PANA ACADEMY

NEPAL ENGINEERING COUNCIL
LICENSE EXAM PREPARATION COURSE

FOR

CIVIL ENGINEERS

PANA ACADEMY

4. Structural Mechanics

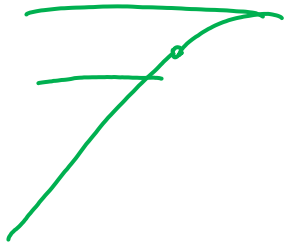
4.6 Indeterminate structures

Sub topics

PANA ACADEMY

- Flexibility Method
- Slope Deflection Method
- Moment Distribution Method
- Stiffness method
- Elementary Plastic analysis

Force Method



PANA ACADEMY

Forces are treated as unknown and corresponding forces are calculated.

Degree of static indeterminacy = Unknown forces

Using compatibility conditions and force displacement relation, unknowns are found.

Consistent deformation method, Flexibility method, Moment area method, Conjugate beam method, Three moment theorem

Displacement Method

PANA ACADEMY

Displacement are treated as unknown and corresponding displacement are computed first.

Degree of kinematic indeterminacy = Unknown displacements

Using equilibrium conditions and force displacement relation, unknowns are found.

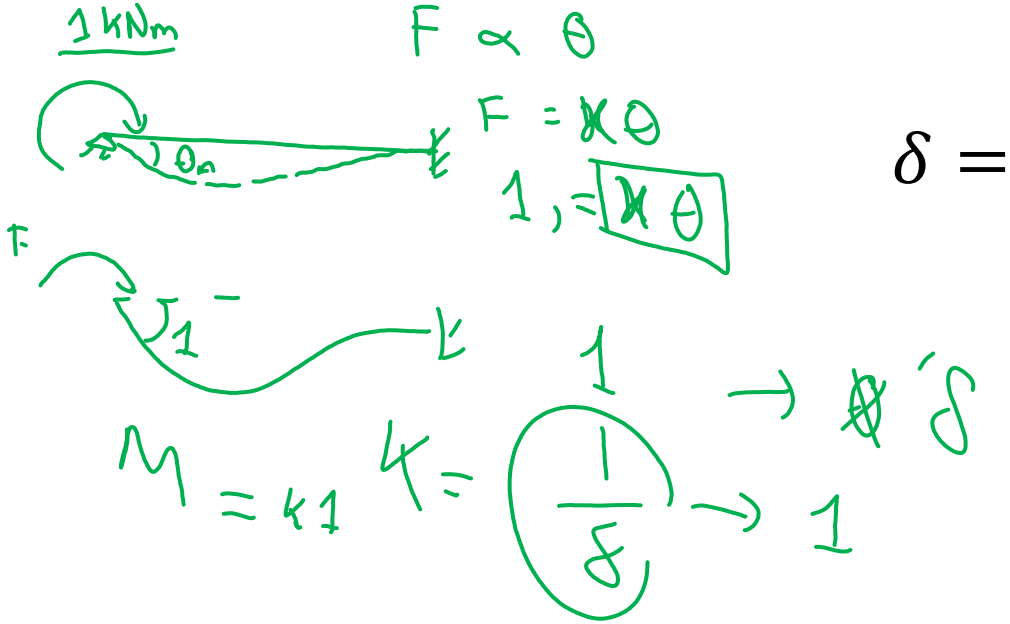
Slope deflection method, moment distribution method, stiffness method, Minimum potential energy method, Kani's method

Flexibility and stiffness

PANA ACADEMY

Flexibility is displacement caused by unit force. Denoted by δ

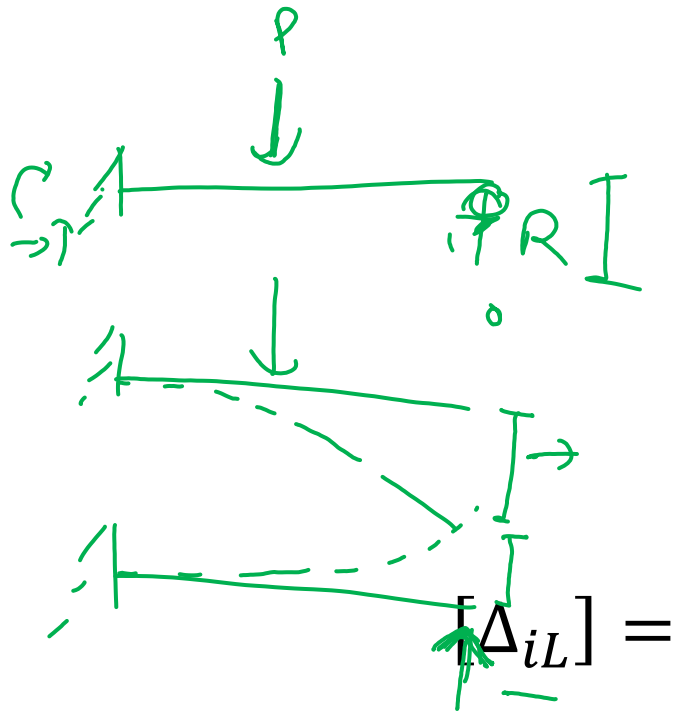
Stiffness is force required for unit displacement. Denoted by k.



$$\delta = \frac{1}{k}$$

$[\delta] = [kJ]^{-1}$

Flexibility Method



$$\{\ [\Delta] = [\Delta_L] + [\delta][P]\ \}$$

$$[P] = [\delta]^{-1}\{[\Delta] - [\Delta_L]\}$$

$$[\Delta_{iL}] = \int \frac{M \times m_i}{EI}\, ds \qquad [\delta_{ij}] = \int \frac{m_i \times m_j}{EI}\, ds$$

$$[\Delta_{iL}] = \int \frac{P_o \times P_i}{AE}\, ds \qquad [\delta_{ij}] = \int \frac{P_i \times P_j}{AE}\, ds$$

Flexibility Method

PANA ACADEMY

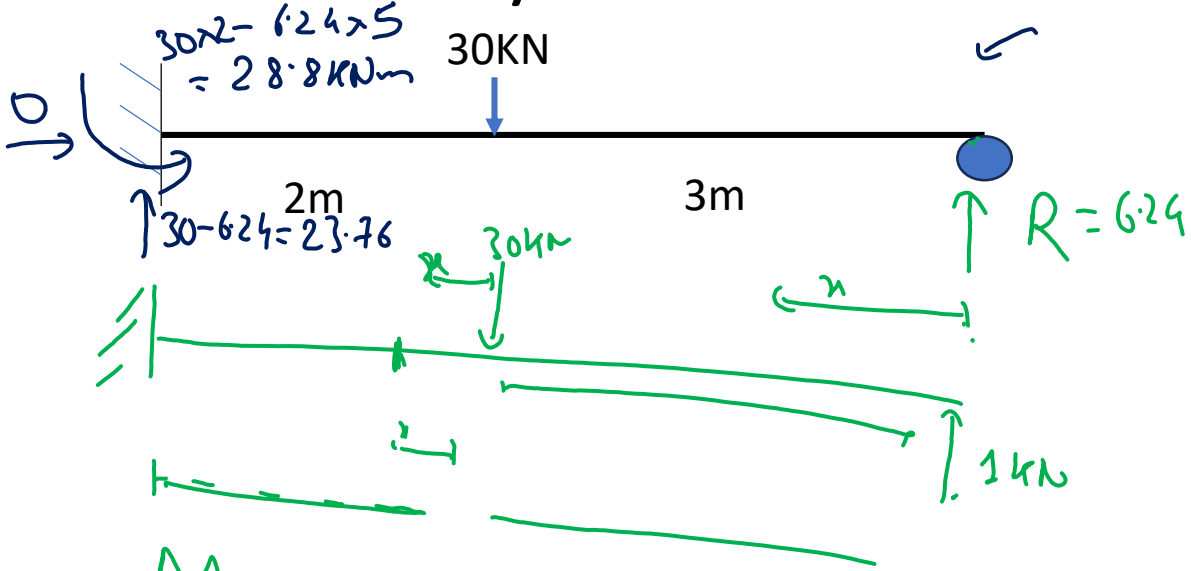
$$[\Delta] = [\Delta_L] + [\delta][P]$$

$$[P] = [\delta]^{-1}\{[\Delta] - [\Delta_L]\}$$

$$[\Delta_{iL}] = \int \frac{M \times m_i}{EI} ds \qquad [\delta_{ij}] = \int \frac{m_i \times m_j}{EI} ds$$

$$[\Delta_{iL}] = \int \frac{P_o \times P_i}{AE} ds \qquad [\delta_{ij}] = \int \frac{P_i \times P_j}{AE} ds$$

