4.1 → AF, SF, BM
4.2 → Stress → Strain
4.3 → Bending

NEPAL ENGINEERING COUNCIL
LICENSE EXAM PREPARATION COURSE

FOR

CIVIL ENGINEERS

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4. Structural Mechanics

4.3 Theory of flexure and columns

Sub topics

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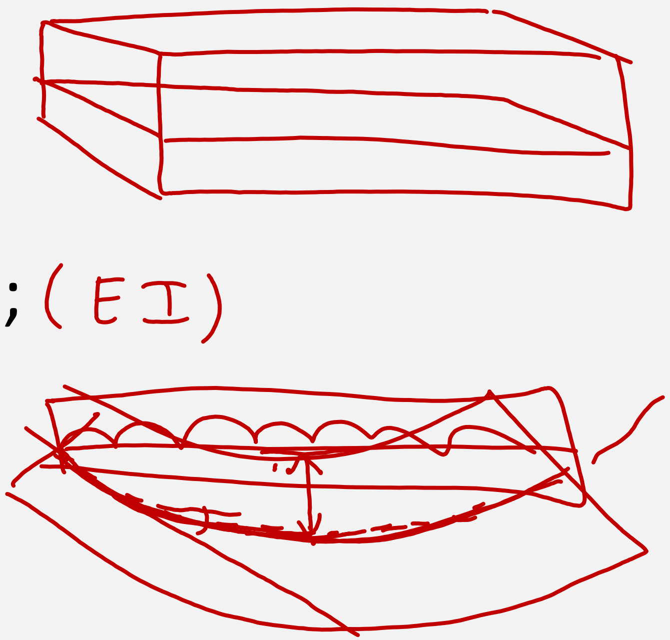
200kN

100

120

- Co-planar and pure bending
- Elastic curve
- Angle of rotation
- Radius of curvature and flexural stiffness; (EI)
- Deflection; Bending stress; (σ)
- Euler's formula for long column.



$$\boxed{\sigma = \frac{My}{I}}$$

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

$\sigma = \frac{My}{I} \rightarrow NA = 0$

Co-planar and pure bending

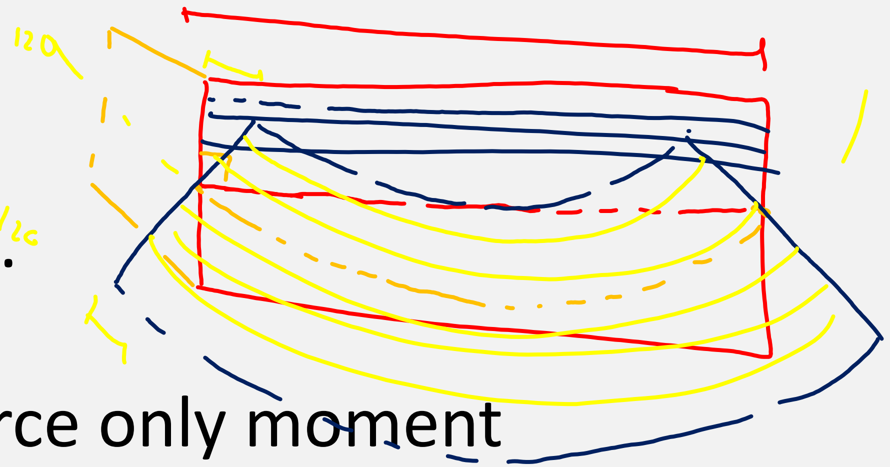
The traverse sections which are plane and normal before bending remain plane and normal to neutral axis.

Every layer is free to expand or contract

Modulus of elasticity has same value for tension and compression.

Bends as arc of circle.

