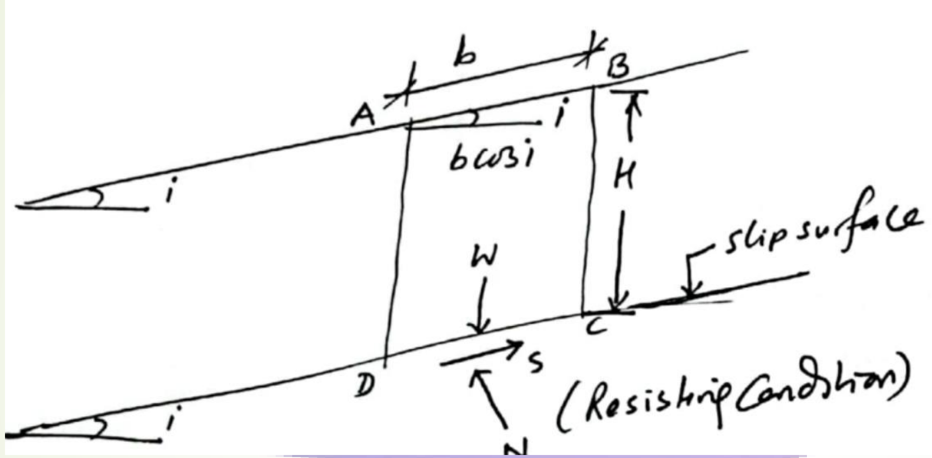
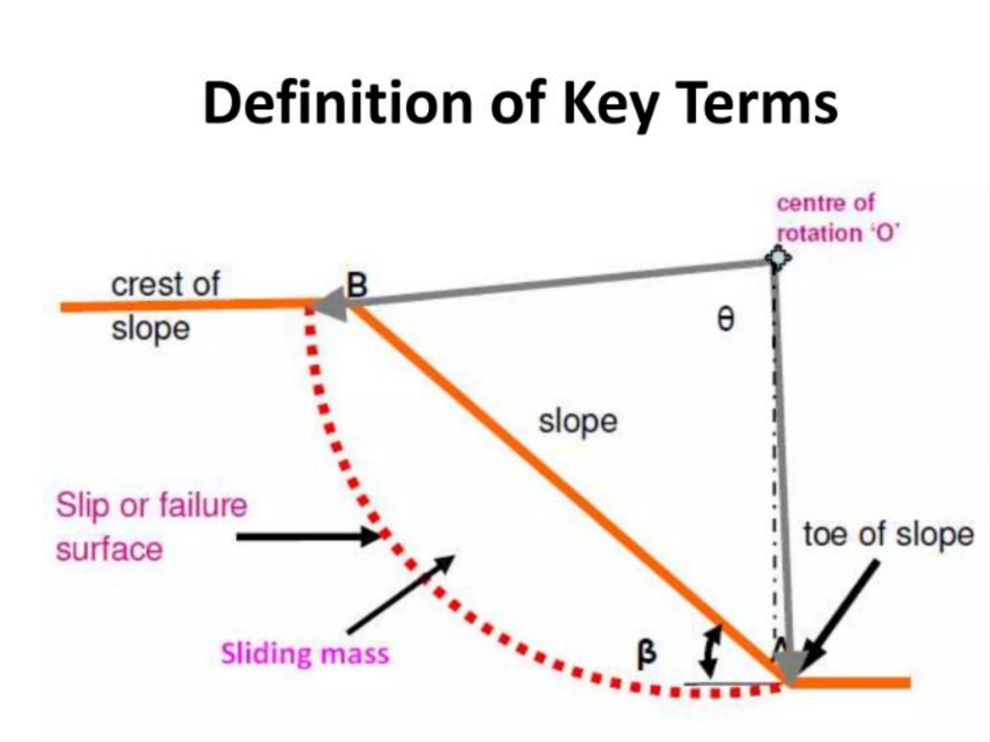
Stability of slopes