Flexibility Method



$M = 0$

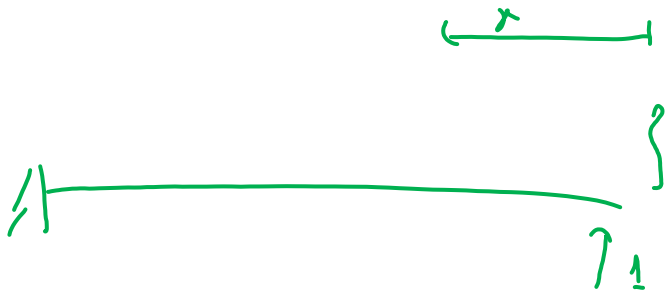
$M = -30x$

$m = 1x$

$m = \frac{1}{2}(3+x)$

$\Delta_L = \int_0^3 \frac{0 \times 1x}{EI} dx + \int_0^3 \frac{-30x(3+x)}{EI} dx$

$= -\frac{260}{EI}$



$M = 1x$ $\quad m = 1x$

$\delta = \int_0^5 \frac{1x \times 1x}{EI} dx$

$= \frac{125}{3EI}$

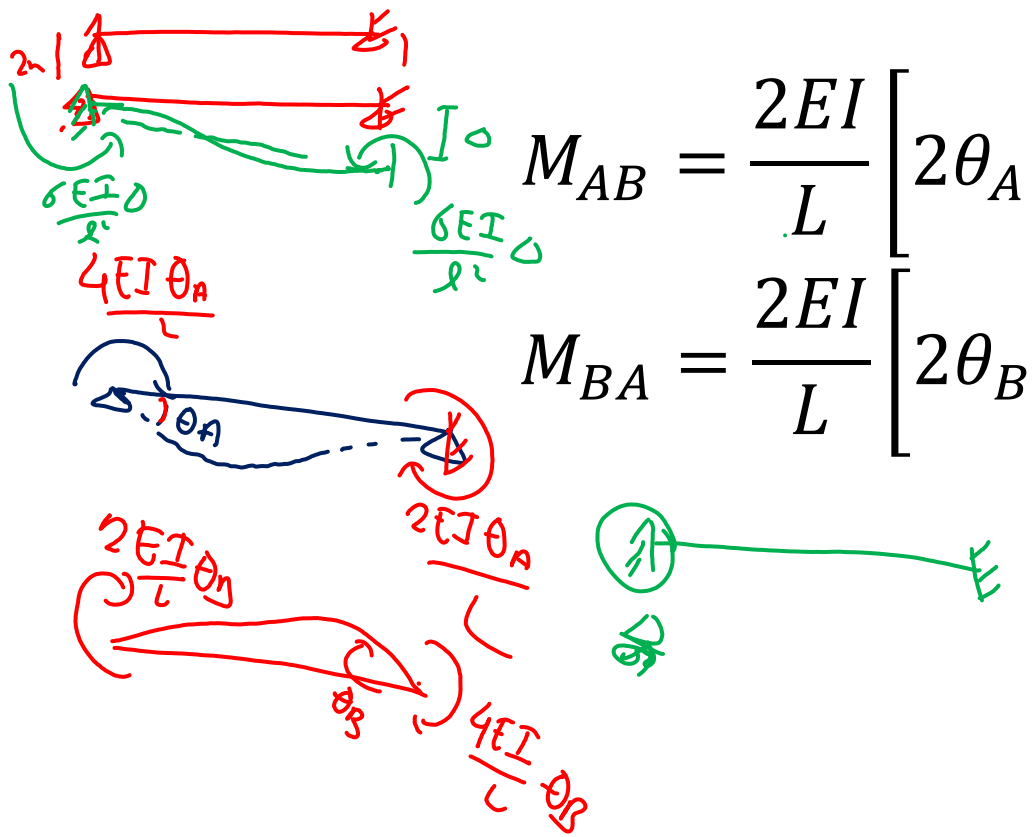
$0 = \Delta_L + \delta R$

$0 = \frac{-260}{EI} + \frac{125}{3EI} R$

$R = \frac{3 \times 260}{125} = 6.24kN$

Slope Deflection Method

FEM – Fixed end moment



$$M_{AB} = \frac{2EI}{L}\left[2\theta_A + \theta_B - \frac{3\Delta}{L}\right] + FEM_{AB}$$

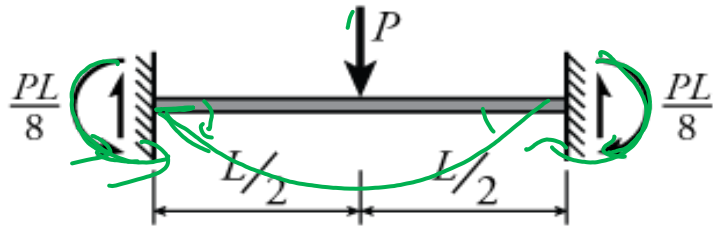
$$M_{BA} = \frac{2EI}{L}\left[2\theta_B + \theta_A - \frac{3\Delta}{L}\right] + FEM_{AB}$$