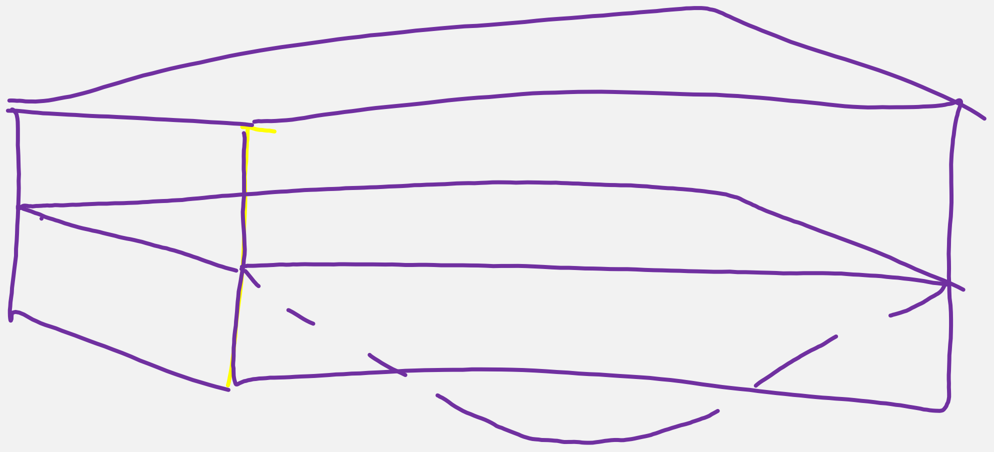
Radius of curvature is large.



Pure bending : No shear force only moment

Elastic curve

Beam subjected to couples
Elastic curve is arch of circle

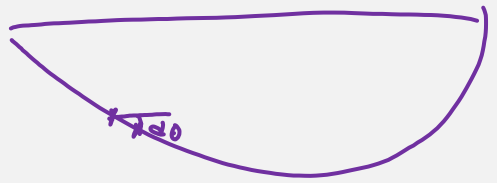


Centroidal axis, neutral plane

Centroidal axis deflects in form of elastic curve or deflected shape.

Angle of rotation

$$d\theta = \frac{ds}{\rho}$$



Radius of curvature and flexural stiffness

(R) (ρ) Radius of curvature is radius of arc forming elastic curve.

Unit m

Curvature is $\frac{1}{\rho}$. $= K$ unit (m^{-1})

x

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

$$\frac{M}{EI} = \frac{1}{R} = K$$

$EI \rightarrow N m^{-2}\ m^4$
$\rightarrow N m^2$

material

$R \propto 1/M$

Section property

Deflection

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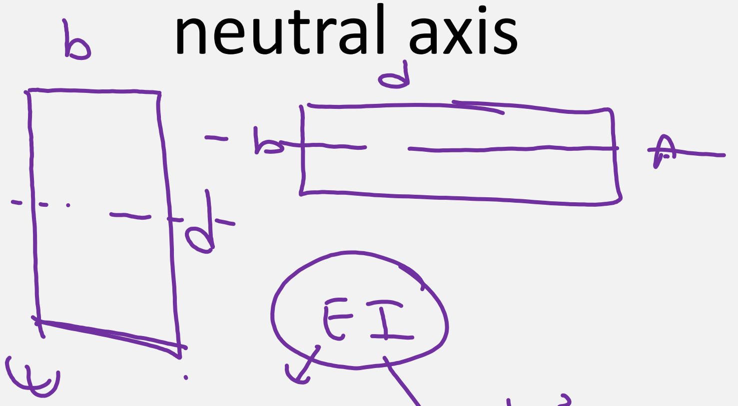
Displacement of the neutral plane from original position

