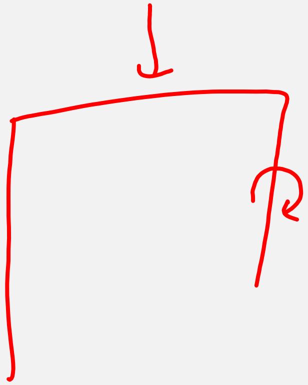

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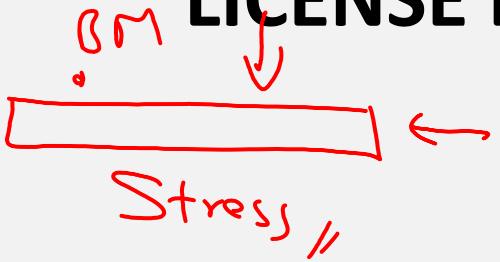


FOR

CIVIL ENGINEERS

PANA ACADEMY

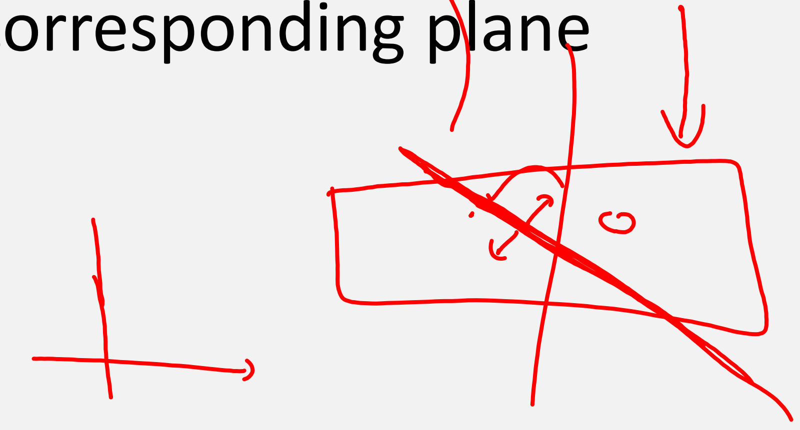
4. Structural Mechanics

4.2 Stress and strain analysis

Sub topics

compress / tension

- Normal and shear stresses
- Principle stresses and principle planes
- Maximum shear stress and corresponding plane
- Stress strain curves
- Torsion



Stress

$\sigma = \frac{F}{A}$

The force of resistance per unit area, offered by a body against deformation is known as stress.

The external force acting on the body is called the load or force. The load is applied on the body while the stress is induced in the material of the body. A loaded member remains in equilibrium when the resistance offered by the member against the deformation and the applied load are equal.

Normal Stress: (AF , BM)

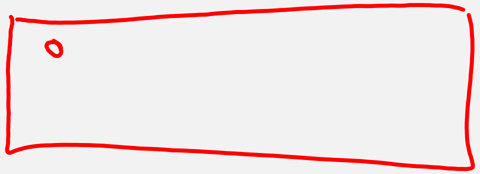
Stress perpendicularly acting to cross section.

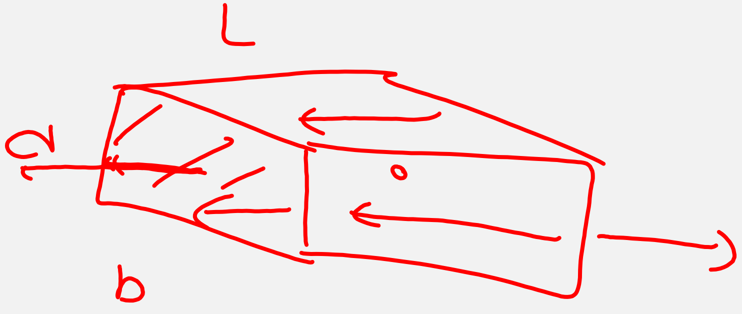
Mainly due to axial forces and bending moments

Force/perpendicular area.

Denoted usually by σ

M, AF





$$\sigma = \frac{F}{bd}$$

Shear stresses (V, τ)

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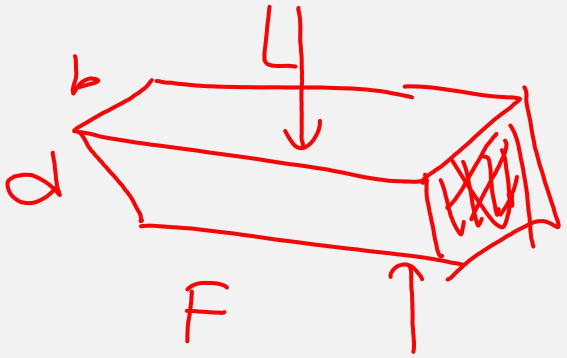
Stress along the cross section or coplanar

Mainly due to shear forces and torsion.

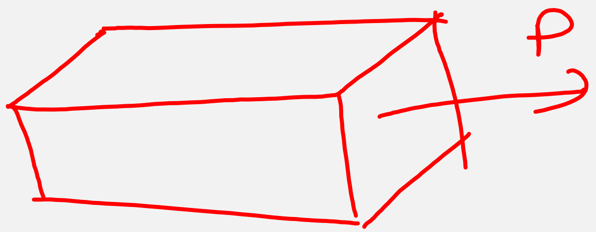
Force/ area parallel to force

Denoted usually by τ

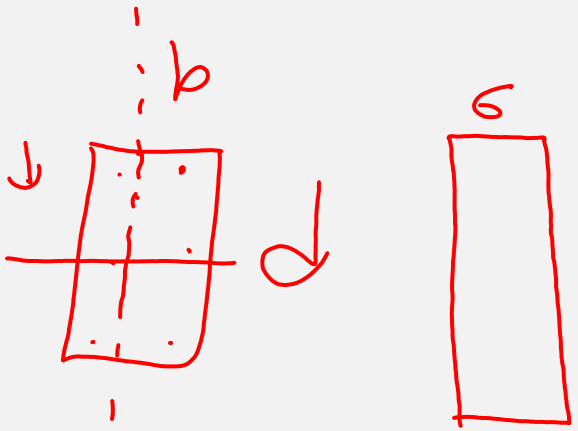


$$\frac{F}{bd}$$

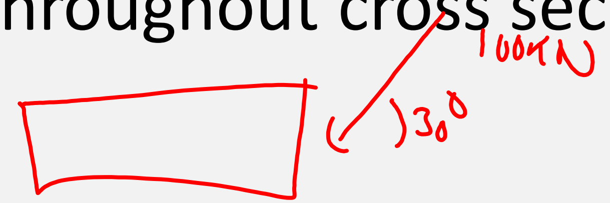
Stress due to axial force



$$\sigma = \frac{P}{A} = \frac{P}{bd}$$



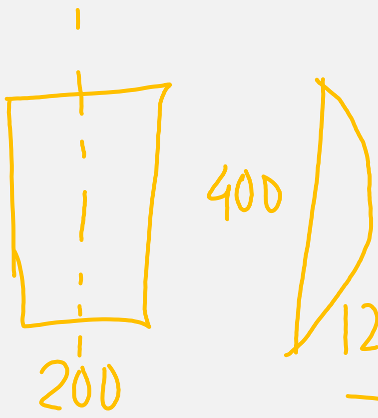
Stress diagram is rectangular with constant stress throughout cross section.