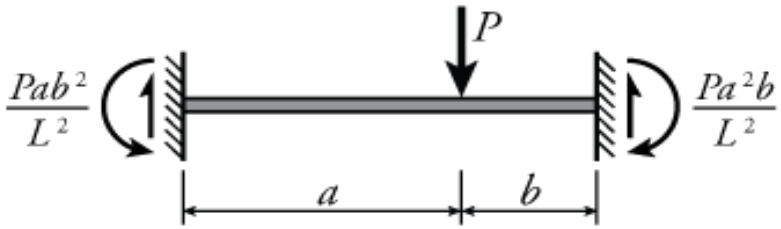
Slope Deflection Method

θ → clockwise

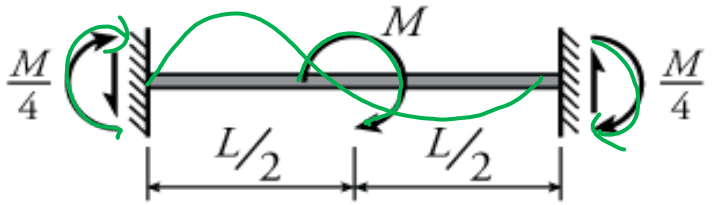
PANA ACADEMY

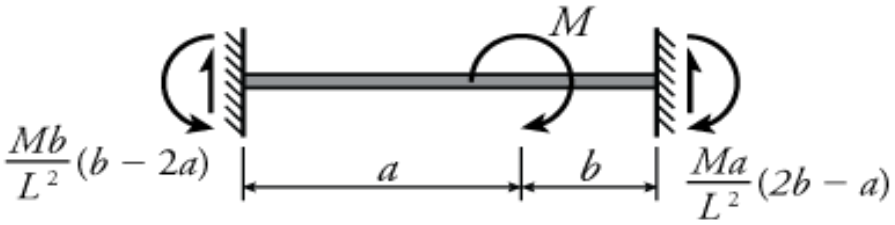
Point Loads

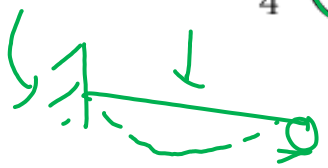




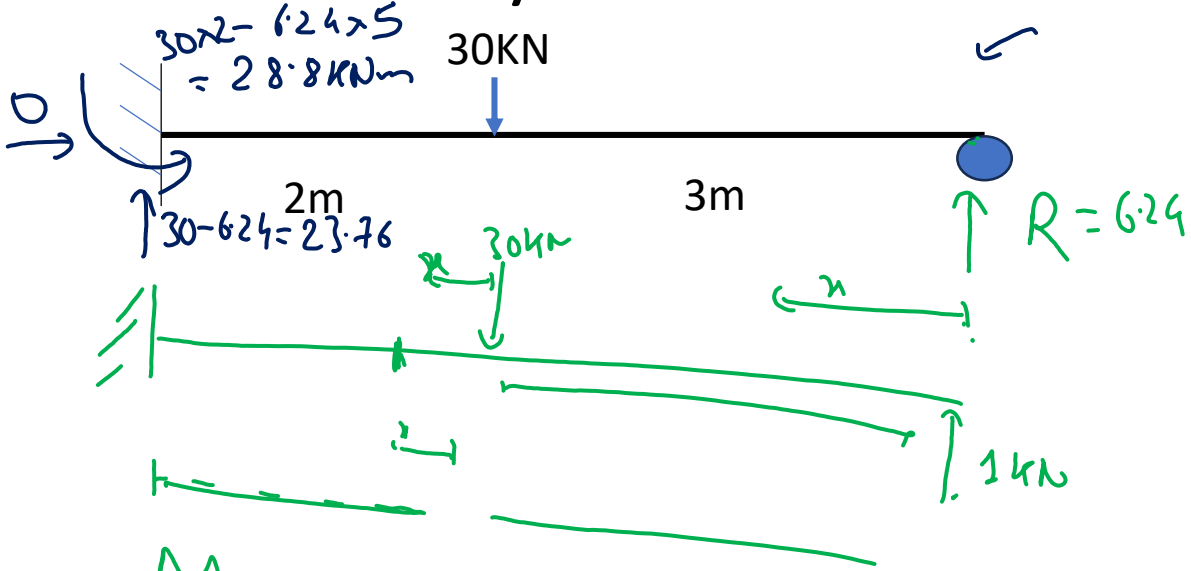
Point Moments







Flexibility Method



$M = 0$

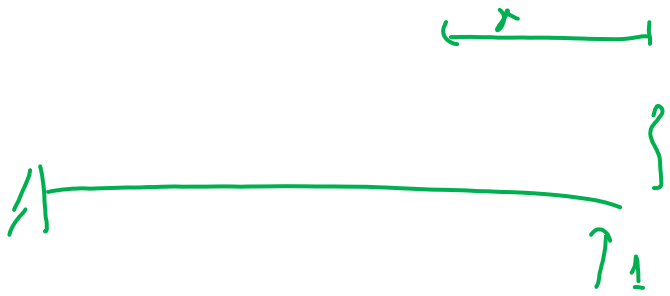
$M = -30x$

$m = 1x$

$m = \frac{1}{2}(3+x)$

$\Delta_L = \int_0^3 \frac{0 \times 1x}{EI} dx + \int_0^3 \frac{-30x(3+x)}{EI} dx$

$= -\frac{260}{EI}$



$M = 1x$ $\quad m = 1x$

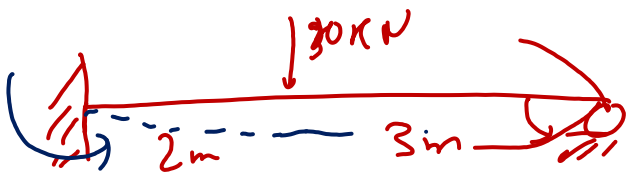
$\delta = \int_0^5 \frac{1x \times 1x}{EI} dx$

$= \frac{125}{3EI}$

$0 = \Delta_L + \delta R$

$0 = \frac{-260}{EI} + \frac{125}{3EI} R$

$R = \frac{3 \times 260}{125} = 6.24kN$



PANA ACADEMY

$$M_{AB} = FEM_{AB} + \frac{2EI}{L}\left(2\theta_A + \theta_B - \frac{3\psi}{L}\right)^{0}$$

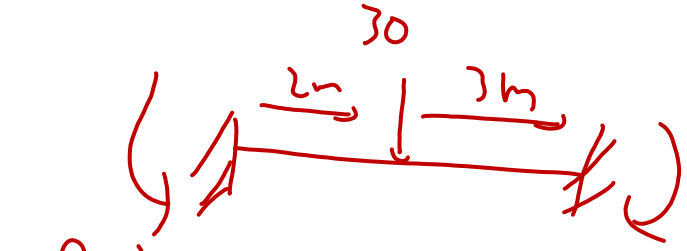
$$M_{AB} = -21.6 + \frac{2EI}{5}\left(\theta_B\right)$$

$$= -21.6 + \frac{2EI}{5} \times \left(\frac{-18}{EI}\right) = -28.8\ kNm$$

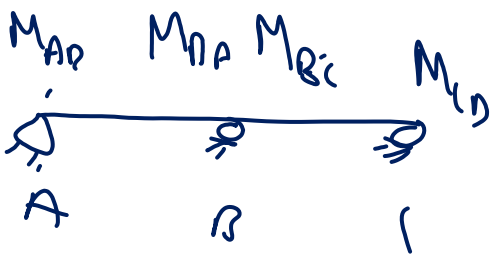
$$M_{BA} = FEM_{BA} + \frac{2EI}{L}\left(\theta_A + 2\theta_B - \frac{3\psi}{L}\right)^{0}$$

$$0 = 14.4 + \frac{2EI}{5}\,2\theta_B$$

$$\theta_B = \frac{-14.4 \times 5}{4EI} = \frac{-18}{EI}$$



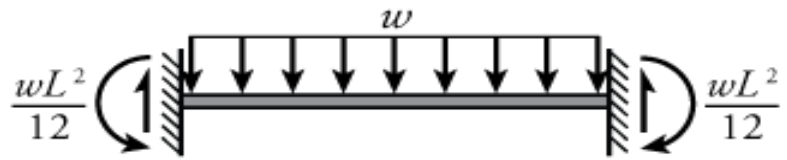
$$\frac{Pab^2}{L^2} = 21.6\ kNm \qquad \frac{Pa^2b}{L^2} = 14.4\ kNm$$