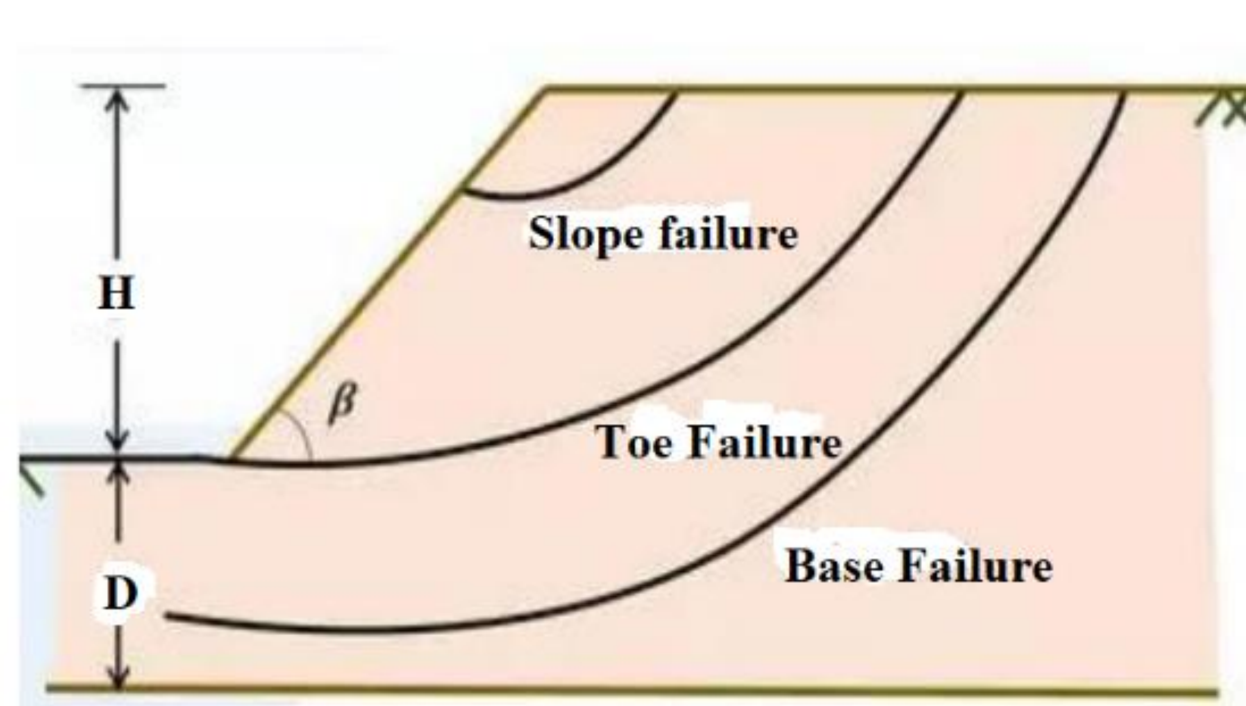
- **Infinite slope**: The slope which is very large in extent, theoretically infinite and the properties of soils will be the same at the identical depth so that a slip surface will be a plane parallel to the slope surface.
- **Finite slope**: The slope which is limited in extent and the properties of soils will not be the same at identical depths so that the slip surface will be curve.

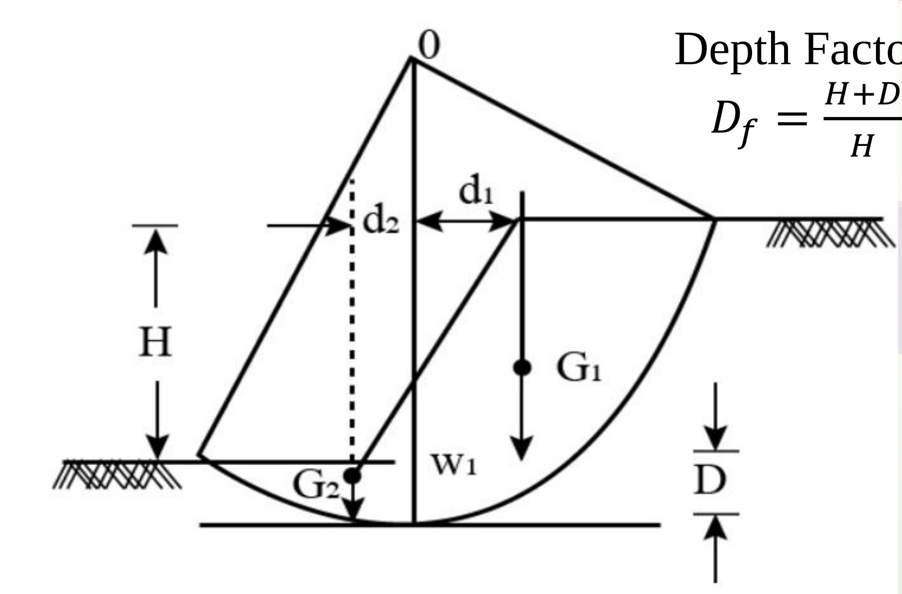




Rotational Failure

- Failure **caused by rotation along the slip surface** is called rotational failure.
- Rotational failures are basically, **toe failures, slope failures and base failure**.
- Failure along the **surface which passes through the toe are called toe failure**, and those along a surface which **intersect the slope above the toe of slope are called slope failure and those which passes below the toe is called base failure**.
- The slope failure occurs when a weak plane exists above the toe.
- The base failure occurs when a week stratum lies beneath the toe.
- If a strong stratum exists below the toe, the slip surface of the base failure is tangential to that stratum. In all other case, the failures are generally toe failures. Toe failures are most common





Depth Factor

$$D_f = \frac{H+D}{H}$$

Slope failure $\mathbf{D_f}$<1.
Base failure $\mathbf{D_f}$ >1.
Toe failure $\mathbf{D_f}$ =1

- **Translational Failure:** These types of failures occur in <u>infinite slopes</u> along a failure surface which is parallel to the slope.