$b \times d = 200mm \times 400mm$

$$\sigma_A = \frac{86.7\ kN}{200 \times 400\ mm^2}$$

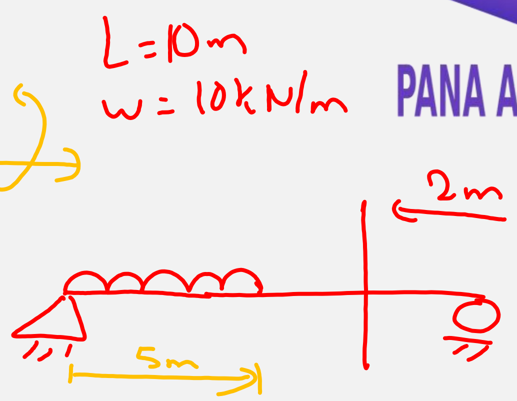
Stress due to shear force



$$\tau = \frac{VQ}{Ib}$$

$$\tau_{avg} = \frac{V}{A}$$

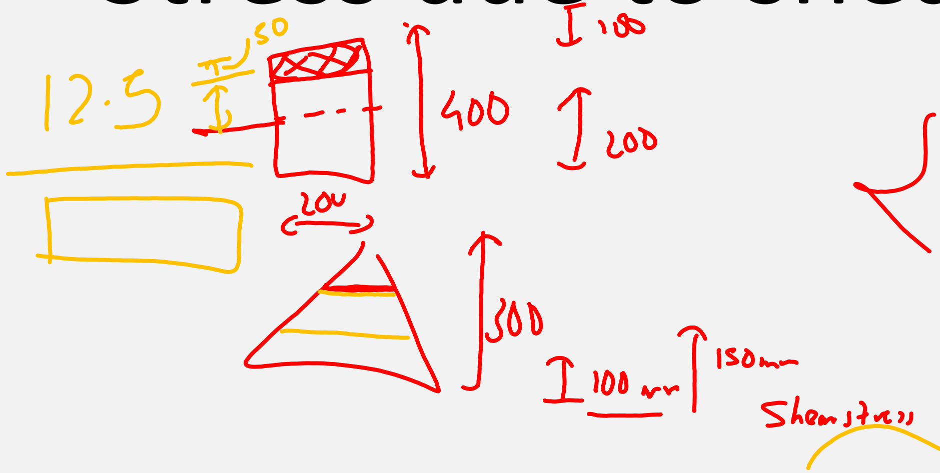
$$Q = A\bar{y}$$



Stress diagram is parabolic with zero stress at top and bottom.

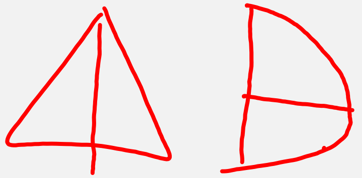
Stress due to shear force

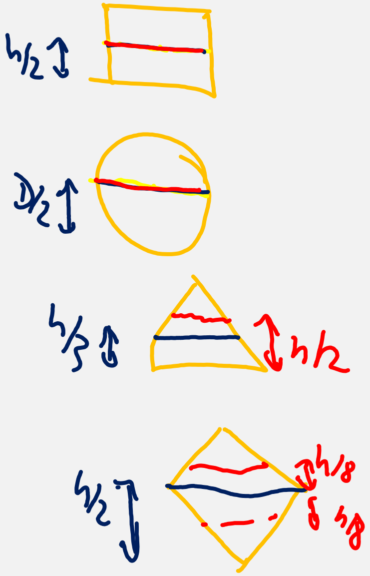
PANA ACADEMY



$$\tau = \frac{VQ}{Ib} = Q = A \times \bar{y}$$

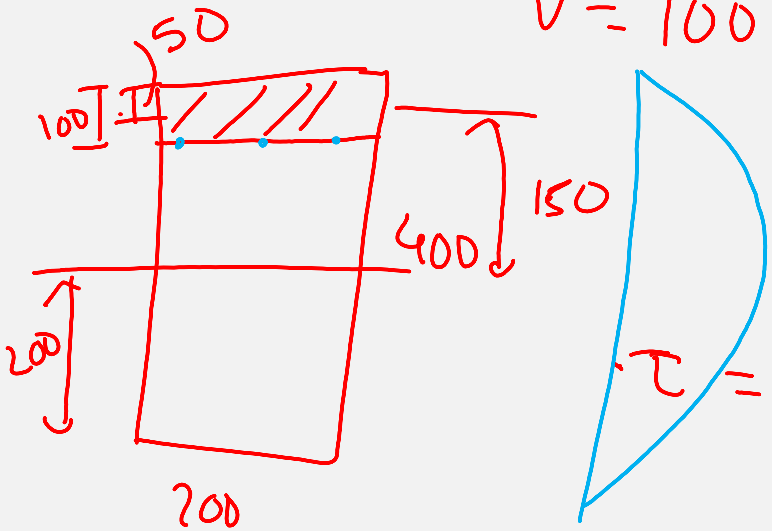
$$\tau_{avg} = \frac{V}{A}$$





Section	τ_{max}/τ_{avg}	*Location*	τ_{NA}/τ_{avg}	*Location*
Rectangle/ Square	3/2 = 1.5	$h/2$	3/2	h/2
Circular	4/3	$h/2$	4/3	h/2
Triangular	3/2	$h/2$	4/3	h/3
Diamond	9/8	$\frac{h}{8}$ *above NA*	1	h/2

$V = 100\,kN$



100×1000

$$\tau = \frac{VQ}{I\,b}$$

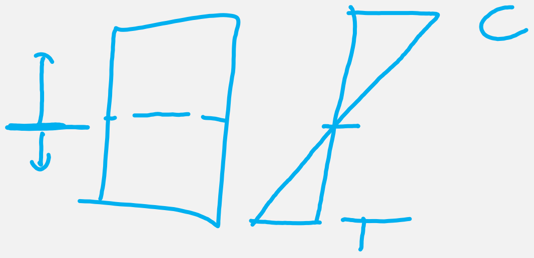
Q: $A \times \bar{y} = 100 \times 200 \times 130 = 3000000$

I: $\frac{200 \times 400^3}{12}$, b: $\times 200$

$\frac{64000000}{3}$

$\tau = 1.41\ N/mm^2$

Stress due to bending



$$\sigma = \frac{My}{I} = \frac{M}{I/y}$$

(Moment)

