PANA ACADEMY

NEPAL ENGINEERING COUNCIL
LICENSE EXAM PREPARATION COURSE

FOR

CIVIL ENGINEERS

PANA ACADEMY

4. Structural Mechanics

4.4 Determinate structure-1

Sub topics

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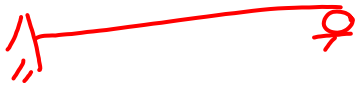
- Degree of determinacy
- Energy methods
- Virtual work method
- Deflection of beams and portal frame

Degree of determinacy



4 unknown

3 eqn of equilibrium

1 extra equ

Static indeterminacy

Number of extra equation required to find all internal and external forces.

Kinematic indeterminacy

Number of extra equation required to find all rotation and deflection.

Degree of determinacy

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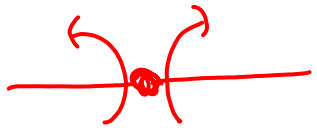
Determinate structure

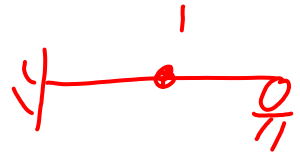
Number of unknown = Number of equation available

equilibrium eqn + extra eqn

Indeterminate structure

Number of unknown > Number of equation available





3+1

4

Degree of static indeterminacy

$$D_s = Unknown - equation\ of\ equilibrium$$

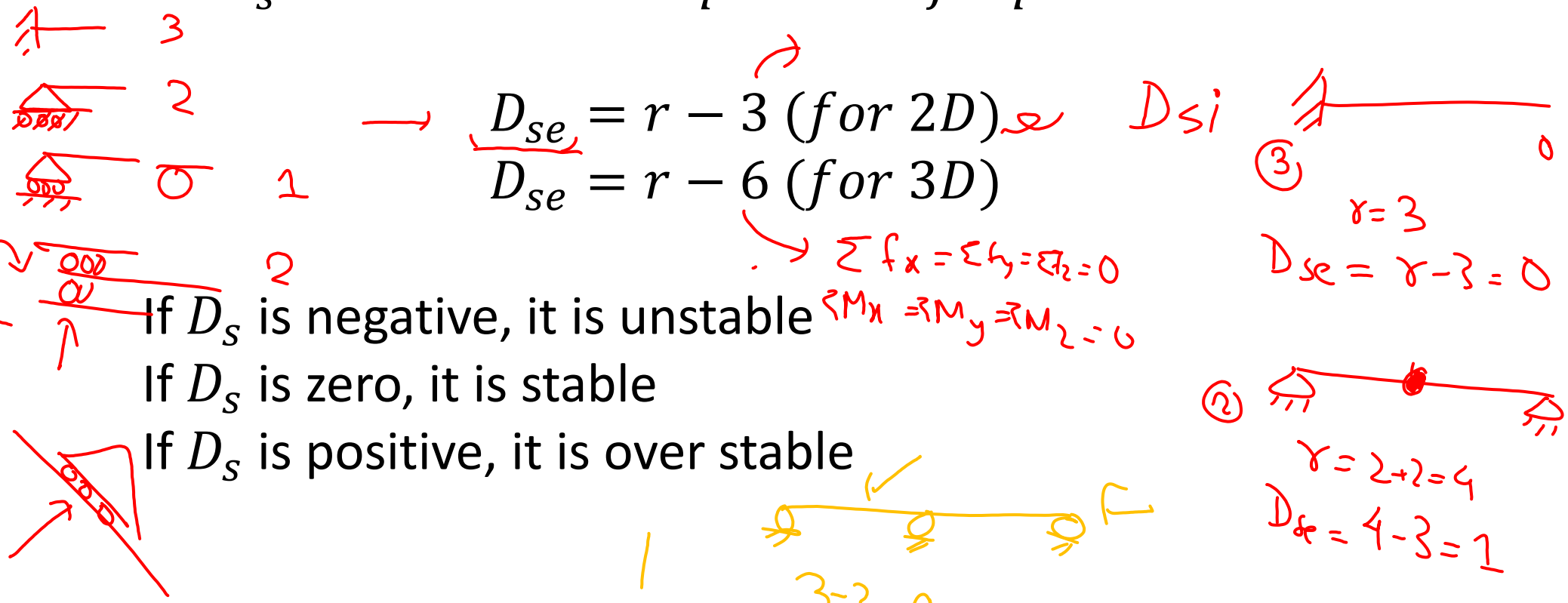
$$D_{se} = r - 3\ (for\ 2D)$$

$$D_{se} = r - 6\ (for\ 3D)$$

If D_s is negative, it is unstable

If D_s is zero, it is stable

If D_s is positive, it is over stable



Degree of static indeterminacy