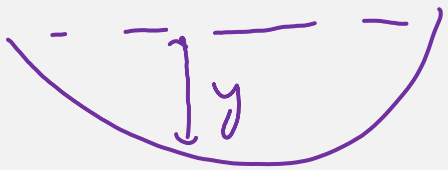
b<d

Slope: Angle in radian made by tangent at any point of neutral axis



$$\theta = \frac{dy}{dx}$$

$$M = EI\frac{d^2y}{dx^2}$$



$y =$

EI

$\frac{bd^3}{12} \rightarrow \frac{b(2d)^3}{12} = 8 \times \frac{bd^3}{12}$

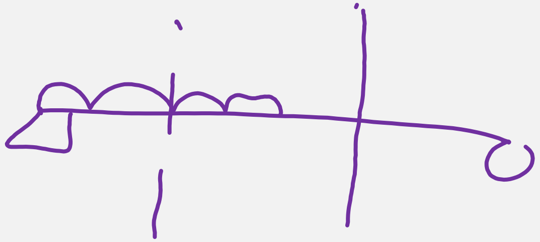
$\frac{2bd^3}{12}$

Flexural formula

$$\frac{M}{\frac{bd^3}{12}} = \frac{\sigma}{d/2}$$

$$\frac{6M}{bd^2} = \sigma$$

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R} = Ek$$



For a section M/I and E/R is constant.

$$\text{const} = \frac{\sigma\,d^2}{b} = \frac{\sigma d^2}{M}$$

$$\frac{\sigma}{y} = \text{con}$$

$$\frac{1}{R} = k =$$

M and d is doubled the max bending stress will be . . . . ?

$\sigma \propto y$

$$\frac{\sigma_1 d_1^2}{M_1} = \frac{\sigma_2 d_2^2}{M_2}$$

$$\frac{M_2}{M_1} \quad \sigma_1 d_1^2 = \sigma_2 (2d_1)^2$$

$$\sigma_2 = \frac{\sigma_1}{4} \times 2 = \frac{\sigma_1}{2}$$

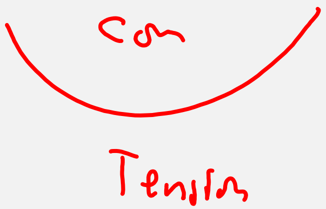
Bending Stress

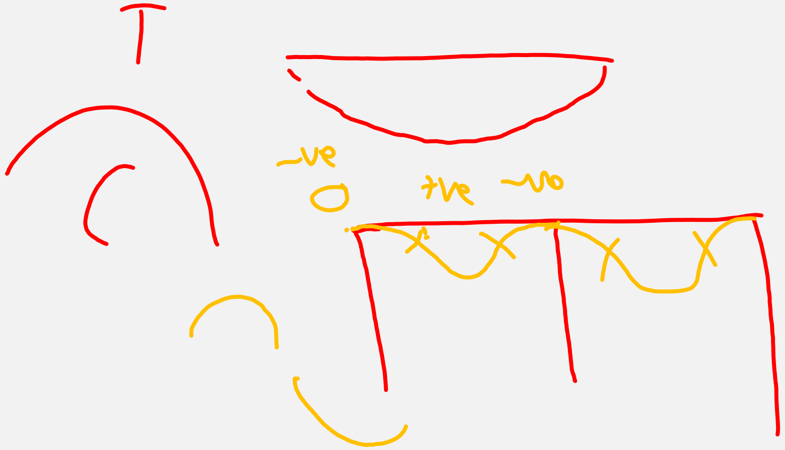
For sagging : tension below NA, compression above NA

For hogging: tension above NA, compression below NA

Proportional to distance from NA

Sag +ve mom





Uniform Strength Beam

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$\sigma_{max} = \sigma_y$

Each section has bending stress equal to allowable stress

$\boxed{\frac{M}{d^2}} = \frac{\sigma_y \times b}{6}$ $\quad \sigma_y = \frac{My}{I} = \frac{M}{Z}$