- **Compound Failure:** It is combination of the rotational and transitional failures. Its surface is curved at the two ends and plane in the middle portion. A compound failure generally occurs when a hard stratum exists at considerable depth below the toe.

15.2 Factor of Safety

The task of the engineer charged with analyzing slope stability is to determine the factor of safety. Generally, the factor of safety is defined as

$$F_s = \frac{\tau_f}{\tau_d} \tag{15.1}$$

where F_s = factor of safety with respect to strength
τ_f = average shear strength of the soil
τ_d = average shear stress developed along the potential failure surface

$$F_s = \frac{c' + \sigma' \tan \phi'}{c'_d + \sigma' \tan \phi'_d}$$

$$F_{c'} = \frac{c'}{c'_d}$$

$$F_{\phi'} = \frac{\tan \phi'}{\tan \phi'_d}$$

Stability Analysis of Infinite Slope

- **Cohesionless soil**

$$Factor\ of\ Safety\ (FoS) = \frac{Shear\ strength\ along\ the\ failure\ surface}{Shear\ stress\ along\ the\ failure\ surface} = \frac{\tan \phi}{\tan i}$$

For the limiting condition, FoS = 1, i.e., $\tan \phi = \tan i$. Thus, the maximum inclination of an infinite slope (i) in a cohesionless soil is equal to angle of shearing resistance (ϕ) of soil.

- **Cohesive soil**

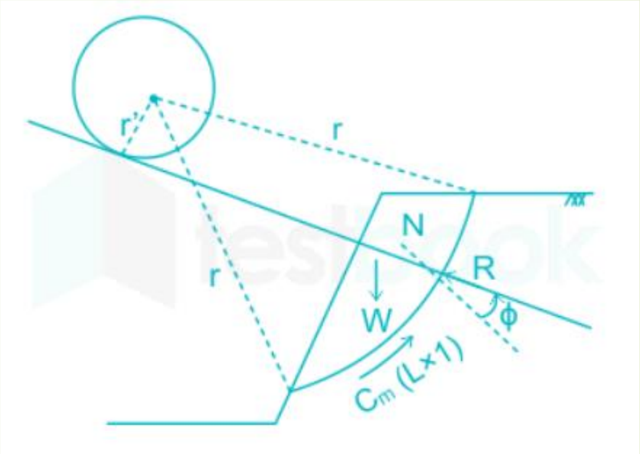
$$FoS = \frac{c' + (\gamma H \cos^2 i) \tan \phi'}{\gamma H \cos i \sin i}$$

Hence, for $C - \phi$ soil, the slope is stable as long as the slope angle $i \leq \phi$. If $i > \phi$, the slope will be stable only upto a limited height known as critical height (H_C) which is obtained by equating FoS to one.

$$H_C = \frac{C'}{\gamma(\tan i - \tan \phi') \cos^2 i}$$

Stability Analysis of Finite Slopes

- Rotational failures are generally encountered in finite slopes.
- Most commonly used limit equilibrium methods for stability of finite slopes are Swedish method (Slip circle method) and Friction Circle method.
- Both of these methods assume that the surface of sliding to be an **arc of a circle**.
- This method is done by dividing the area within the slip circle into a number of vertical slices.
- A series of slip circle are selected and the circle along which the factor of safety is minimum is the likely failure plane.
- **Taylor's Stability charts** are based on the total stresses using the Friction circle method.



Friction circle method:

- Based on total stress analysis in which shearing angle (ϕ) is used to analyze the stability of finite slopes.
- Radius of friction circle is R sinϕ, where R is the radius of sliding mass.

Stability Number ($\boldsymbol{S_n}$)

A dimensional parameter is used for the analysis of stability of $C-\phi$ soil, which is defined as the ratio of mobilized cohesion and γH is called stability number. Theoretically, it's maximum value is 0.5 but practically its value is observed to be 0.25-0.30.

$$\boldsymbol{S_n} = \frac{C_m}{\gamma H} = \frac{C}{F_C \gamma H} = \cos^2 i\,(\tan i - \tan \phi')$$

- The reciprocal of stability number is called **stability factor.**
- This method is applicable for c-ϕ and ϕ=0(pure clay) soil.

56. If the failure of the finite slope occurs through the toe, it is called:
   a. Slope failure
   b. Face failure
   c. Base failure
   d. Toe failure

57. A plane inclined at an angle ϕ to the horizontal at which the soil is expected to stay without sliding is:
   a. ϕ-line
   b. Repose line
   c. Natural slope line
   d. All of the above

58. For a clay slope of height 35 meters, stability number 0.05, bulk density 20 kN/m^2 and cohesion 25 kN/m^2 , the critical height of the slope will be:

   a. 10.0 m   b. 12.5 m   c. 25.0 m   d. 35 m

59. For a base failure, the Depth factor (D_f) is

   a. Zero   b. 1   c. Between 0 and 1   d. Greater than one.