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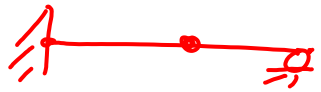
$$D_s = D_{se} + D_{si}$$

$$D_{se} = External\ static\ indeterminacy$$
$$D_{si} = Internal\ static\ indeterminacy$$

Degree of static indeterminacy

$$D_s = (3m + r) - (3j + a)$$



r=4

Dse = r - j = 1

Di=3*loop

'a' is extra equations from internal hinge

$$a = m' - 1$$

m' is number of member joining at that internal hinge

Internal guided roller provides 1 extra equation

Internal unguided roller provides 2 extra equation

Link provides 2 extra equation

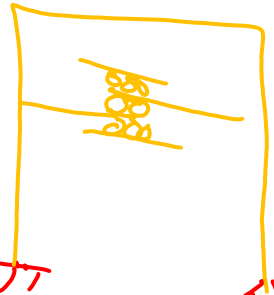
m = 1

j = 2

$D_i = (3 \times 1 + 4) - (3 \times 2 + 0)$

$= 1$

$D_{si} = D_s - D_{se} = 1 - 1 = 0$



r = 3 + 2 = 5

m = 6, j = 6

$D_{se} = 5 - 3 = 2$

$D_s = 3$

$D_{si} = 1$

Degree of static indeterminacy

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$$D_s = (3m + r) - (3j + a)$$

Di=3*loop

'a' is extra equations from internal hinge

$$a = m' - 1$$

m' is number of member joining at that internal hinge

Internal guided roller -1 extra equation

Internal unguided roller - 2 extra equation

Link - 2 extra equation

Degree of static indeterminacy

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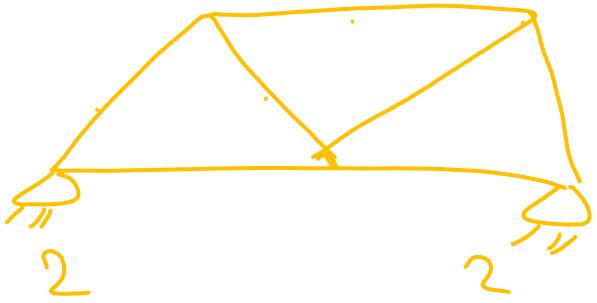
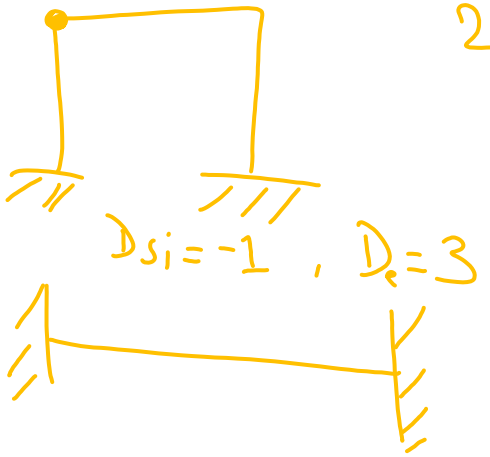
For Truss,