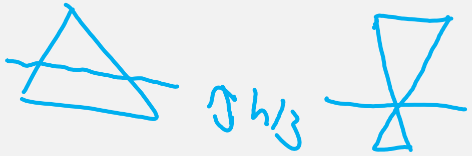
Sagging Moment +ve

$$\sigma_{max} = \frac{Moment}{section\ modulus} = \frac{M}{Z}$$

$M = 12.5 \times 2$

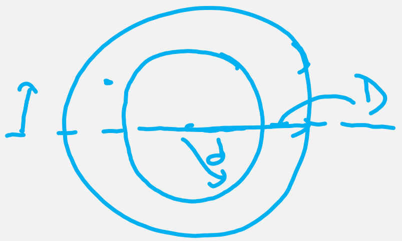
$Z = I/y_{max}$

Bending stress zero at NA



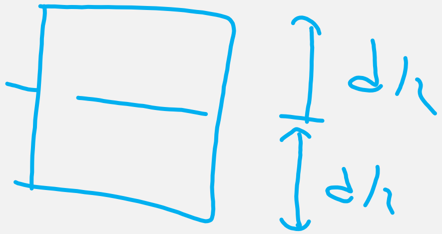
Stress due to bending

$$Z_{hollow\ circle} = \frac{I}{y_{max}} = \frac{\frac{\pi}{64}(D^4 - d^4)}{D/2} = \frac{\pi}{32D}(D^4 - d^4)$$

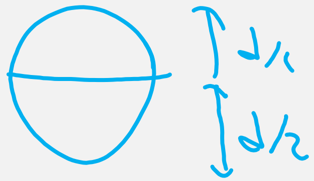


$$\sigma = \frac{My}{I}, \sigma_{max} = \frac{Moment}{section\ modulus}$$

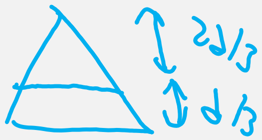
$$Z_{rect} = \frac{I}{y_{max}} = \frac{bd^3/12}{d/2} = bd^2/6$$



$$Z_{circle} = \frac{I}{y_{max}} = \frac{\pi d^4/64}{d/2} = \frac{\pi d^3}{32}$$



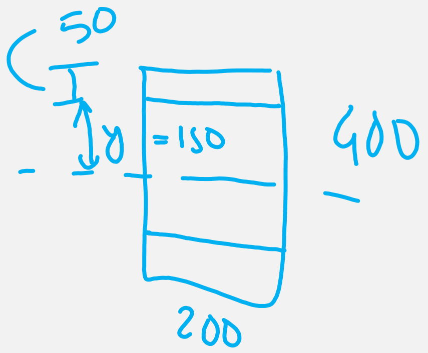
$$Z_{triangle} = I/y_{max} = \frac{bd^3/36}{2d/3} = bd^2/24$$



Stress due to bending

1x

$$\sigma = \frac{My}{I}, \sigma_{max} = \frac{Moment}{section\ modulus}$$



$M = 25\ kNm = 25 \times 1kN \times 1m = 25 \times 1000N \times 1000mm$

$= 25 \times 10^6\ Nmm$

$$\sigma_{max} = \frac{25 \times 10^6 \times 200}{\frac{400^3 \times 200}{12}} = 4.69\ N/mm^2$$

$$\sigma_{50mm\ from\ top} = \frac{25 \times 10^6 \times 150}{\frac{400^3 \times 200}{12}} =$$

Stress due to torsion

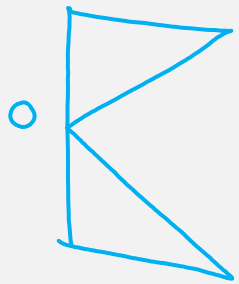
$J = \frac{\pi d^4}{32}$

$$\tau = \frac{Tr}{J}$$

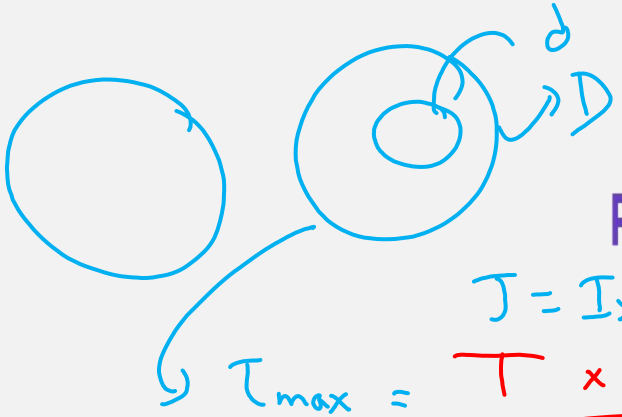
$$\tau_{max} = \frac{Torsion}{Polar\ section\ modulus}$$

$\frac{J}{r_{max}}$

Zero at axis of torque.



$\tau_{max} = \frac{T}{\frac{\pi D^4}{32} / \frac{D}{2}} = \frac{16T}{\pi D^3}$



PANA ACADEMY

$J = I_x + I_y$

$\tau_{max} = \frac{T \times D/2}{\frac{\pi}{32}(D^4 - d^4)}$

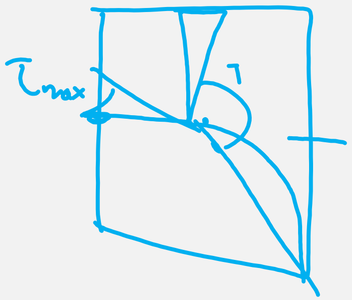
$= \frac{16TD}{\pi(D^4 - d^4)}$

$\tau_{min} = \frac{16Td}{\pi(D^4 - d^4)}$

Stress due to torsion

PANA ACADEMY

$$\tau = \frac{Tr}{J}, \tau_{max} = \frac{Torsion}{Polar\ section\ modulus}$$



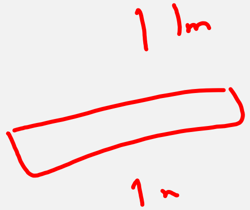
warp

$\tau_{corner} = 0$

$\tau_{centre} = 0$

τ_{max} is at middle edge of longer side

Strain



$$\frac{0.1\,m}{1\,m}$$

When a body is subjected to some external force, there is some change of dimension of the body.
The ratio of change of dimension of the body to the original dimension is known as strain.
Strain is dimensionless

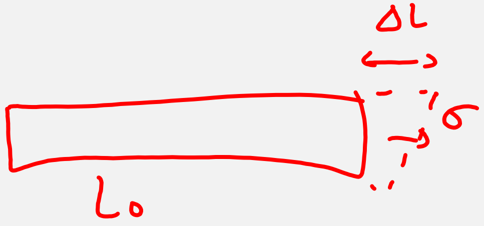
$$\frac{\text{Change}}{\text{Original configuration}} = \frac{\text{final} - \text{initial}}{\text{intial}}$$