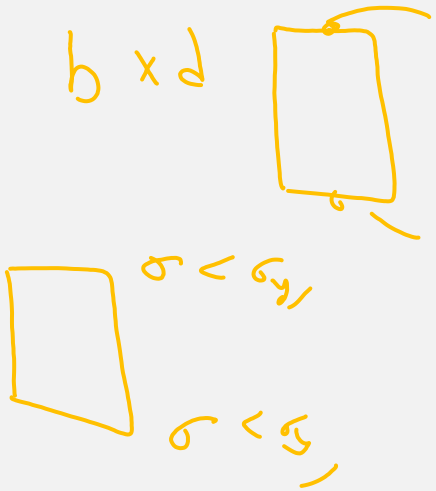
$$\sigma_{max} = \sigma_y$$

breadth

Either by varying depth or ~~height~~ we can make beam of uniform strength

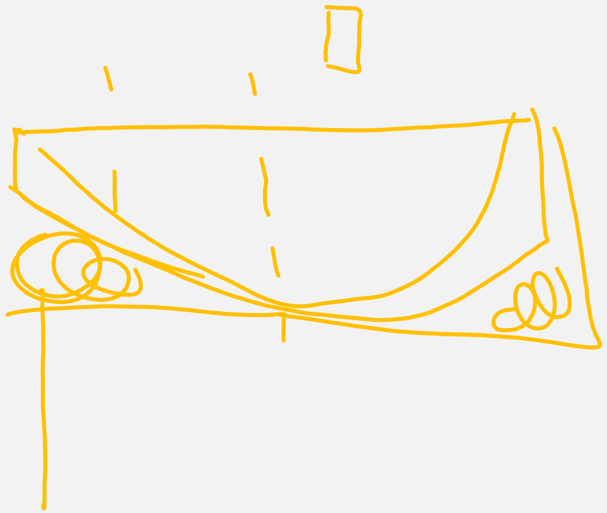
$\sigma_y = \frac{M}{bd^2/6}$

$bd^2 \propto M$



$b \times d$

$\sigma < \sigma_y$



$\frac{M_1}{d_1^2} = \frac{M_2}{d_2^2}$ $\quad M_2 \to M_1/2$

$d_2 = \frac{d_1}{\sqrt{2}}$

Euler's formula for long column

buckling

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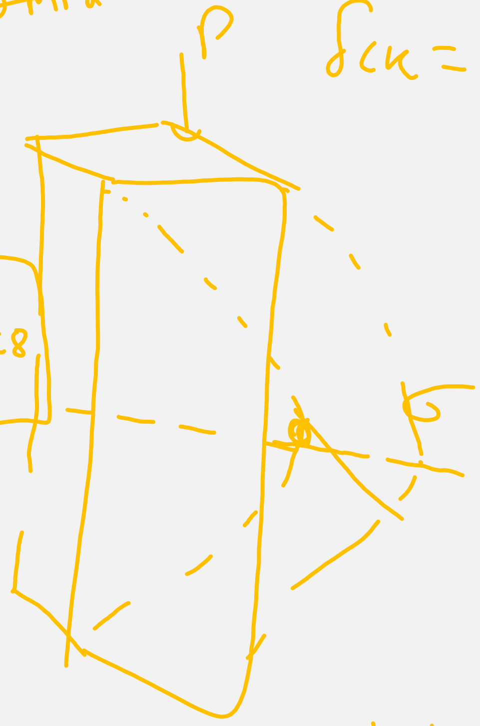
~~20 MPa~~

$f_{ck} = 20\ N/mm^2$

Slenderness ratio: $\lambda = \dfrac{effective\ length}{radius\ of\ gyration}$



Short column fails by $\boxed{\text{crushing}}$ $\lambda < 32$

$\frac{l}{d} < 8$

$$P_c = \sigma_c . A$$

20

2-4

$\sigma = \boxed{\frac{P}{A}} + \boxed{\frac{M}{Z}}$

Long column fails by $\boxed{\text{buckling}}$ $\lambda > 120$

$\sigma \quad \frac{l}{d} > 30$

$$P_c = \frac{\pi^2}{l_e^2} . EI$$

$1000 mm^2$

$k = \sqrt{\frac{I}{A}}$ $\quad \boxed{l_e} = KL$ $\quad 32 - 120$ $\quad f_{ck}$ $P = 20000$