$$D_s = (m + r) - 2j \quad = (7+4) - 2\times 5 = 1$$

$$\longrightarrow D_{se} = r - 3\ (for\ 2D) \quad = 4-3 = 1$$

$$D_{si} = m - 2j + 3 \quad = 1-1 = 0$$

$$D_{ki} = 2j - r = 2\times 5 - 4 = 6$$

Degree of kinematic indeterminacy

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For Truss,

$$D_k = 2j - r$$

Degree of kinematic indeterminacy

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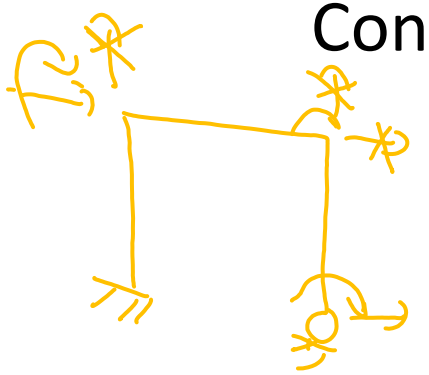
Considering Axial Deformation in Frame

$$D_k = 3j - r + a$$

Neglecting Axial Deformation in Frame / inextensible

$$D_k = 3j - r + a - m$$

Consider horizontal reaction



Degree of kinematic indeterminacy

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Considering Axial Deformation in Beam