Strain

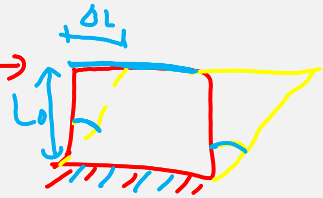
PANA ACADEMY

Longitudinal strain $\varepsilon = \Delta L / L_0$

Length is doubled → strain = $(2L-L)/L = 1$



Shearing strain $\gamma = \tan\theta = \theta^c$



Volumetric strain $\varepsilon_v = \Delta V/V$

$\varepsilon_v = \varepsilon_x + \varepsilon_y + \varepsilon_z = 3\varepsilon_x$

Temperature strain

$\varepsilon = \alpha.\Delta T$

20C

$\Delta T = 5°C$

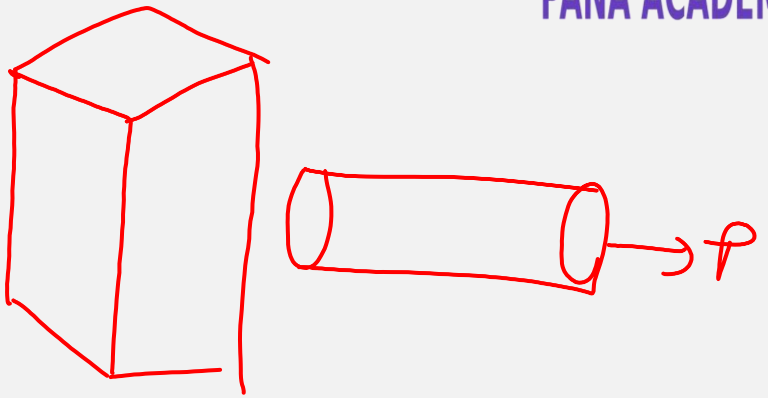
25°C

Elongation due to load

Cylindrical bar

$$e = \frac{PL}{AE}$$

$$e_{circ} = \frac{PL}{\frac{\pi d^2}{4} E} = \frac{4PL}{\pi E d^2}$$



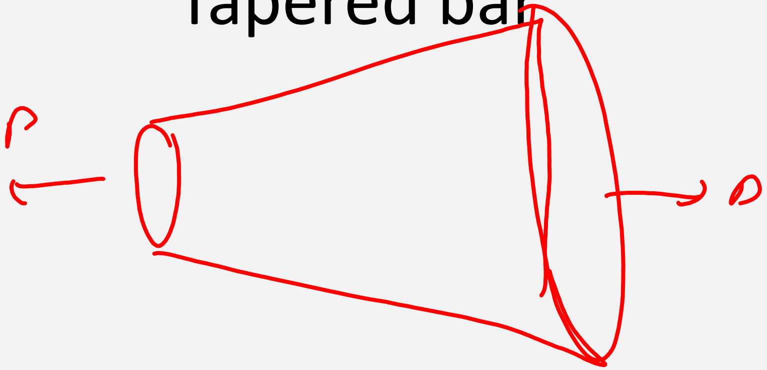
$$\sigma = \varepsilon E$$

$$\sigma \propto \varepsilon$$

$$\frac{P}{A} = \frac{\Delta L}{L} E$$

$$\Delta L = \frac{PL}{AE}$$

Tapered bar



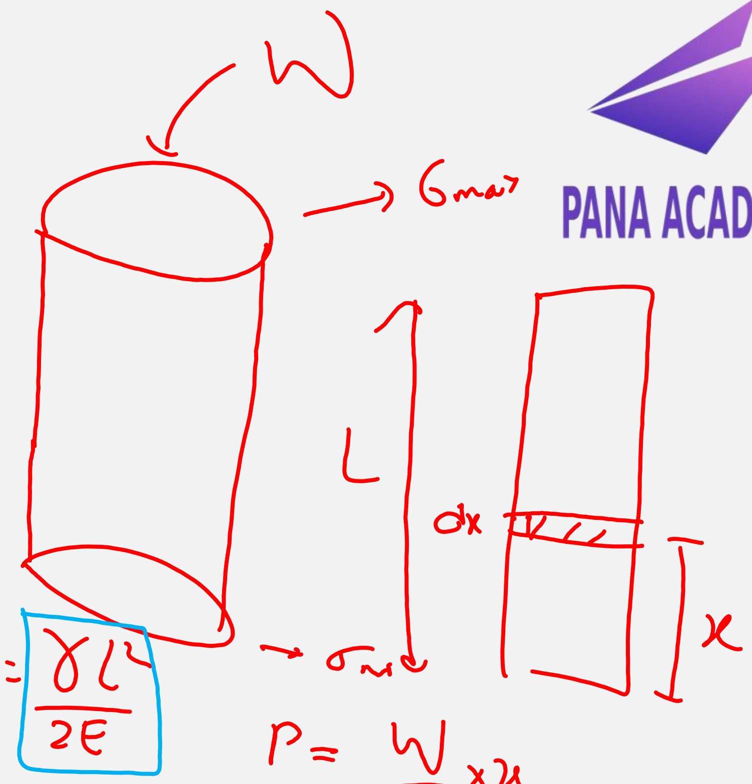
$$e = \frac{4PL}{\pi d_1 d_2 E}$$

Elongation due to self weight

PANA ACADEMY

Cylindrical bar

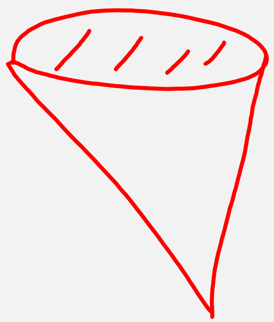
$$e = \frac{WL}{2AE} = \frac{\gamma V L}{2AE} = \frac{\gamma A L \times L}{2AE} = \frac{\gamma L^2}{2E}$$



$$P = \frac{W}{L} \times x$$

$$e_x = \frac{Pl}{AE} = \frac{\frac{Wx}{L}\,dx}{AE}$$

Conical bar



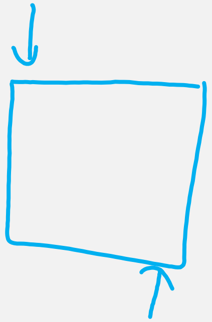
$$e = \frac{WL}{2AE} = \frac{\gamma V L}{2AE} = \frac{\gamma \times \frac{1}{3} A \times L \times L}{2AE} = \frac{\gamma L^2}{6E}$$

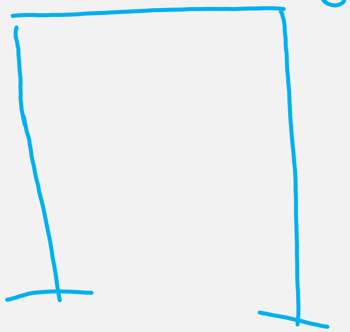
Principle stress

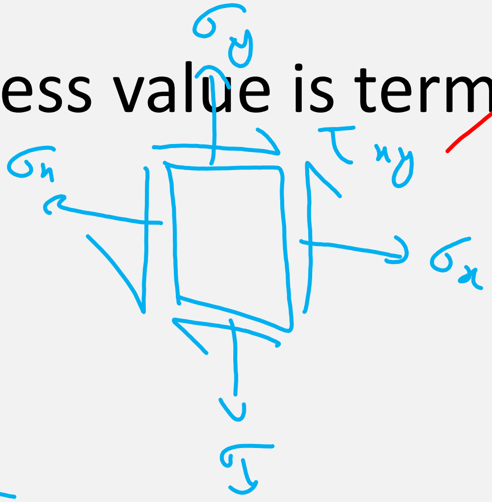
Principal stress is the maximum or minimum normal stress which may be developed on a loaded body.