Euler's formula for long column

$\boxed{0.7 = \frac{1}{\sqrt{2}}}$

$P \times k^2 \times l^2 = \text{constant}$

$$P_c = \frac{\pi^2}{l_e^2}.\,EI = \frac{\pi^2}{K^2 L^2} EI$$

mm^4

m

Buckles in plane of major axis

$l_e^2 = k^2 L^2$

$P k^2 = c$

Effective length (l) depends on end condition of member

P_1

P_2

Case I → both end fixed

Case IV one end fixed other hinged

length doubled

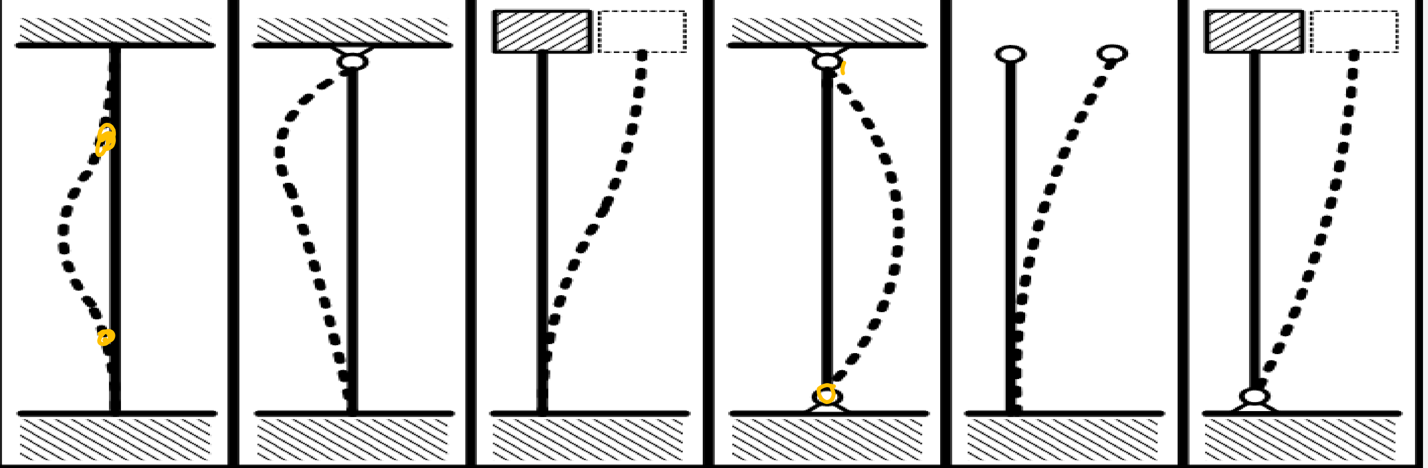
$P_1 k_1^2 l_1^2 = P_2 k_2^2 l_2^2$

$P_2 = P_1/8$

Euler's formula for long column

$L_e = K L$

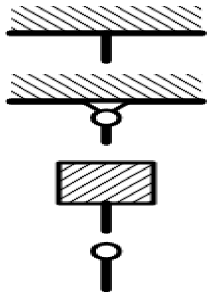
$P = \frac{\pi^2 EI}{K^2 L^2}$

Buckled shape of column shown by dashed line						
Theoretical K value	0.5	0.7 $= \frac{1}{\sqrt{2}}$	1.0	1.0	2.0	2.0
Recommended design value K	$\boxed{0.65}$	0.80	1.2	1.0	2.10	2.0