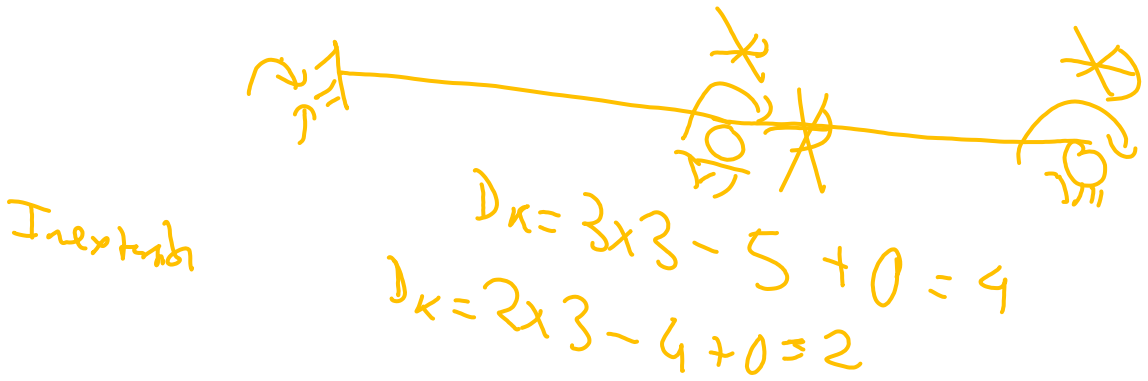
$$D_k = 3j - r + a$$

Neglecting Axial Deformation in Beam

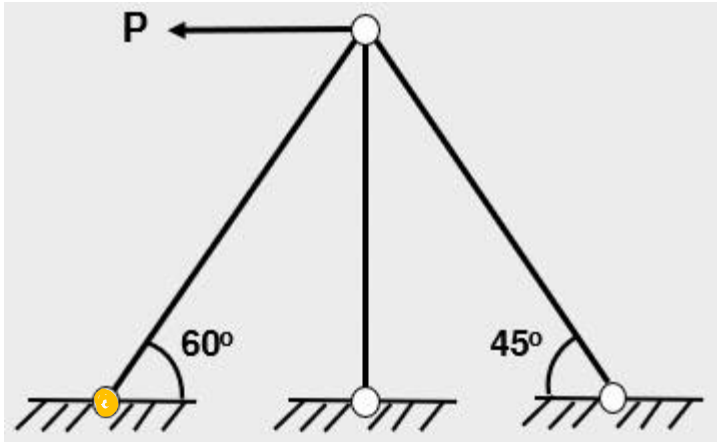
Inextensible

$$D_k = 2j - r + a$$

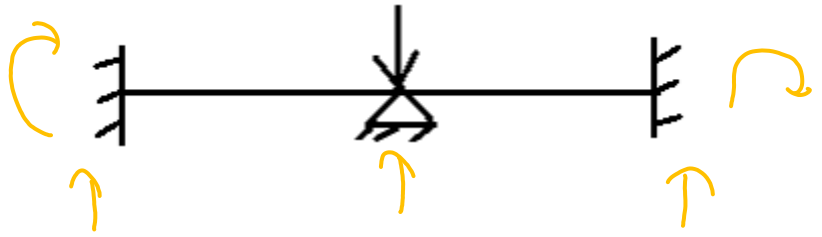
Do not consider horizontal reaction



Degree of indeterminacy



$D_s = r - 3 = 6 - 3 = 3$

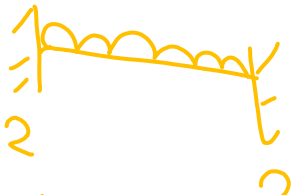


$5 - 2 = 3$

$(3+1) - 1 = 3$

$D_s = (m+r) - 2j = 3 + 6 - 2\times4 = 1$

$8 - 3 = 5$



$D_j = 1 - 3 = -2$

$(2+1) - 2 = 2$

Energy methods

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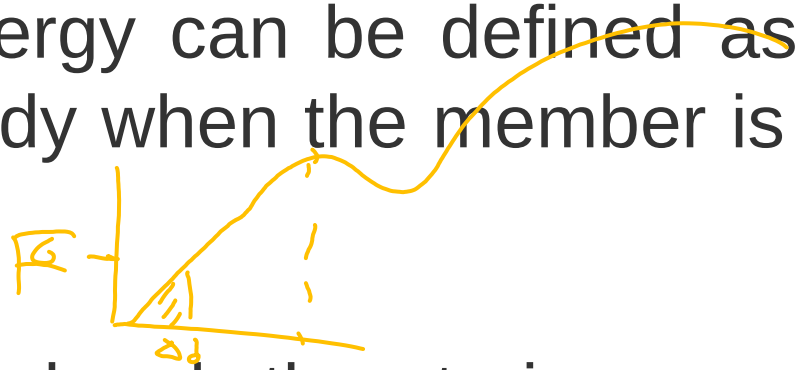
Strain energy can be defined as the energy stored within an elastic body when the member is deformed under the action of the load.



Ability to absorb the strain energy within elastic limit is called resilience.

Maximum energy absorbed is Proof Resilience.

$$U_{max} = \frac{\sigma^2 Vol}{2E}$$

Maximum energy per unit volume is modulus of resilience.

Energy methods

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Stress due to load:

$$\sigma = \frac{W}{A}$$

$\sigma = P/A$

impact or suddenly loads

$$\sigma = 2\frac{W}{A}$$

$\sigma = 2 \times \sigma_{nor}$

When object of weight 'W' falls from height 'h'

$$\sigma = \frac{W}{A}\left[1 + \sqrt{1 + \frac{2hAE}{Wl}}\right]$$

Energy methods

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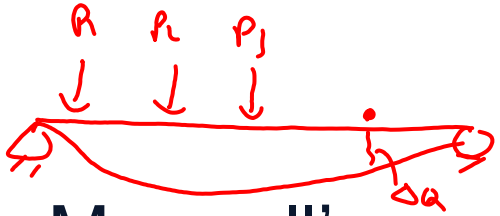
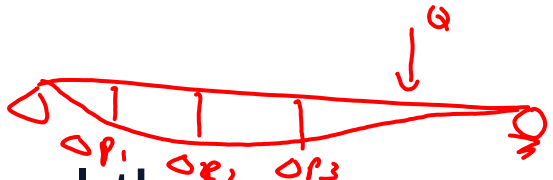
Betti's Law

$$P.\Delta_Q = Q.\Delta_P$$

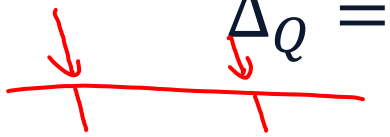




Maxwell's reciprocal theorem

The deflection at 'i' due to a unit load at 'j' is the same as the deflection at 'j' if a unit load was applied at 'i'.