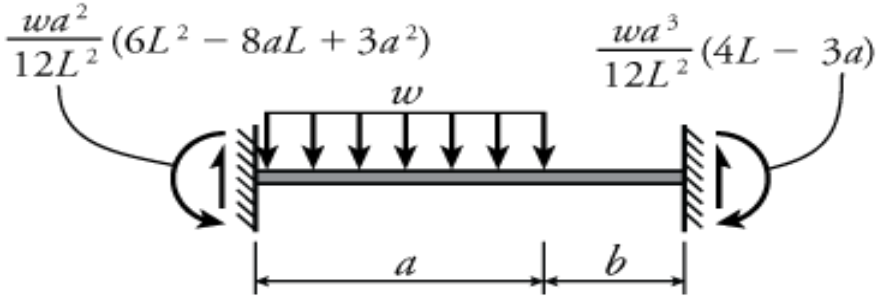


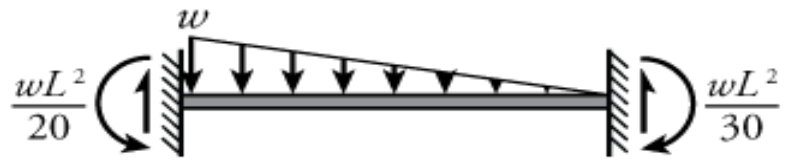
Slope Deflection Method

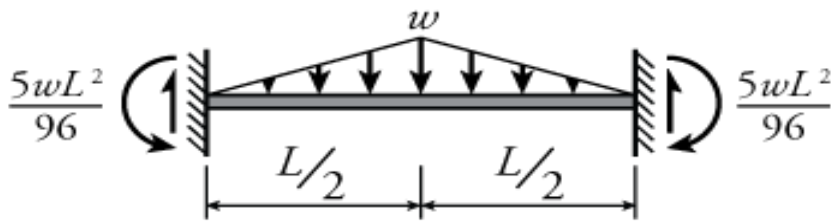
PANA ACADEMY

Distributed Loads





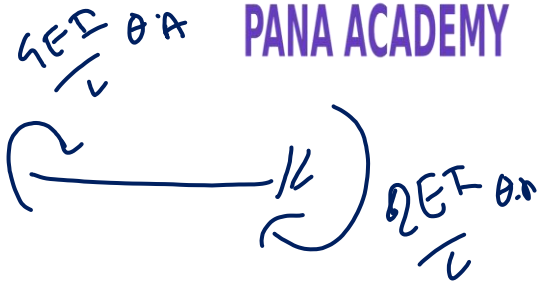




Support Movements

Moment Distribution Method

Hardy Cross

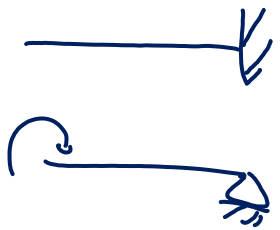


Carry over factor:

For a member with opposite end fixed or continuous, CO=0.5

For a member with opposite end hinged, CO=0

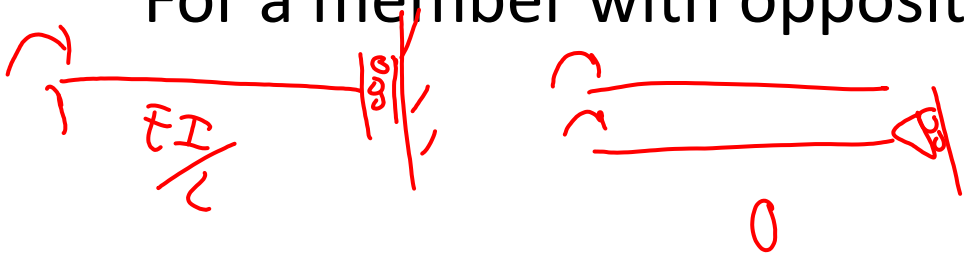
$\frac{EI/L}{4E/L}$ = Relative stiffness = 1/4



Rotational stiffness:

For a member with opposite end fixed or continuous, $K = \frac{4EI}{L}$

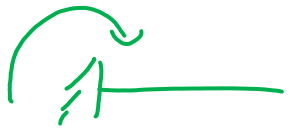
For a member with opposite end hinged, $K = \frac{3EI}{L}$



Moment Distribution Method

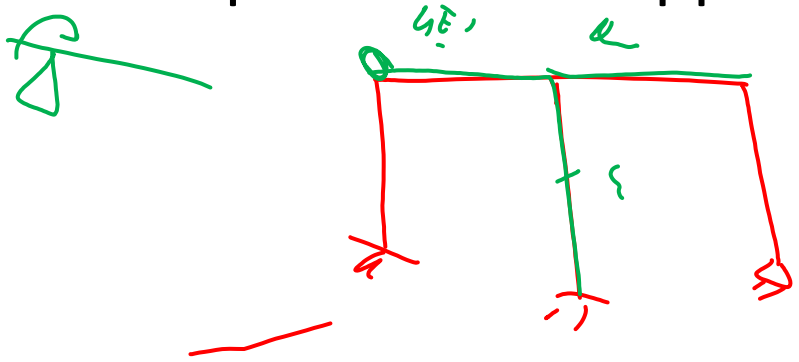
Distribution factor:

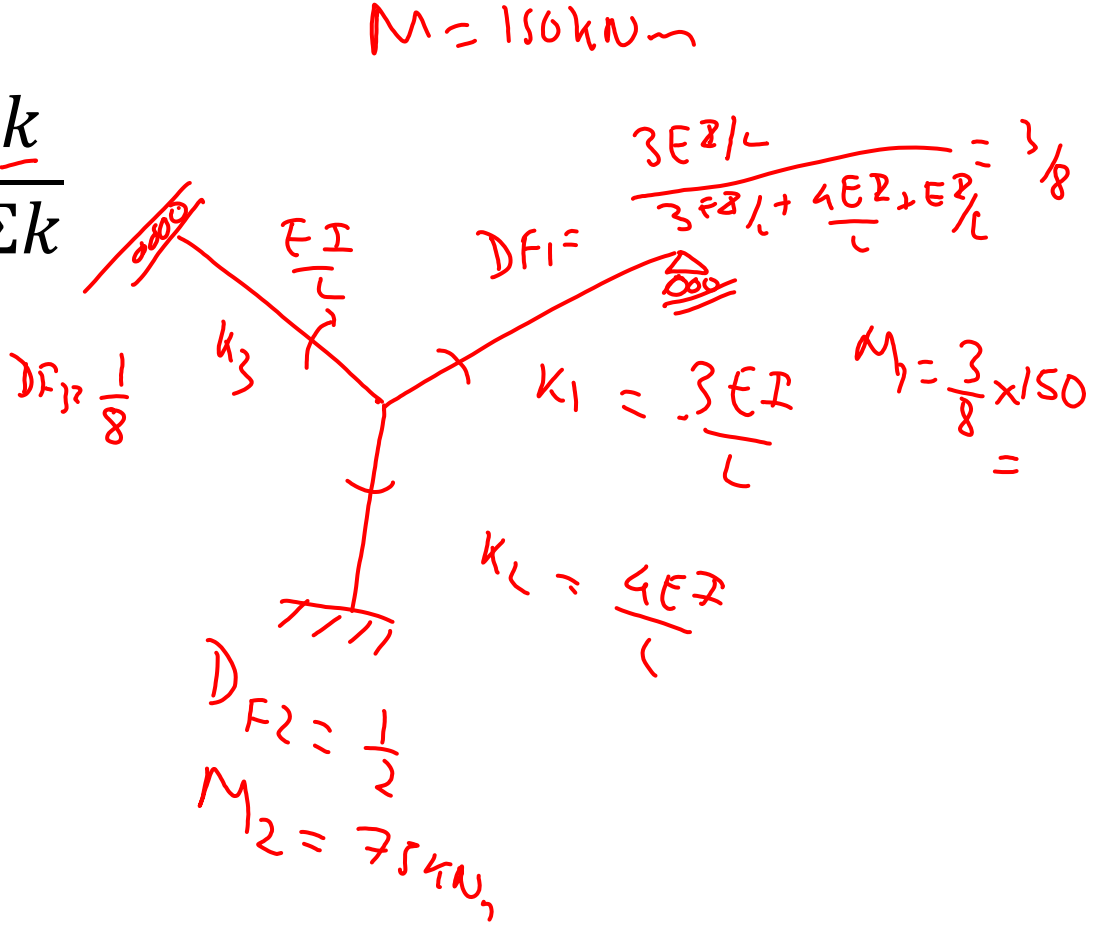
$$DF = \frac{k}{\Sigma k}$$



For fixed end support DF= 0

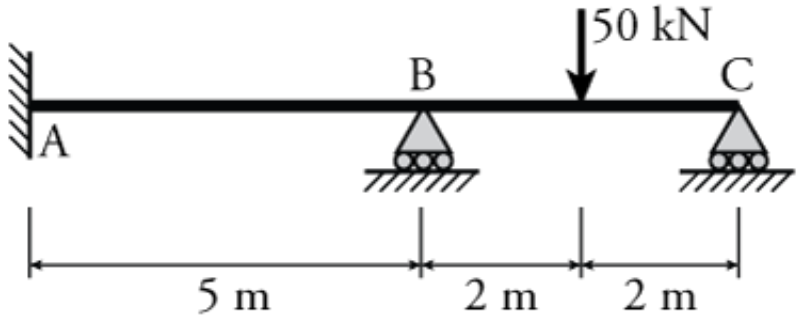
For pinned end support DF= 1





Moment Distribution Method

PANA ACADEMY

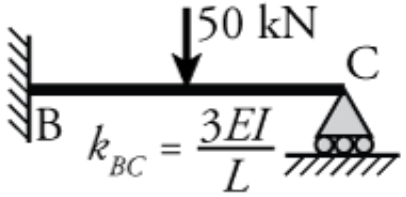


a) Fix all Interior Nodes

Fix-Fix

A $k_{BA} = \frac{4EI}{L}$ B

Fix-Pinned



$k_{BC} = \frac{3EI}{L}$

b) Fixed End Moments

None

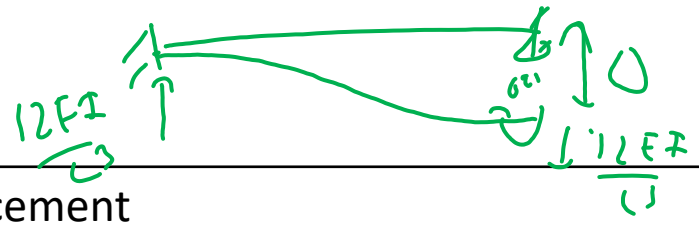
$FEM_{BC} = +25$ kNm $FEM_{CB} = -25$ kNm

Moment Distribution Method

PANA ACADEMY

Joint	Fixed A		B			Pinned C
Member	AB		BA	BC		CB
DF	0		0.516	0.484		1.0
FEM				25.0		-25.0
Balance B			-12.9	-12.1		
Carry-Over	-6.45	$\leftarrow$ CO = $\frac{1}{2}$			CO = 0 $\rightarrow$	0
Balance C						25.0
Carry-Over				12.5	$\leftarrow$ CO = $\frac{1}{2}$	0
Balance B			-6.45	-6.05		
Carry-Over	-3.23	$\leftarrow$ CO = $\frac{1}{2}$			CO = 0 $\rightarrow$	0
SUM $\sum$	-9.68		-19.35	19.35		0

Stiffness



Displacement	Stiffness
Axial	$\frac{AE}{L}$
Bending (far end fixed)	$\frac{4EI}{L}$
Bending (far end hinged)	$\frac{3EI}{L}$
Transverse (far end fixed)	$\frac{12EI}{L^3}$
Transverse (far end hinged)	$\frac{3EI}{L^3}$
Torsional	$\frac{GJ}{L}$

PANA ACADEMY

Stiffness Method

PANA ACADEMY

$$[\mathrm{P}] = [P'] + [K][\Delta]$$

Stiff →

Flexibility meth →

$$[\Delta] = [K]^{-1}\{[\mathrm{P}] - [P']\}$$

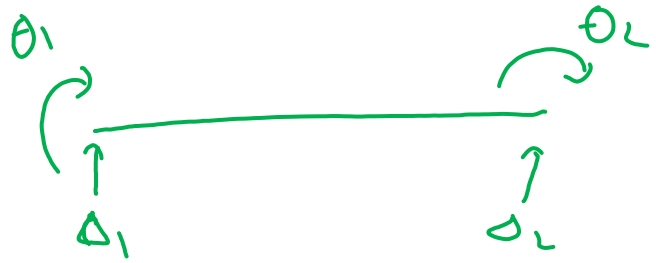
$F_{xB} = 0$

$M_B = 0$

Stiffness method

PANA ACADEMY

For axial ,K=AE/L



$$\begin{Bmatrix} V_1 \\ M_1 \\ V_2 \\ M_2 \end{Bmatrix} = (EI) \begin{bmatrix} 12/L^3 & 6/L^2 & -12/L^3 & 6/L^2 \\ 6/L^2 & 4/L & -6/L^2 & 2/L \\ -12/L^3 & -6/L^2 & 12/L^3 & -6/L^2 \\ 6/L^2 & 2/L & -6/L^2 & 4/L \end{bmatrix} \begin{Bmatrix} \Delta_1 \\ \theta_1 \\ \Delta_2 \\ \theta_2 \end{Bmatrix}$$

Plastic analysis

120kN

100kN → 150

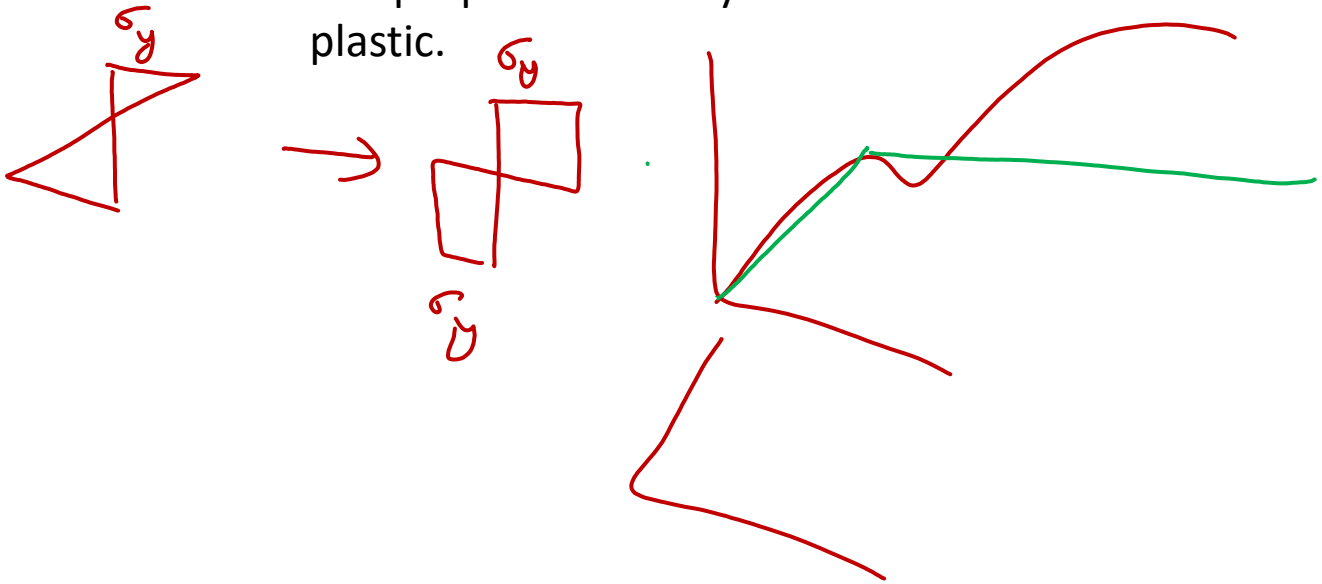
PANA ACADEMY

Plastic analysis is defined as the analysis in which the criterion for the design of structures is the ultimate load. The material goes beyond the elastic zone into plastic region.

(σ_y) Yield stress is reached at each fiber at section irrespective of distance from neutral axis.

Analysis and design of indeterminate structure. Economic.

For purpose of analysis the strain stress curve is considered to be perfectly elasto-plastic.