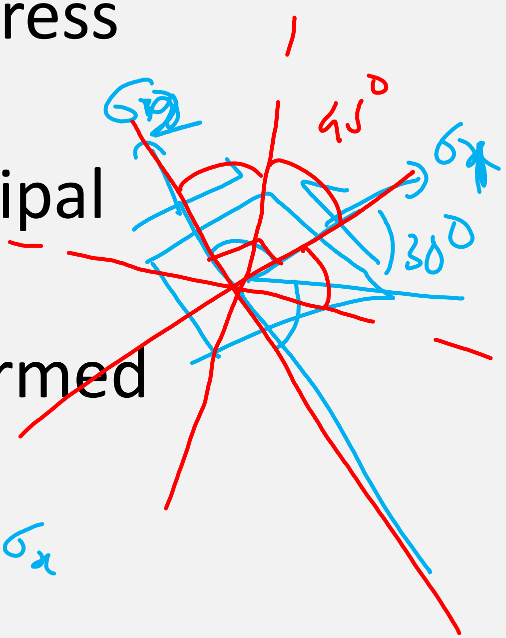
It is classified as major principal stress and minor principal stress.

On the plane of principal stress shear stress value is termed as zero.









Principle stress

PANA ACADEMY

When principal stress gets on the major principal plane, it is called major principal stress and when it is found on the minor principal plane, it is known as minor principal stress.

These are denoted as σ1 and σ2, respectively.

Principle stress

PANA ACADEMY

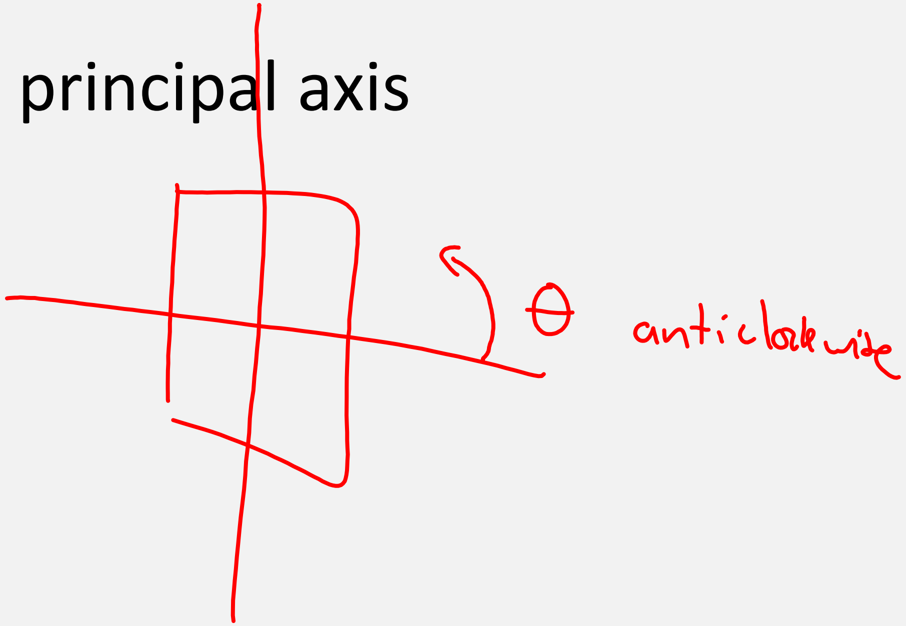
σ_x = Stress in x direction

σ_y = Stress in the y direction

τ_n = normal shear stress

Θ = inclination angle of stress to the principal axis

X and Y are the axes of the plane.



Principle stress (mohr circle)

PANA ACADEMY

At principle plane
Shear stress is 0
$\tau_{x'y'} = 0$

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

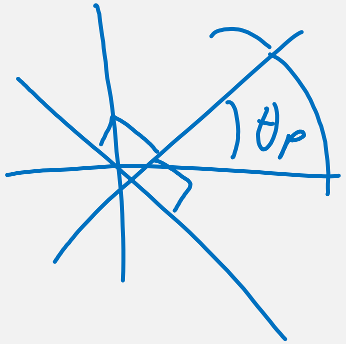
$$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = R$$

R → radius of Mohr circle

$\sigma_{x'} = \sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} = \frac{\sigma_1 + \sigma_2}{2}$

$$tan2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$



Principle stress

PANA ACADEMY

$$\sigma'_x = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} cos2\theta + \tau_{xy} sin2\theta$$

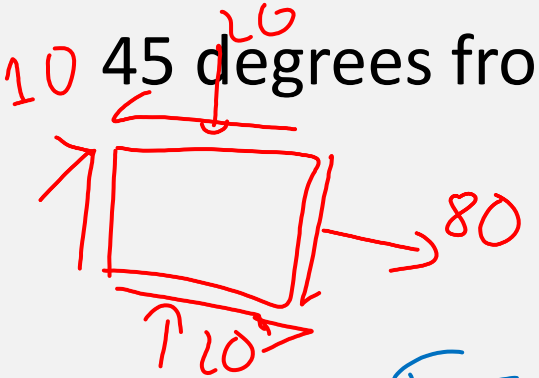
$$\sigma'_y = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2} cos2\theta - \tau_{xy} sin2\theta$$

$$\tau'_{xy} = -\frac{\sigma_x - \sigma_y}{2} sin2\theta + \tau_{xy} cos2\theta$$