60. The largest value of Taylor's stability number is

   a. 0.261
   b. 0.562
   c. 1.911
   d. 2.911

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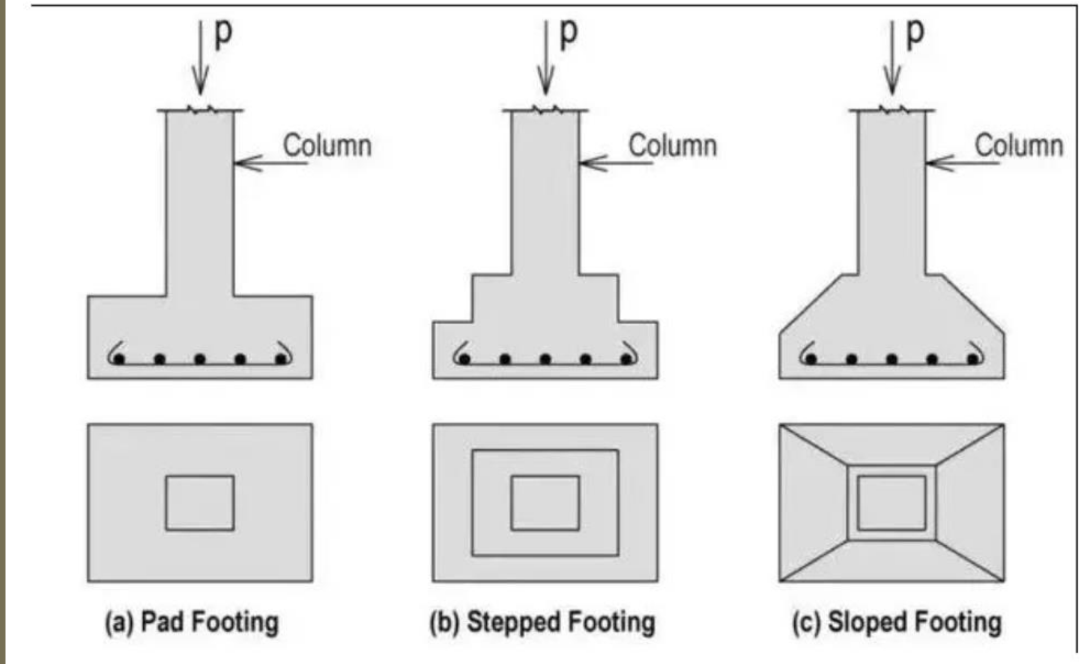
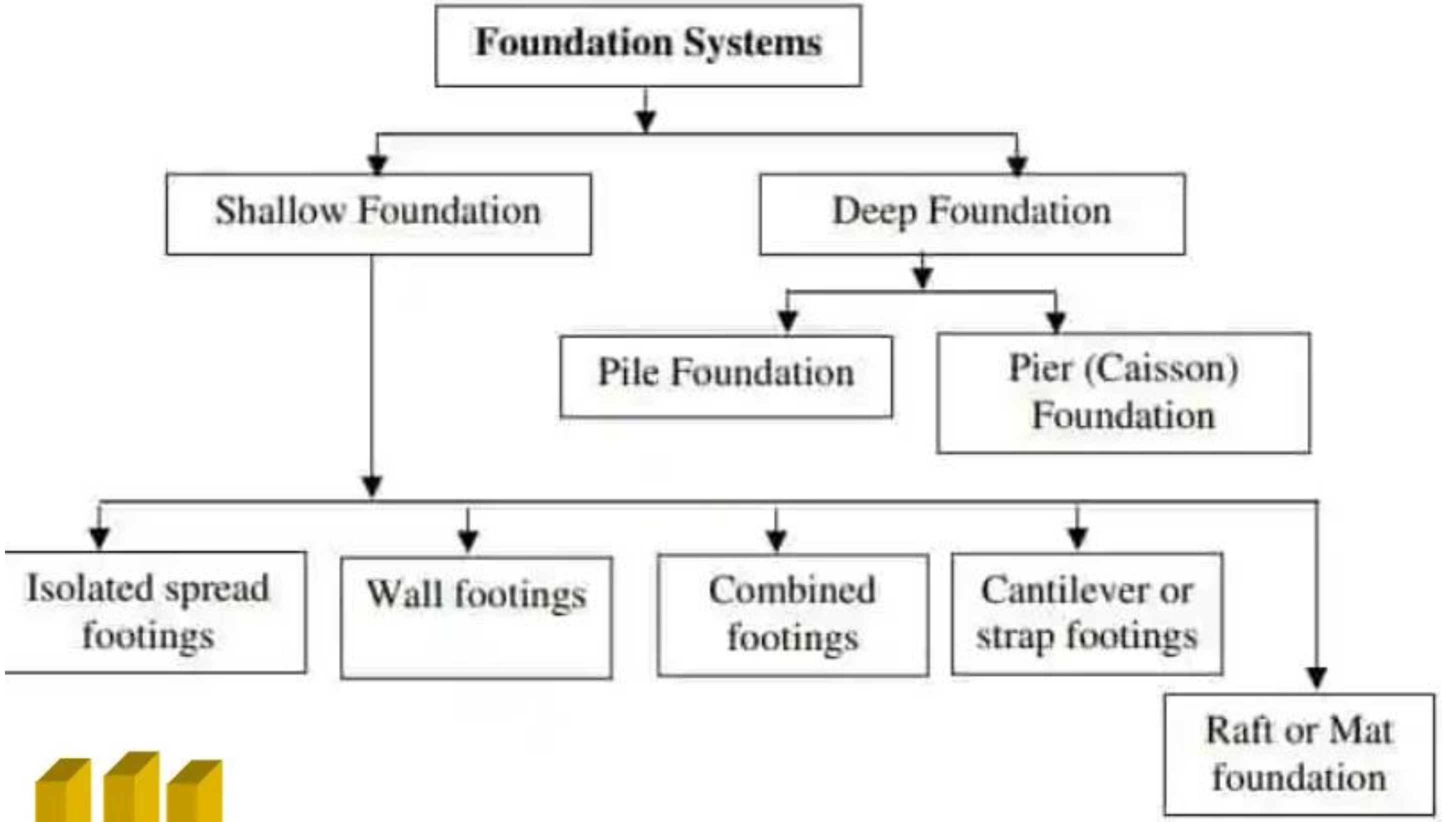
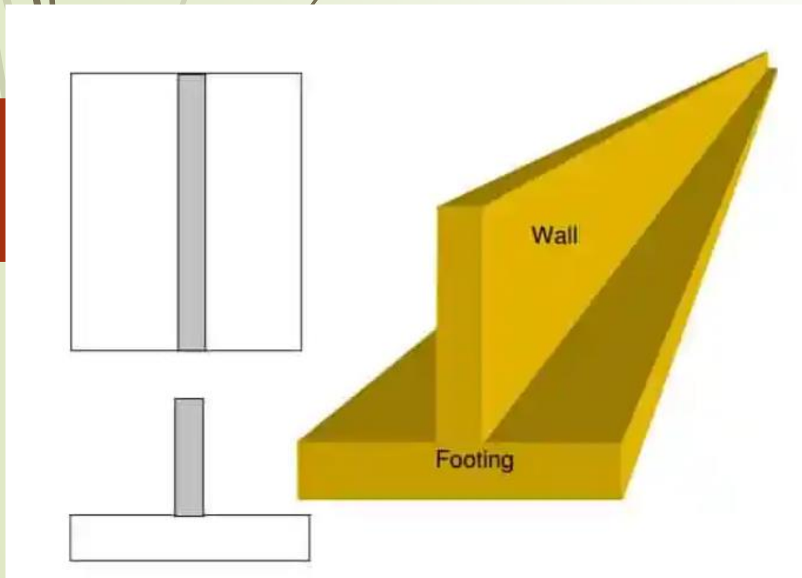
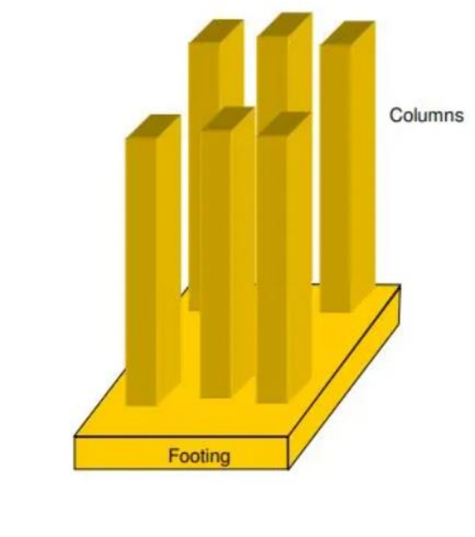