Theorem of Plastic analysis:

PANA ACADEMY

- Lower bound theorem

  The structure is safe and statically admissible for load W such that it is less than or equal to Collapse load (W_c).

$$W \leq W_c$$

- Upper bound theorem

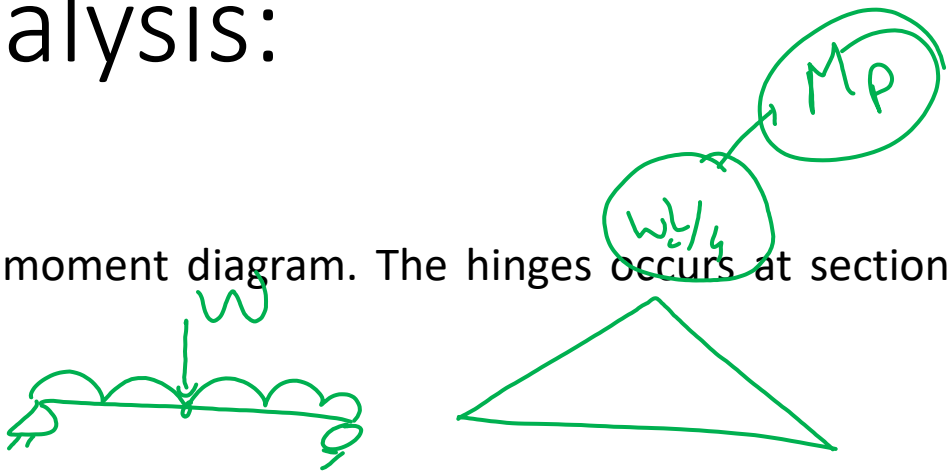
  The mechanism of collapse is the one which causes the structure to fail plastically and has the lowest collapse load.

$$W \geq W_c$$

- Uniqueness method

$$W = W_c$$

Methods of Plastic analysis:



- Static method

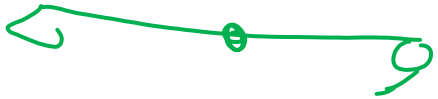
  For structure with known bending moment diagram. The hinges occurs at section of highest ordinate of BMD.

  Based on lower bound theorem.

- Mechanism method

  The plastic hinge is considered at different section for failure mechanism. Internal work done at internal hinge is equal to external work by load.

  Based on upper bound theorem.



Internal work done = $\Sigma M_p \theta$

External work done = $W\delta \ or \ \frac{1}{2} wL\delta$

Collapse mechanism:

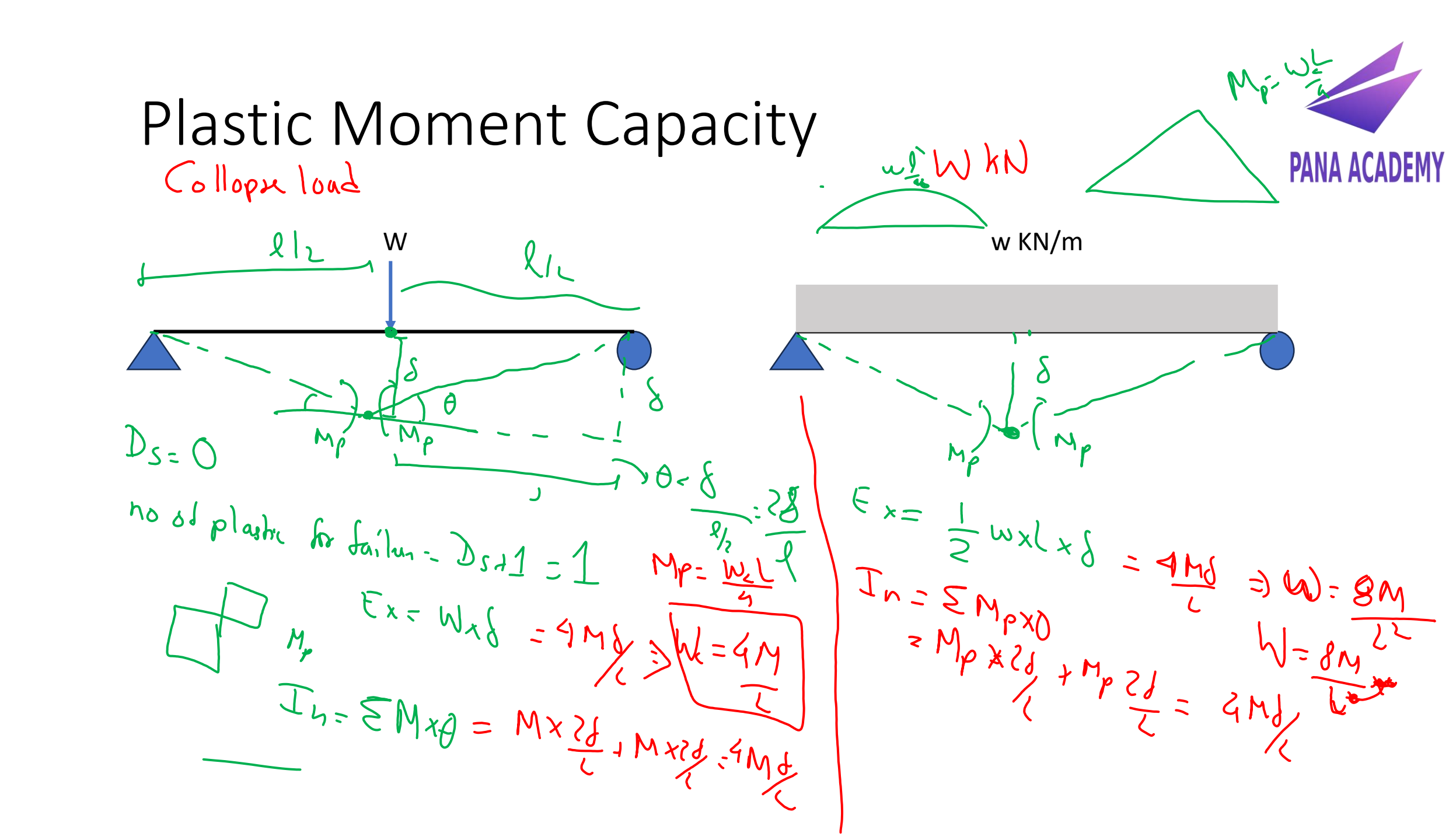
PANA ACADEMY

- Beam mechanism
  Plastic hinge is formed at support and/or at some point on span.

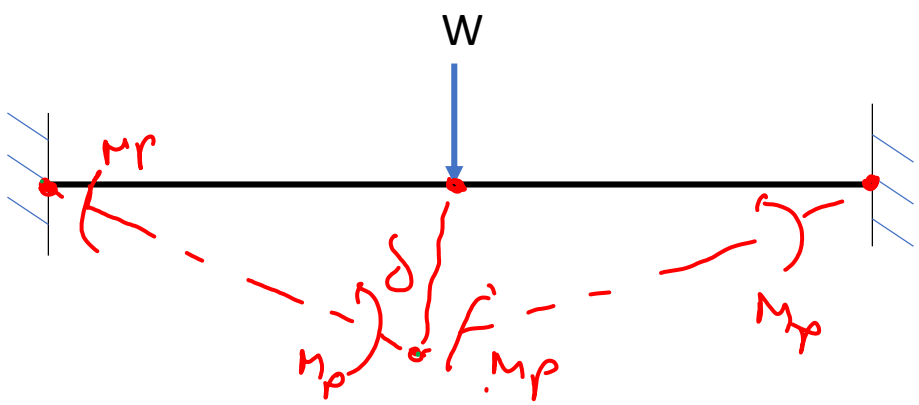
- Sway mechanism
  The sway occurs by formation of hinge at joints of column in frame.