θ is counter clockwise rotation

Plane of maximum shear

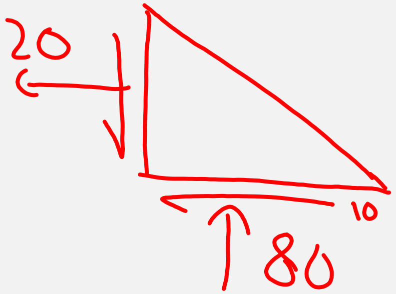
- 45 degrees from principle plane



$\sigma_x = 80$

$\sigma_y = -20$

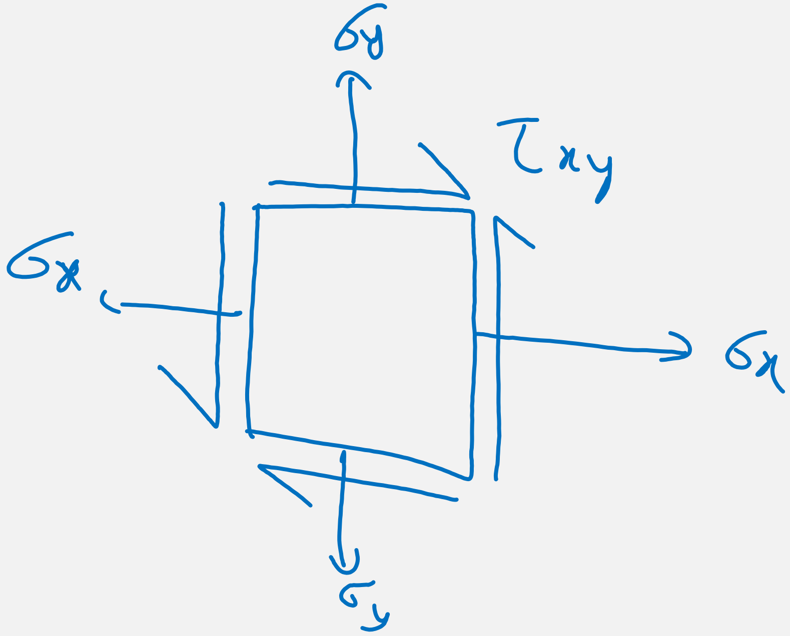
$\tau_{xy} = -10$



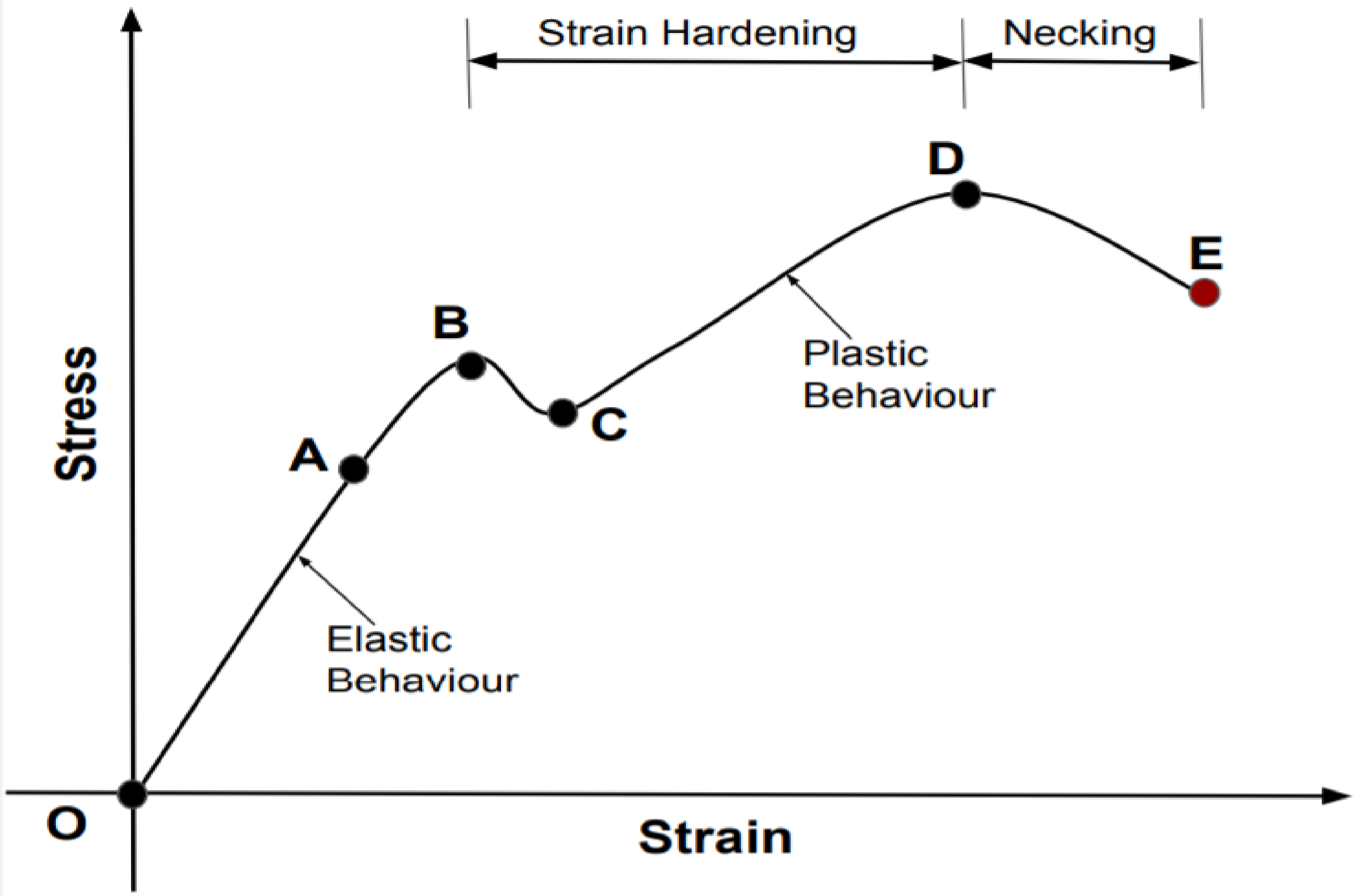
$\sigma_x = 20$

$\sigma_y = -80$

$\tau_{xy} = 10$



Stress-Strain Curve



OA : Proportional Limit
B : Upper Yield Stress Point
C : Lower Yield Stress Point
D : Ultimate Stress Point
E : Fracture

Stress-Strain Curve

PANA ACADEMY

Modulus of Resilience:

Signifies the ability of material to store or absorb energy without permanent deformation.

Modulus of Toughness:

Shows the ability of material to absorb energy upto fracture.

It is a measure of the impact loading a structure can withstand before failure.

MCQs

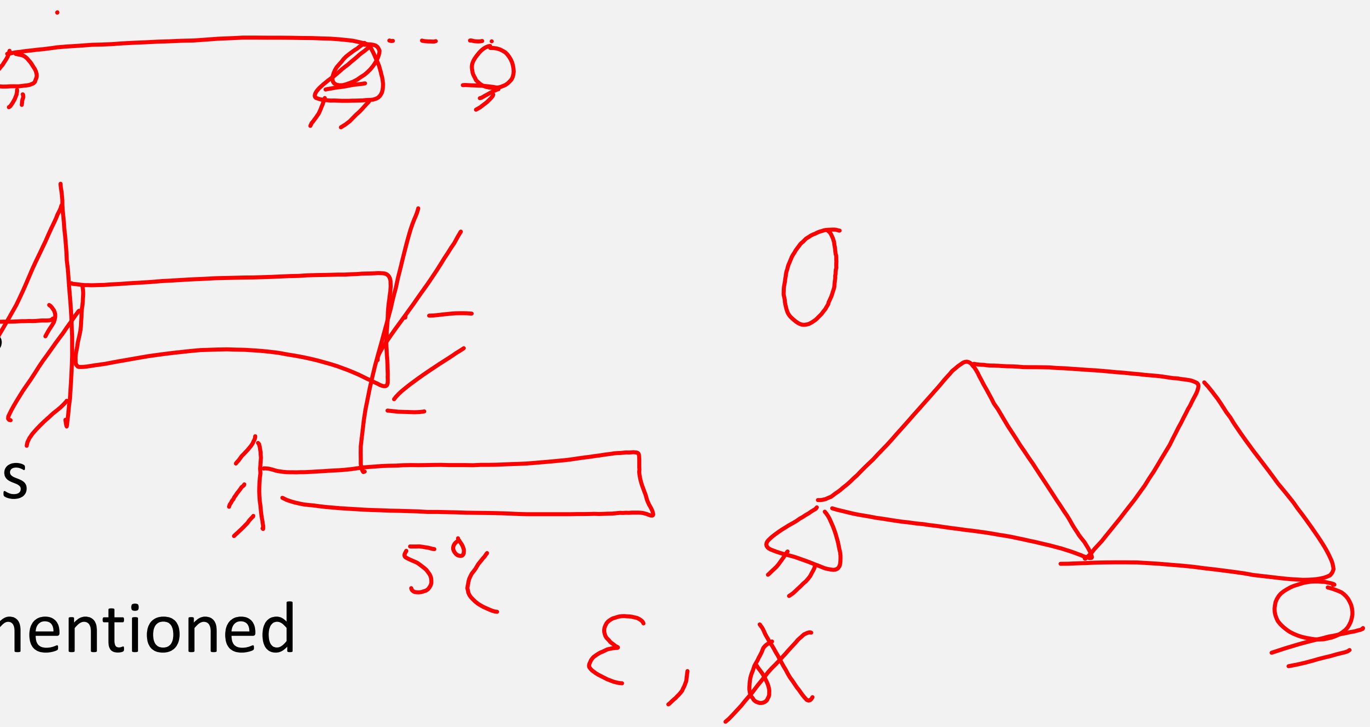
The stress which acts in a direction perpendicular to the area is called