Types of Isolated Spread Footing

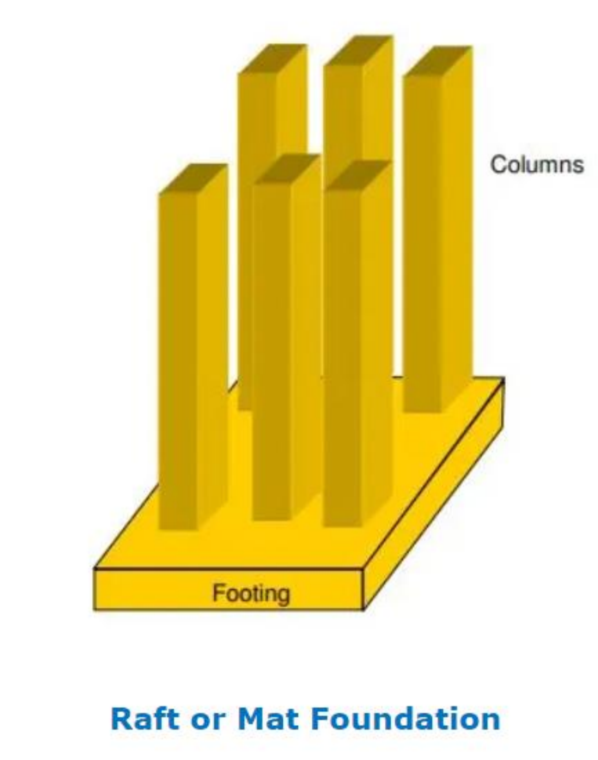


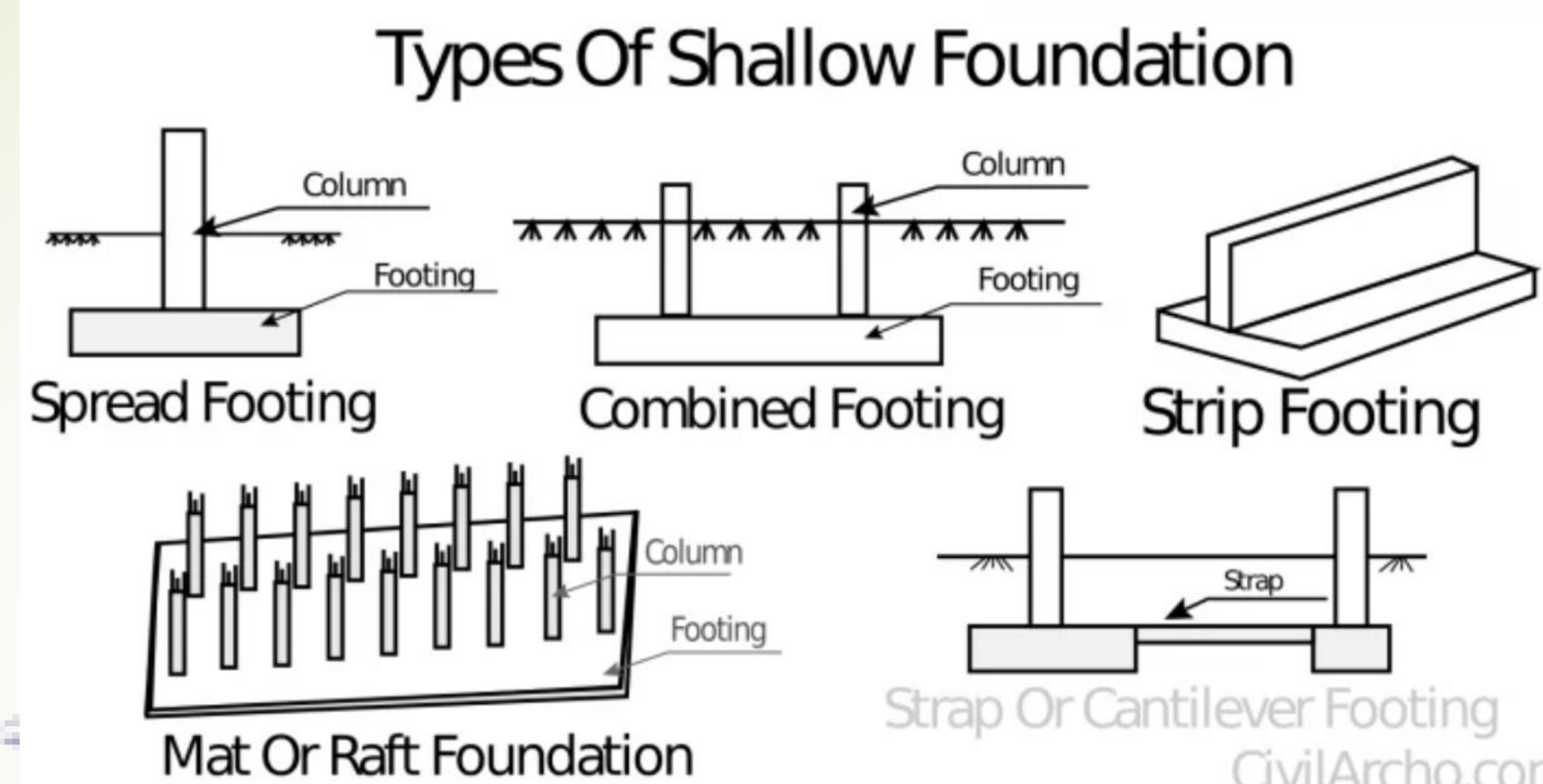




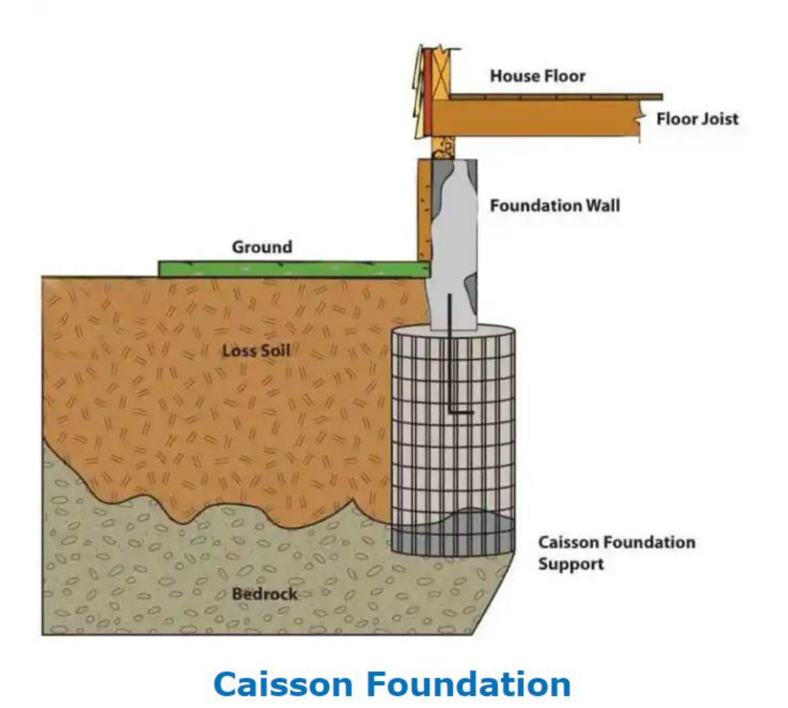
Raft or Mat Foundation

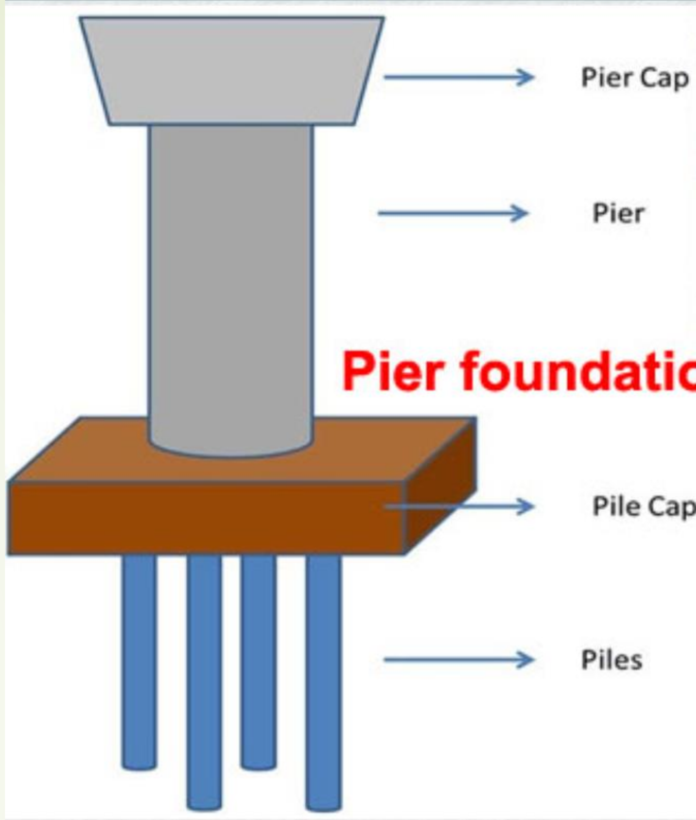
- **Isolated footing**: - below a column
- **Strip foundation**: The foundation whose length is considerably greater than its width is called strip or continuous foundation.
- **Combined foundation**: It supports two columns, and is provided when two columns are such closely spaced that their individual column footing overlap, or when the **property line is close to one column** and spread footing will be eccentrically loaded when kept entirely within the property line. It can be either rectangular or trapezoidal in plan.