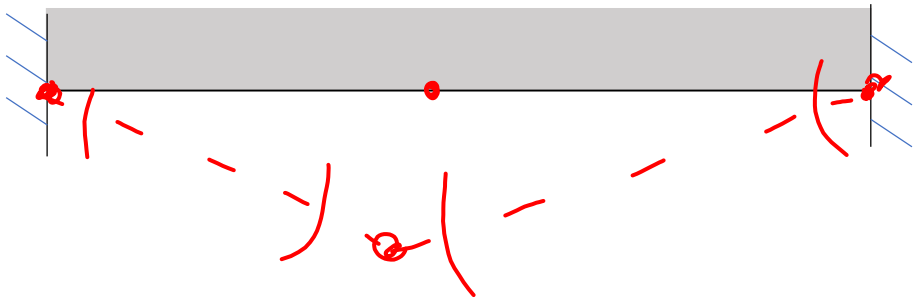
- Combined mechanism
  Mixed form of beam and sway mechanism

Plastic Moment Capacity



Plastic Moment Capacity





$D_s = 2$

$PH = D_s M = 2+1 = 3$

$Ex = W\delta$

$I_n = M_p * \frac{2\delta}{L} + M_p\frac{2\delta}{L} + M_p\frac{2\delta}{L} + M_p\frac{2\delta}{L}$

$= \frac{8M_p\delta}{L}$

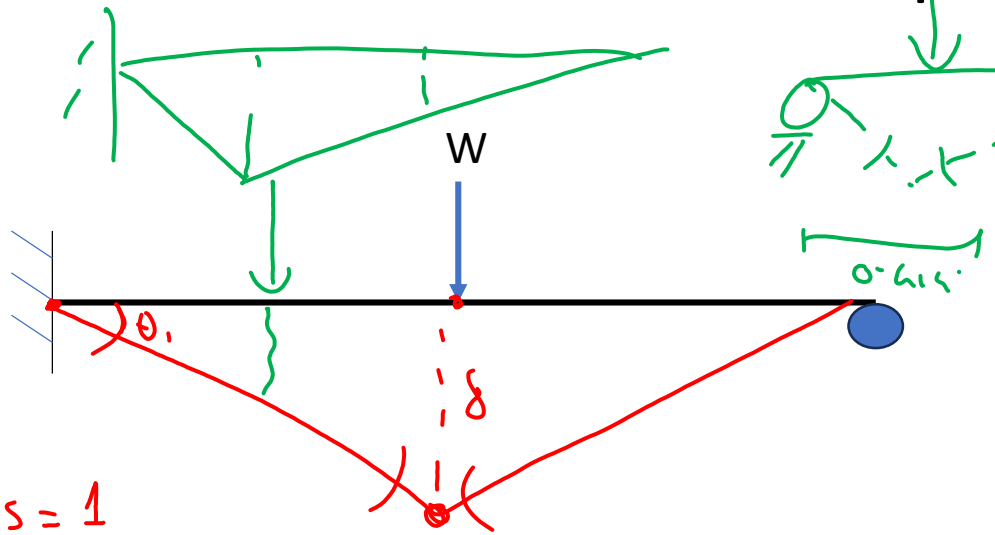
$W_L = \frac{8M_p}{L}$

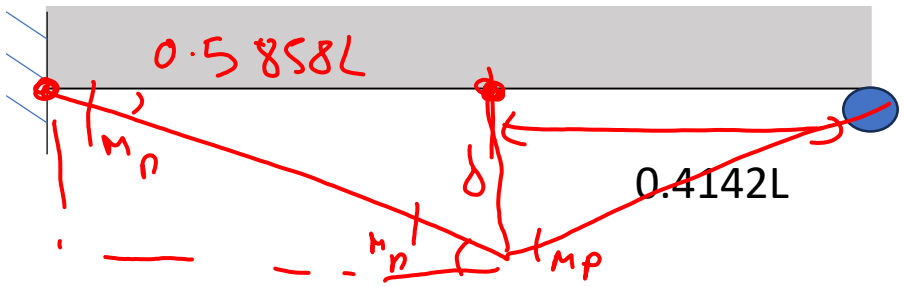
$M_p = \frac{W_c L}{8}$

$w = \frac{16M_p}{L^2}$

$w = \frac{16M_p}{L}$

Plastic Moment Capacity





$D_S = 1$

$P_M = 1+1 = 2$

$E_x = W \times \delta$

$I_n = M_p \times \frac{2\delta}{L} + M_p \times \frac{2\delta}{L} + M_p \times \frac{2\delta}{L}$

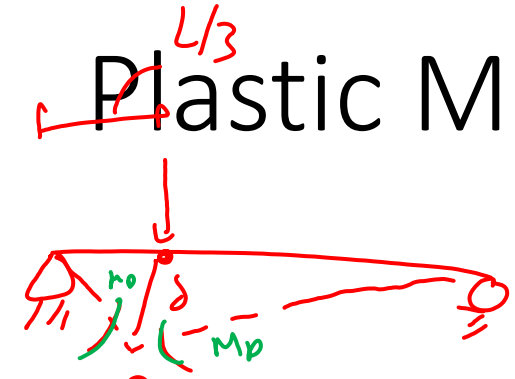
$$M_p = 0.0858\, W_c L^2$$

$$W_c = 11.655\, M_p L^2$$

$E_x = \frac{1}{2} W \times \delta$

$I_n = M_p \frac{\delta}{0.5858L} + M_p \frac{\delta}{0.4142L} + M_p \frac{\delta}{0.5858L}$

Plastic Moment Capacity



$M_p = ?$

$E_x = W \times \delta$

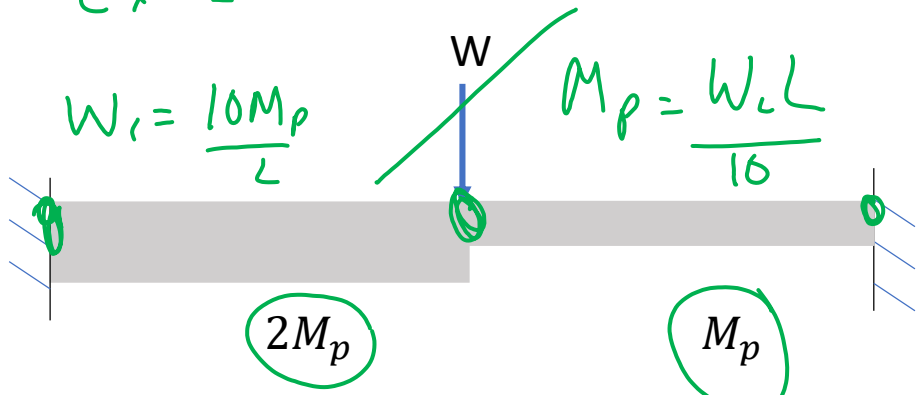
$I_h = -\frac{M_p \times 3\delta}{L} + \frac{M_p \times 3\delta}{2L}$