a) Shear stress

b) Normal stress

c) Thermal stress

d) None of the mentioned



MCQs

PANA ACADEMY

Which type of stress does in a reinforced concrete is taken by the concrete?

a) Shear stress

b) Compressive stress

c) Tensile stress

d) Bending stress

MCQs

PANA ACADEMY

The stress induced in a body, when subjected to two equal and opposite forces which are acting tangentially across the resisting section resulting the shearing of the body across its section is called ______________

a) Bending stress

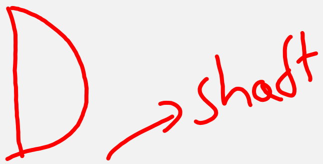
b) Compressive stress

c) Shear strain

d) Shear stress

MCQs

PANA ACADEMY



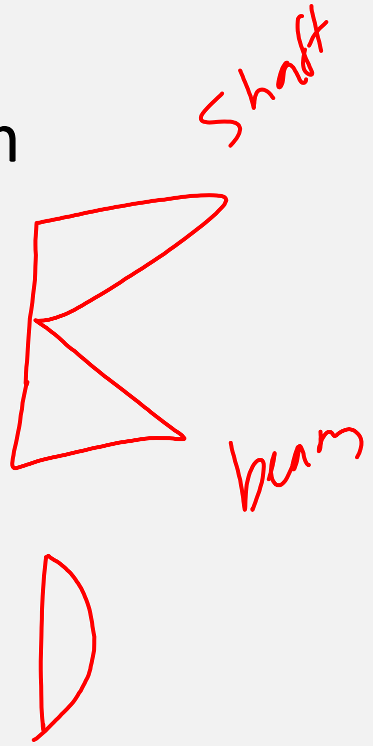
The transverse shear stress acting in a beam of rectangular cross-section, ~~subjected to a transverse shear load, is~~ ______________

a) variable with maximum at the bottom of the beam

b) variable with maximum at the top of the beam

c) Uniform

d) Variable with maximum on neutral axis ✓



MCQs

PANA ACADEMY

The phenomenon of slow growth of strain under a steady tensile stress is called

a) Yielding

b) Creeping → long time

c) Breaking ↳ fatigue → reversible load

d) None of the mentioned

MCQs

PANA ACADEMY

A rod 150cm long and of diameter 2cm is subjected to an axial pull of 20kN. What will be the stress?

a) 60 N/mm^2

b) 65 N/mm^2

c) 63.6 N/mm^2

d) 71.2 N/mm^2

$e = \frac{PL}{AL}$

$\sigma = \frac{P}{A} \rightarrow \frac{\pi d^2}{4}$

MCQs

PANA ACADEMY

The stress in a rod is 70 N/mm^2 and the modulus of elasticity is 2 x 10^5 N/mm^2. what will be the strain in the rod?

a) 0.00052

b) 0.00035

c) 0.00030

d) 0.00047

$\sigma = \varepsilon E$

N/mm^2

MCQs

A solid circular shaft of diameter d is subjected to a torque T. the maximum shear stress induced in the shaft is

a) 0

b) $16T/\pi d^3$

c) $32T/\pi d^3$

d) None

MCQs

PANA ACADEMY

A solid circular shaft of diameter d is subjected to a uniform moment M. The maximum bending stress induced in the shaft is

a) 0

b) $16M/\pi d^3$

c) $32M/\pi d^3$

d) $64M/\pi d^3$

MCQs

The maximum shear force for square beam of side 300mm, if shear force is 90 kN is

a) $1.00\ N/mm^2$

b) $1.33\ N/mm^2$

c) $1.50\ N/mm^2$ ✓

d) $3.00\ N/mm^2$

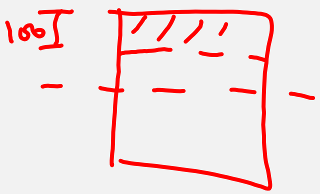
$\tau_{avg} = \frac{90 \times 1000}{300 \times 300} = 1\ N/mm^2$ ✗

$\tau_{max} = 1.5 \times 1 = 1.5$

PANA ACADEMY

MCQs

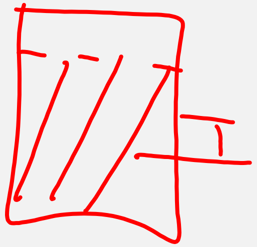
The shear force (stress) at 100mm form top for square beam of side 300mm, if shear force is 90 kN is