$$\Delta_Q = \Delta_P$$



Energy methods

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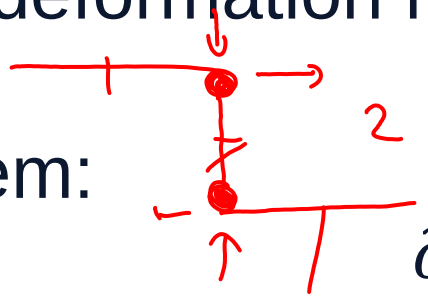
Castigliano's theorem

System should behave elastically

force deformation relations are linear

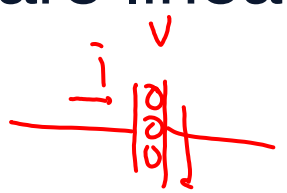


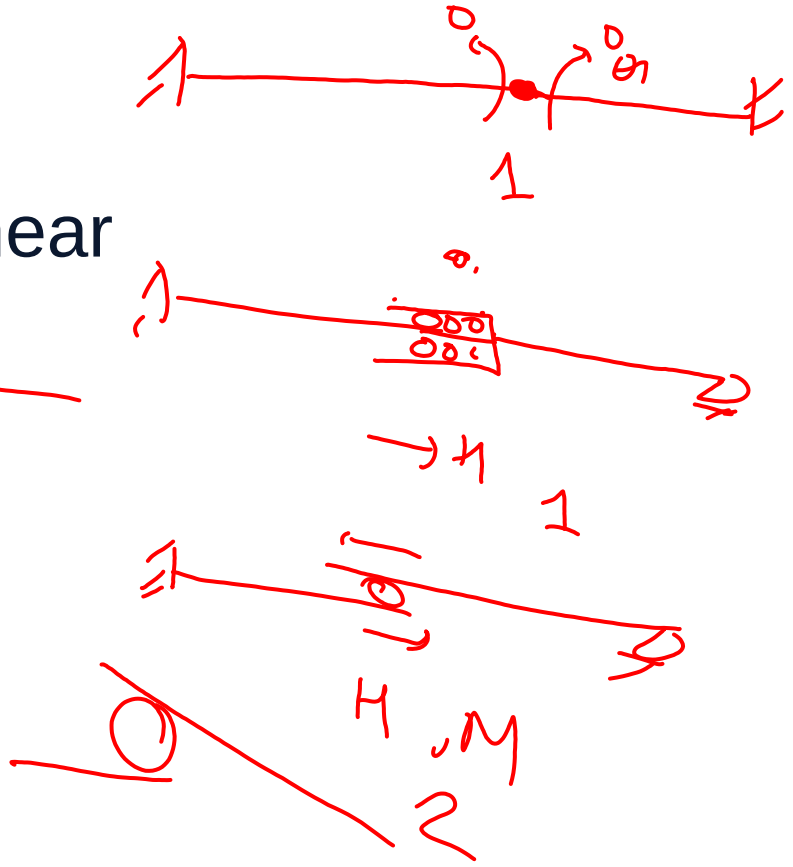
First Theorem:

$$\frac{\partial U}{\partial u_n} = P_n$$

Second Theorem:

$$\frac{\partial U}{\partial P_n} = u_n$$





Real work method

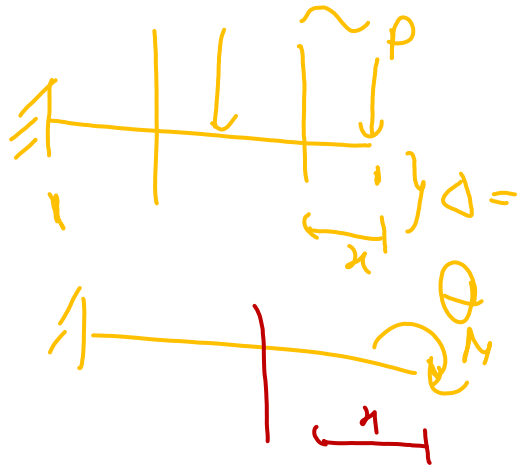
External work done:

$$W = \frac{1}{2} P\Delta \text{ or } \frac{1}{2} M\theta$$

Internal work done:

Calculate from strain energy

$$W = \frac{1}{2}\int \frac{M^2}{EI}\,dx$$



Strain Energy Method

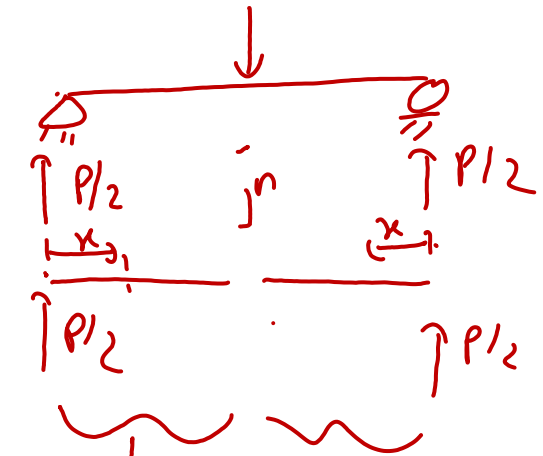
$$U = \frac{1}{2}\int \frac{P^2}{EA}\,dx$$

$$U = \frac{1}{2}\int \frac{M^2}{EI}\,dx$$

$$U = \frac{1}{2}\int \frac{T^2}{GJ}\,dx$$

$$U = \frac{1}{2}\int k\frac{V^2}{AG}\,dx$$

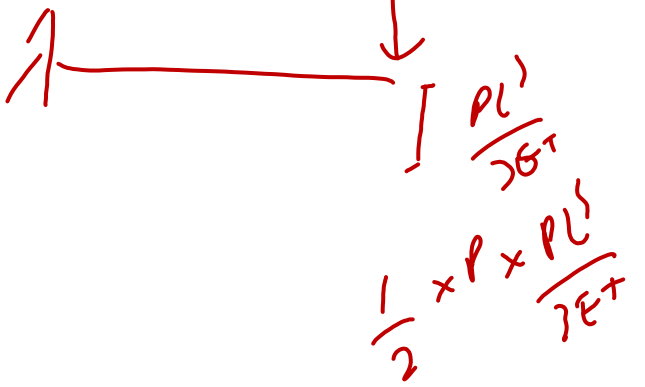
K = 1.2 for rectangle, 10/9 for circular



$$\frac{1}{2}\int_0^{L/2}\frac{(Px/2)^2}{EI}\,dx + \frac{1}{2}\int_0^{L/2}\frac{(\frac{P}{2}x)^2}{EI}\,dx$$

$$\frac{2P^2}{8EI}\int_0^{L/2} x^2\,dx$$

$$\frac{P^2}{4EI}\cdot\frac{1}{3}\cdot\frac{L^3}{8} = \frac{P^2L^3}{96EI}$$



$$\frac{1}{2}\times P\times\frac{PL^3}{3EI}$$

$$\frac{1}{2}P\Delta = \frac{P^2L^3}{96EI}$$

$$\Delta = \frac{PL^3}{48EI}$$

Strain Energy

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Resilience

It is the property of materials to absorb energy and to resist shock and impact loads.
It is measured by the amount of energy absorbed per unit volume within elastic limit this property is essential for spring materials.

Proof resilience
It is defined as the maximum strain energy stored in a body.
So, it is the quantity of strain energy stored in a body when strained up to the elastic limit (ability to store or absorb energy without permanent deformation).

Modulus of resilience
It is defined as proof resilience per unit volume.
It is the area under the stress-strain curve up to the elastic limit.