End condition key

fixed

hinge



Rotation fixed and translation fixed

Rotation free and translation fixed

Rotation fixed and translation free

Rotation free and translation free

Tortional formula

$J = \frac{\pi d^4}{32}$

$$\frac{T}{J} = \frac{\tau}{r} = \frac{G\phi}{L}$$

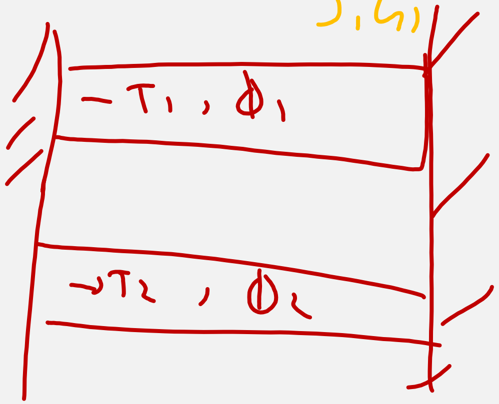
$\phi = \frac{LT}{JG}$

$\phi_1 = \frac{L_1 T_1}{J_1 G_1}$

Members in parallel connection

ϕ constant ,T additive

$\phi = \phi_1 = \phi_2 \qquad T = T_1 + T_2$

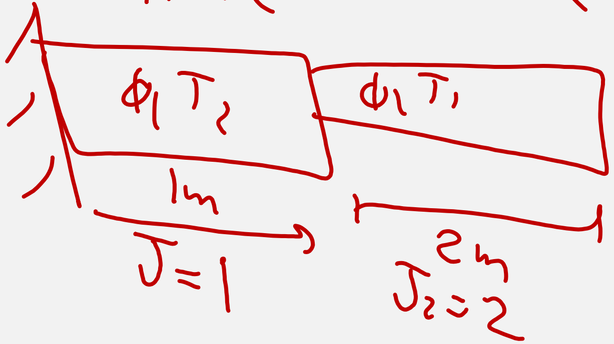


Members in series connection

ϕ additive ,T constant

$\rightarrow \phi = \phi_1 + \phi_2 \qquad T = T_1 = T_2$

$G = 10$



$T = 10$, Angle of twist

Relation of modulus

$(-1, 0.5) \rightarrow$ limiting value of ν

- $E = 2G(1+\mu) = 3K(1-2\mu)$

$K=1,\ G=2,\ E=?=3.6$

$$\boxed{E = \frac{9KG}{3K+G}}$$

- μ=0.25-0.3 for steel
- μ=0.15-0.2 for concrete

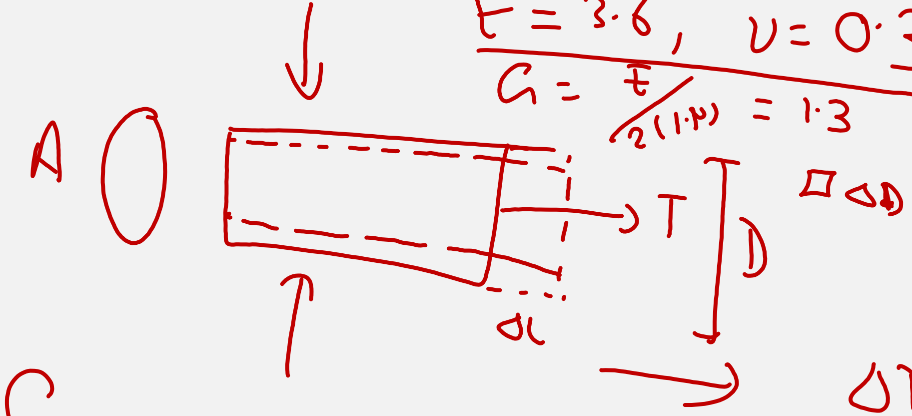
$$\nu = \frac{\text{lateral strain}}{\text{longitudinal strain}}$$

$E = 3.6,\ \nu = 0.3$

$G = \frac{E}{2(1+\mu)} = 1.3$

$$= -\frac{\frac{\Delta D}{D}}{\frac{\Delta L}{L}}$$

$\boxed{0-0.5}$



$G=$

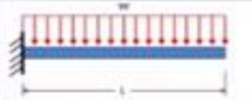
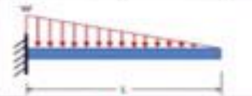
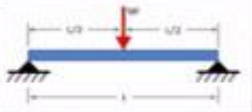
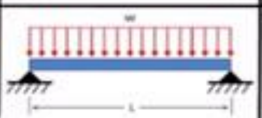
$\Delta D =$ final − initial