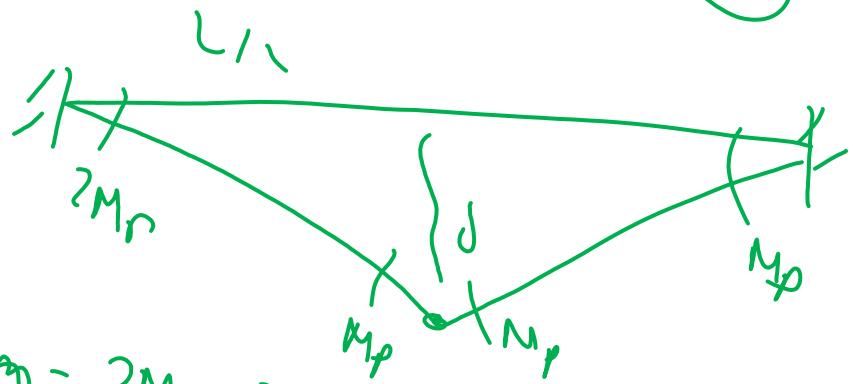
$= \frac{M_p\, 9\delta}{2L}$

$W = \frac{9M_p}{2L} = 4.5\frac{M_p}{L}$

$M_p = \frac{WL}{4.5}$

$E_x = W\delta$

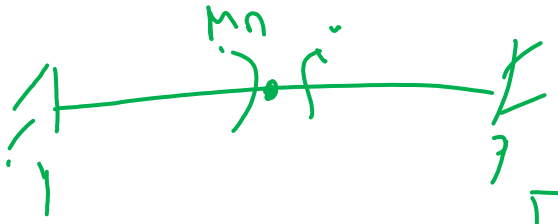


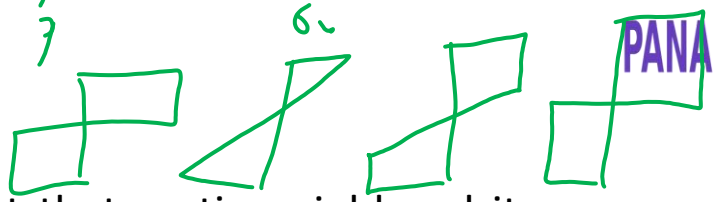


$I_m = 2M_p \times 2\delta/L + 3M_p \times 2\delta/L$

$= 10M_p\delta/L$

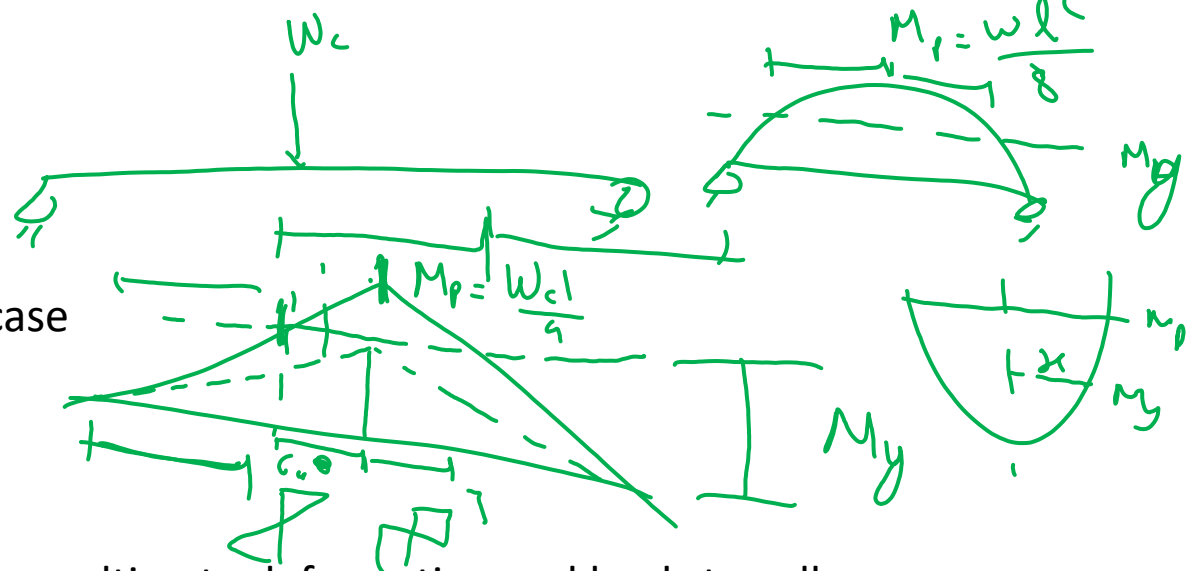
Terminology:





- Plastic hinge:
  It is hinge formed at a section when all the fibers at that section yield and it can no further take more moment

- Length of plastic hinge:
  Effected length due to plastic hinge.
  Point load case
  For rectangular beam, $\frac{l}{3}$ point load case
  For I beam beam, $\frac{l}{6}$



- Collapse load:
  Load that makes some section undergo ultimate deformation and leads to collapse.

Terminology:

PANA ACADEMY

- Factor of safety:
  Ratio of yield moment to working load is called FOS.

$$FOS = \frac{M_y}{M_a}$$

- Load factor :
  Ratio of collapse load to working load is called load factor.

$$\lambda = \frac{Collapse\ load}{Allowable\ load} = \frac{M_p}{M_a}$$

- Plastic moment capacity:
  Maximum moment structure can sustain.

$$M_p = \sigma_y Z_p$$

  Z_p is plastic section modulus.

Terminology:

PANA ACADEMY

Shape factor:

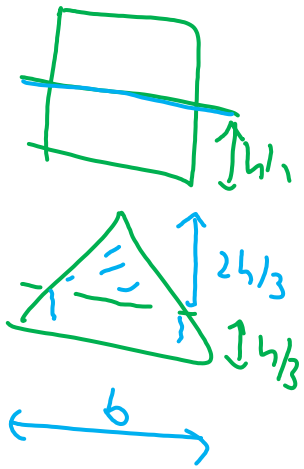
$$\text{Shape factor} = \frac{M_p}{M_y} = \frac{Z_p}{Z}$$

$$\frac{b'}{b} = \frac{2h/3}{h}$$

$$b' = 2b/3$$

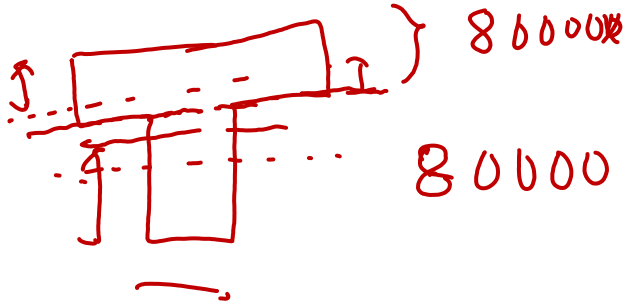
$$A = 1/2 \times \frac{2b}{3} \times \frac{2h}{3} = \frac{4}{9}\frac{bh}{2}$$

$$= \frac{bd^2/4}{bd^2/6} = 1.5 \text{ (for rectangle)}$$



Cross section	Shape Factor
Rectangular	1.5
Triangular	2.34
Solid Circular	1.7
Hollow Circular	1.27
I section	1.12-1.14
T section	1.8
Diamond	2

Terminology:



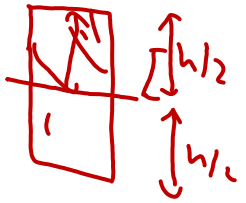
PANA ACADEMY

Shape factor:

$$\text{Shape factor} = \frac{M_p}{M_y} = \frac{Z_p}{Z}$$

$Z_p = 80000 \times 100 + 80000 \times 200 =$

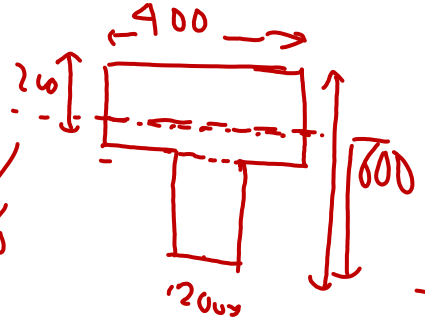
$$Z_p = \frac{A}{2}[\overline{y_1} + \overline{y_2}] = \Sigma A_i \overline{y_i} = b \times \frac{h}{2} \times \frac{h}{4} + b \times \frac{h}{2} \times \frac{h}{4}$$



$Z_e = \dfrac{bh^3/12}{h/2} = bh^2/6$

SF = 1.5

$Z_p = bh^2/4$

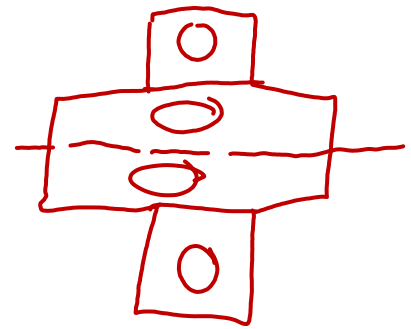


$= \dfrac{bh^2}{4}$

$\bar{y} = \dfrac{\bar{y}_1 A_1 + \bar{y}_2 A_2}{A_1 + A_2}$

$I = I_1 + I_2$

$Z_e =$



PANA ACADEMY

Thank YOU !!!