Strain Energy stored on axial load

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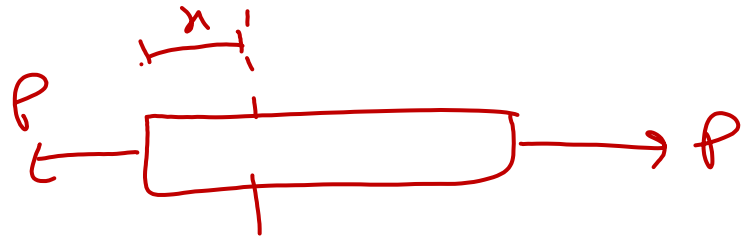
$$U = \frac{1}{2}\int \frac{P^2}{EA}\,dx$$



$$= \frac{1}{2}\frac{P^2}{EA}\int_0^L dx$$

$$= \frac{P^2 L}{2AE} =$$

Strain Energy stored on axial load

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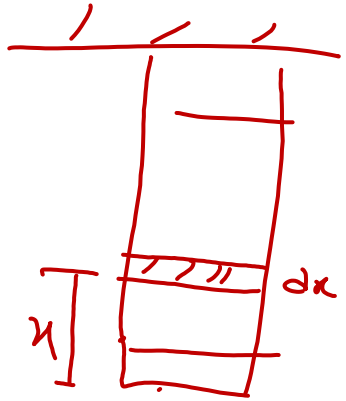
$$U = \frac{1}{2}\int \frac{P^2}{EA}\,dx$$

$$= \frac{1}{2AE}\int_0^{\ell} \frac{W^2x^2}{\ell^2}\,dx$$

$=$



$e = \frac{WL}{2AE}$

$P = \frac{W}{\ell}x$

Strain Energy stored on Beam

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$$U = \frac{1}{2}\int \frac{M^2}{EA}\,dx$$

Virtual work method

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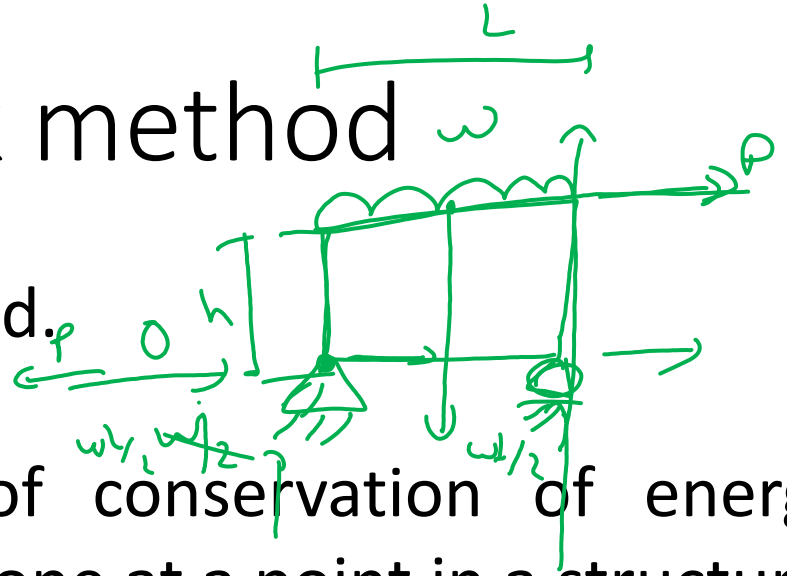
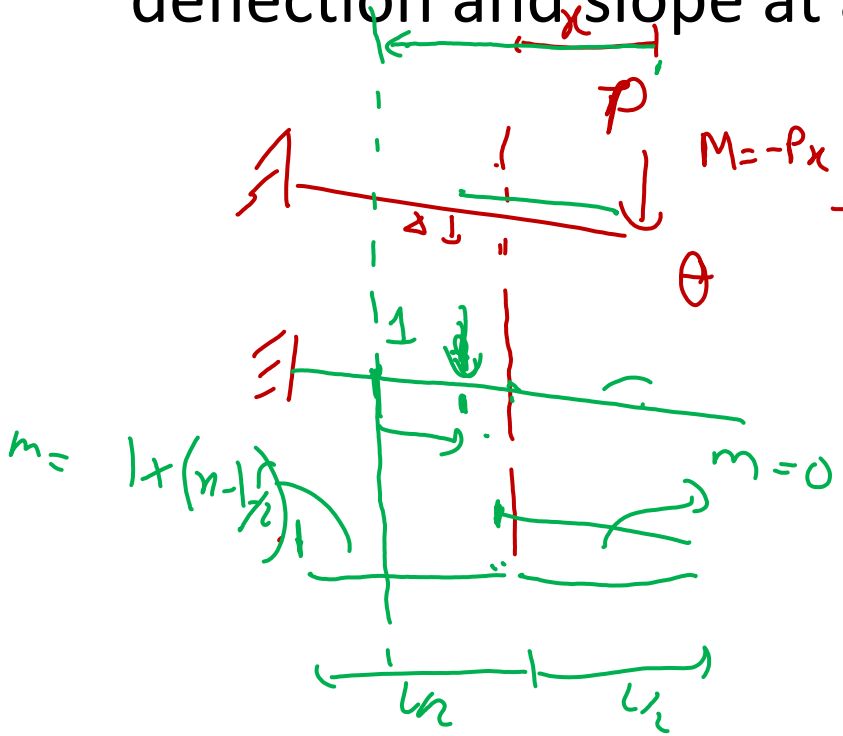
Unit Load method.