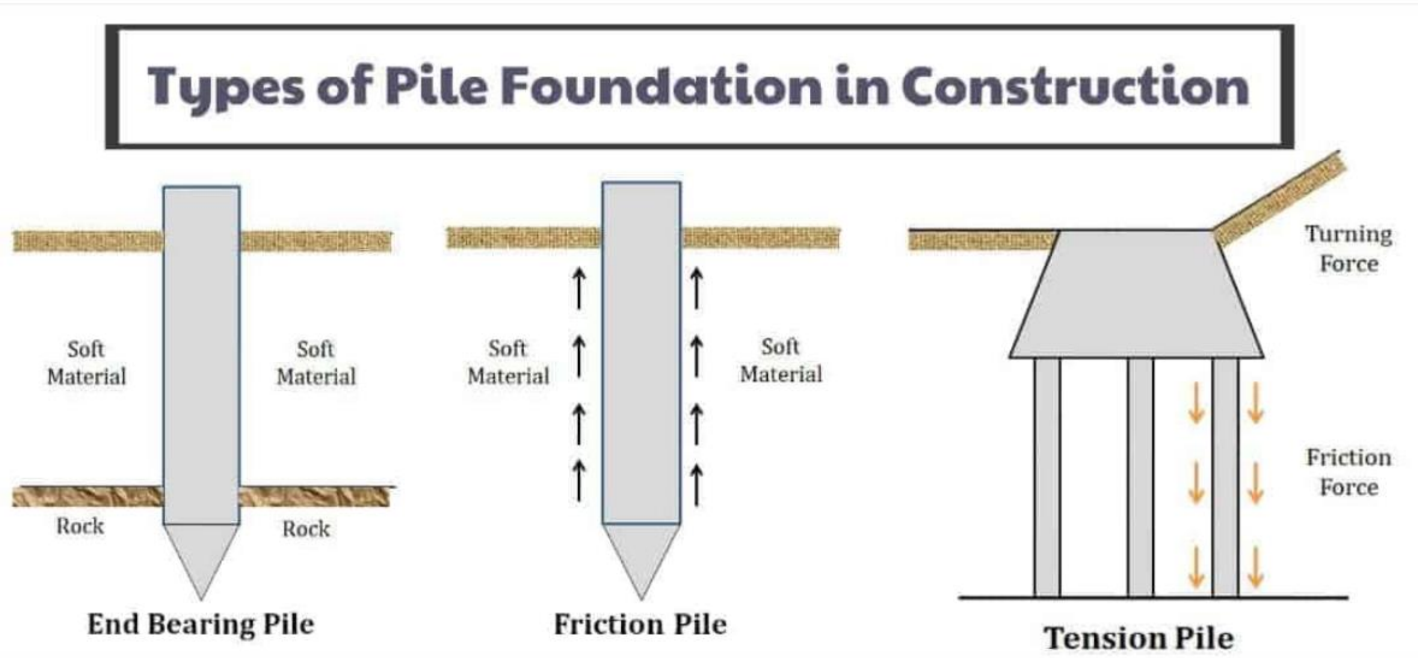




- **Strap foundation**: When two or more isolated footings connected by a beam (called strap) it is called strap foundation. The strap acts as connecting beam and doesn't take any soil reaction. Strap is designed as rigid beam. A strap footing is more economical than a combined footing when the allowable soil pressure is relatively high and the distance between the column is large.
- **Mat foundation**: It is large footing which covers the entire area below a structure and supports the arrangements of all column and wall. It is required when the allowable soil pressure is low or where the columns and walls are so close that individual footing would overlap or nearly touch each other **(Area of footings is more than 50% of total covered area)**. They are useful in reducing **differential settlement** in non-homogenous soil or when there is **large variation of load on individual columns**. **Hence, it can be used when structure is heavy and water table is near the base of the structure**.







- **Pile foundation**: Piles are further classified as:
  - ✓ On the basis of material [Steel, Concrete, Timber, Composite Piles]
  - ✓ On the basis of load transfer [End bearing, friction and combined piles]
- **Pier foundation:** It is a deep and cylindrical foundation large diameter that transfer large superimposed load to a firm stratum below by means of bearing.
- **Cassion foundation**: It is type foundation in a shape of hollow prismatic box which is built above the ground level and then sunk to required depth as a single unit. Its types are open Cassion, pneumatic Cassion and floating Cassion

Factors affecting choice of foundation:

- Location and depth criteria
- Bearing capacity criteria
- Settlement criteria
- Function and load carried by structure
- Subsurface condition
- Service life
- Environmental considerations
- Need of client

61. A raft foundation is preferred for:
    a. Columns of industrial buildings
    b. Load bearing walls of multistoried buildings
    c. Columns of a building which are closely spaced
    d. None
62. When the area of all the footings covers more than 50% of the total area of the structure, then the foundation that is preferrable is:
    a. Isolated foundation
    b. Combined footings
    c. Raft foundation
    d. None of these
63. Shallow footing is one whose depth is:
    a. Always equal to width
    b. Less than the width
    c. More than the width
    d. None of the above

CCBAB

64. Which of the following, is a type of shallow footing?

    a. Spread footing b. Pile foundation c. Pier Foundation d. Well Foundation

65. When do strap footings are used in foundation?
    a) To transfer load of an isolated column
    b) Distance between the columns are long
    c) Two column loads are unequal
    d) All of the mentioned