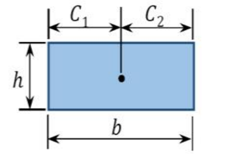
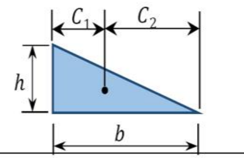
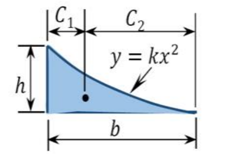
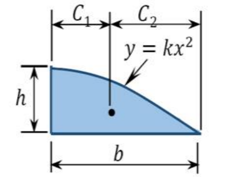
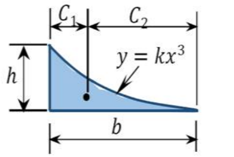
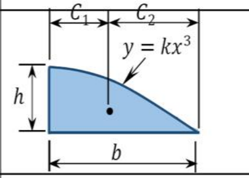
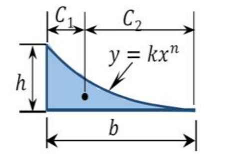
$$E = 2G(1+\nu) = 3K(1-2\nu)$$

Deflection

Area of M/EI diagram gives slope

Moment of area of M/EI diagram gives deflection

SR. NO.	TYPE OF BEAM	MAX. BM	SLOPE	DEFLECTON
1		M	$\theta = \frac{ML}{EI} = \frac{ML}{EI}$	$\delta = \theta \times \frac{L}{2} = \frac{ML^2}{2EI}$
2		WL	$\theta = \frac{ML}{2EI} = \frac{WL^2}{2EI}$	$\delta = \theta \times \frac{2L}{3} = \frac{WL^3}{3EI}$
3		$\frac{WL^2}{2}$	$\theta = \frac{ML}{3EI} = \frac{WL^3}{6EI}$	$\delta = \theta \times \frac{3L}{4} = \frac{WL^4}{8EI}$
4		$\frac{WL^2}{6}$	$\theta = \frac{ML}{4EI} = \frac{WL^3}{24EI}$	$\delta = \theta \times \frac{4L}{5} = \frac{WL^4}{30EI}$
5		$\frac{WL}{4}$	$\theta = \frac{ML}{4EI} = \frac{WL^2}{16EI}$	$\delta = \theta \times \frac{L}{3} = \frac{WL^3}{48EI}$
6		$\frac{WL^2}{8}$	$\theta = \frac{ML}{3EI} = \frac{WL^3}{24EI}$	$\delta = \theta \times \frac{5L}{16} = \frac{5WL^4}{384EI}$

Geometric Shape		Area	Centroid	
			C_1	C_2
Rectangle		bh	$\frac{b}{2}$	$\frac{b}{2}$
Triangle		$\frac{bh}{2}$	$\frac{b}{3}$	$\frac{2b}{3}$
Parabolic spandrel		$\frac{bh}{3}$	$\frac{b}{4}$	$\frac{3b}{4}$
		$\frac{2bh}{3}$	$\frac{3b}{8}$	$\frac{5b}{8}$
Cubic spandrel		$\frac{bh}{4}$	$\frac{b}{5}$	$\frac{4b}{5}$
		$\frac{3bh}{4}$	$\frac{2b}{5}$	$\frac{3b}{5}$
General spandrel		$\frac{bh}{n+1}$	$\frac{b}{n+2}$	$\frac{b(n+1)}{n+2}$

Deflection

Area of M/EI diagram gives slope

Moment of area of M/EI diagram gives deflection

$$\frac{1}{2}\times\frac{L}{2}\times\frac{WL}{4EI}\times\frac{L}{2}\times\frac{2}{3}=\frac{WL^3}{48EI}$$