uses the law of conservation of energy to obtain the deflection and slope at a point in a structure



$$\Delta = \int \frac{Mm}{EI}\,dx$$

$$\Delta = \sum \frac{PpL}{AE}$$

Deflection of beams and portal frame

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Using virtual load methods and strain energy method

Deflection of beams and portal frame

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Using virtual load methods and strain energy method

MCQs

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Which of the following is correct equation for DSI of frame ?

a. 3m+r+3j
b. 3m+r-3j
c. 3m-r-3j
d. Depend on Strutcure.

$6m + r - 6j$

MCQs

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How many degree of freedom are counted for a roller support while calculating DKI ?

a. 0
b. 1
c. 2
d. 3

R = 1 , DIF = 3 - R

extra R = −e

DOF = 3 + extra

MCQs

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If j represents number of joints, r represents number of external force and Structure has 2j-r members , Then Stability of structure will be ,

2D truss →
$D_s = (m+r) - 2j$

a. Stable
b. Unstable
c. Depends on structure
d. Depend on magnitude of load

MCQs

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The ability of material to absorb energy when elastically deformed and to return it when unloaded is called

a. Resilience
b. Proof Resilience → max value of strain energy in elastic zone
c. Plasticity
d. Malleability

MCQs

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Which of the following methods are included in ENERGY Methods ?

a. Castigliano Method
b. Betti’s Method
c. Maxwell Reciprocal theorem
d. All of these

MCQs

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If 4 reactions are acting on the beam then the system is

a. unstable and indeterminate
b. stable and indeterminate
c. Stable and Determinate
d. Can’t Say

MCQs

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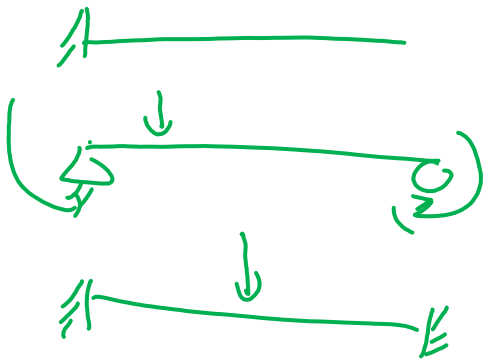
The maximum deflection of a cantilever beam of length l with a point load W at the free end is

a. $WL^3/3EI$

b. $WL^3/8EI$

c. $WL^3/16EI$

d. $WL^3/32EI$



MCQs

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The strain energy stored due to flexural forces is:

a) $U = \frac{1}{2}\int \frac{P^2}{EA}dx$

b) $U = \frac{1}{2}\int \frac{M^2}{EI}dx$ ✓✓

c) $U = \frac{1}{2}\int \frac{T^2}{GJ}dx$

d) $U = \frac{1}{2}\int k\frac{V^2}{AG}dx$

rect → 1.2

circle → 10/9 → 1.11

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Thank YOU !!!