SR. NO.	TYPE OF BEAM	MAX. BM	SLOPE	DEFLECTON
1		M	$\theta=\frac{ML}{EI}=\frac{ML}{EI}$	$\delta=\theta\times\frac{L}{2}=\frac{ML^2}{2EI}$
2		WL	$\theta=\frac{ML}{2EI}=\frac{WL^2}{2EI}$	$\delta=\theta\times\frac{2L}{3}=\frac{WL^3}{3EI}$
3		$\frac{WL^2}{2}$	$\theta=\frac{ML}{3EI}=\frac{WL^3}{6EI}$	$\delta=\theta\times\frac{3L}{4}=\frac{WL^4}{8EI}$
4		$\frac{WL^2}{6}$	$\theta=\frac{ML}{4EI}=\frac{WL^3}{24EI}$	$\delta=\theta\times\frac{4L}{5}=\frac{WL^4}{30EI}$
5		$\frac{WL}{4}$	$\theta=\frac{ML}{4EI}=\frac{WL^2}{16EI}$	$\delta=\theta\times\frac{L}{3}=\frac{WL^3}{48EI}$
6		$\frac{WL^2}{8}$	$\theta=\frac{ML}{3EI}=\frac{WL^3}{24EI}$	$\delta=\theta\times\frac{5L}{16}=\frac{5WL^4}{384EI}$

Geometric Shape		Area	Centroid	
			C_1	C_2
Rectangle		bh	$\frac{b}{2}$	$\frac{b}{2}$
Triangle		$\frac{bh}{2}$	$\frac{b}{3}$	$\frac{2b}{3}$
Parabolic spandrel	$y=kx^2$	$\frac{bh}{3}$	$\frac{b}{4}$	$\frac{3b}{4}$
	$y=kx^2$	$\frac{2bh}{3}$	$\frac{3b}{8}$	$\frac{5b}{8}$
Cubic spandrel	$y=kx^3$	$\frac{bh}{4}$	$\frac{b}{5}$	$\frac{4b}{5}$
	$y=kx^3$	$\frac{3bh}{4}$	$\frac{2b}{5}$	$\frac{3b}{5}$
General spandrel	$y=kx^n$	$\frac{bh}{n+1}$	$\frac{b}{n+2}$	$\frac{b(n+1)}{n+2}$

MCQs

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Based on the governing equation for the theory of pure bending which of the following is valid

a. Bending moment = the product of bending stress and the section modulus

b. Bending strength = product of modulus of elasticity and curvature

c. Moment resistance = the product of flexural rigidity and the curvature

9845690003

d. Bending stress = flexural rigidity per unit radius

MCQs

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If a constant section beam is subjected to uniform bending moment throughout, it bends as

a. zig zag
b. catenary
c. circular arc
d. parabolic arc

MCQs

σ

What is the stress developed in bending a 10 mm diameter steel rod of $E = 2 \times 10^5$ N/mm^2 to 2000 mm diameter?

a. 500 N/mm^2

✓ b. 1000 N/mm^2

c. 1500 N/mm^2

d. 2000 N/mm^2

$$\frac{\sigma}{y} = \frac{E}{R} \qquad \frac{2\times10^5\ N/mm^2}{2000/2}$$

MCQs

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Circular beam of uniform strength can be made by varying the diameter in such way that

a. M/Z is constant

b. σ/Y is constant

c. M/R is constant

d. E/R is constant

$M = \sigma Z$

M/Z

MCQs

Which is the following statement is true ?

a. The strength of a fibre is proportional to its distance from neutral axis.
b. The sum of all the compressive force above neutral layer must be equal to the sum of tensile force below neutral layer.

c. The vertical plane through which load is applied to avoid torsion in the cross section is called load plane.

d. All of the above are true.

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MCQs

The section modulus of a rectangle with breadth B and depth d will be:

a. $Bd^3/6$

b. $Bd^3/12$

c. $Bd^2/6$

d. $Bd^2/12$

MCQs

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What is the effective length of compressive member when both end are hinged?

a. 0.65L

b. L

c. 2L

d.1.5L

MCQs

If the value of flexural stiffness of 3 m long column fixed at both end is 2000 Nm2 . The maximum permissible axial load is

a.1.5 KN

b. 1.7 KN

c. 2.2 KN

d. 3 KN

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Thank YOU !!!