$\tau = \frac{VQ}{Ib}$ where $Q = A \times \bar{y}$

a) $1.00\ N/mm^2$

b) $1.33\ N/mm^2$ ✓



c) $1.50\ N/mm^2$

d) $3.00\ N/mm^2$

MCQs

For a plane stress problem, the minimum normal stress is

$\sigma_x = +80$

$\sigma_y = +20$

$\tau_{xy} = -25$

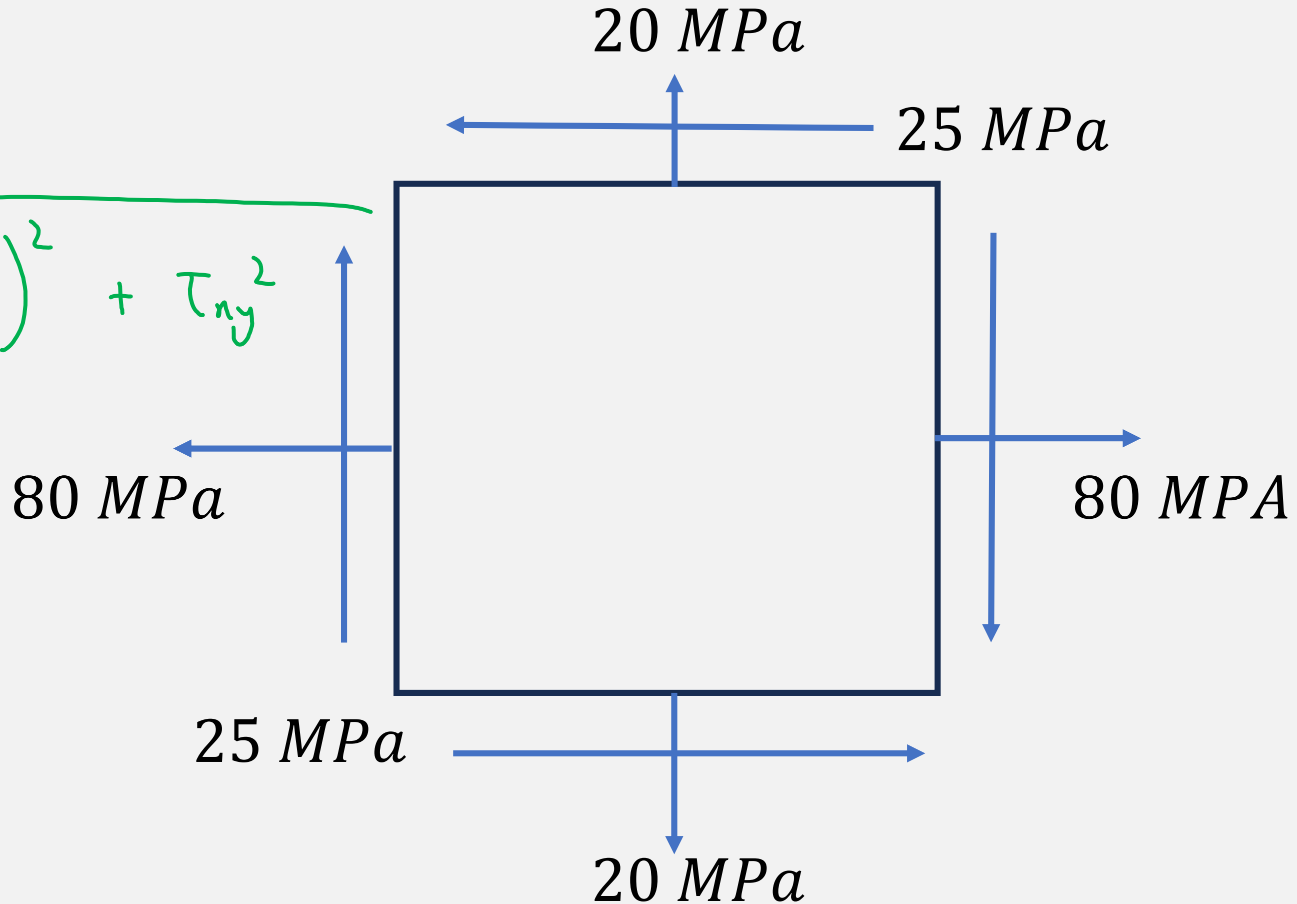
a) -10.9

b) 10.9 ✓

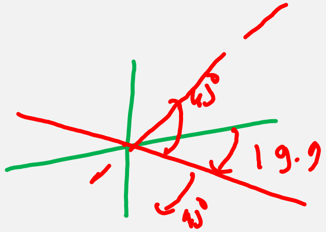
$$\sigma_2 = \frac{\sigma_x+\sigma_y}{2} - \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

c) 81.9

d) 100.0



MCQs



PANA ACADEMY

For a plane stress problem, how much angle in degree the stress element should be rotated in counter clockwise to get plane of maximum shear stress

a) 19.9

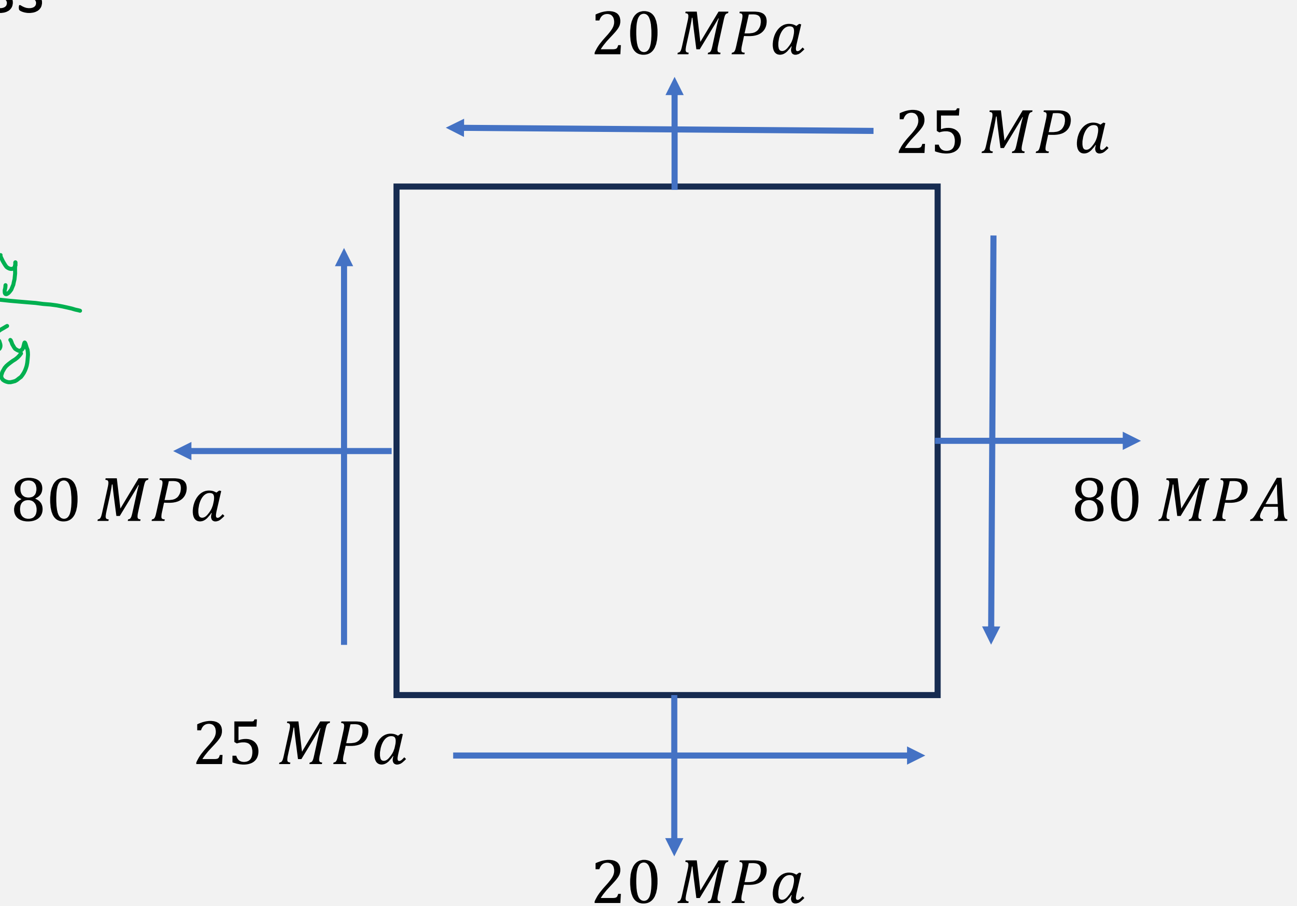
b) 25.1

c) 31.7

d) 48.3

$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$

$\theta_p = -19.9$



MCQs

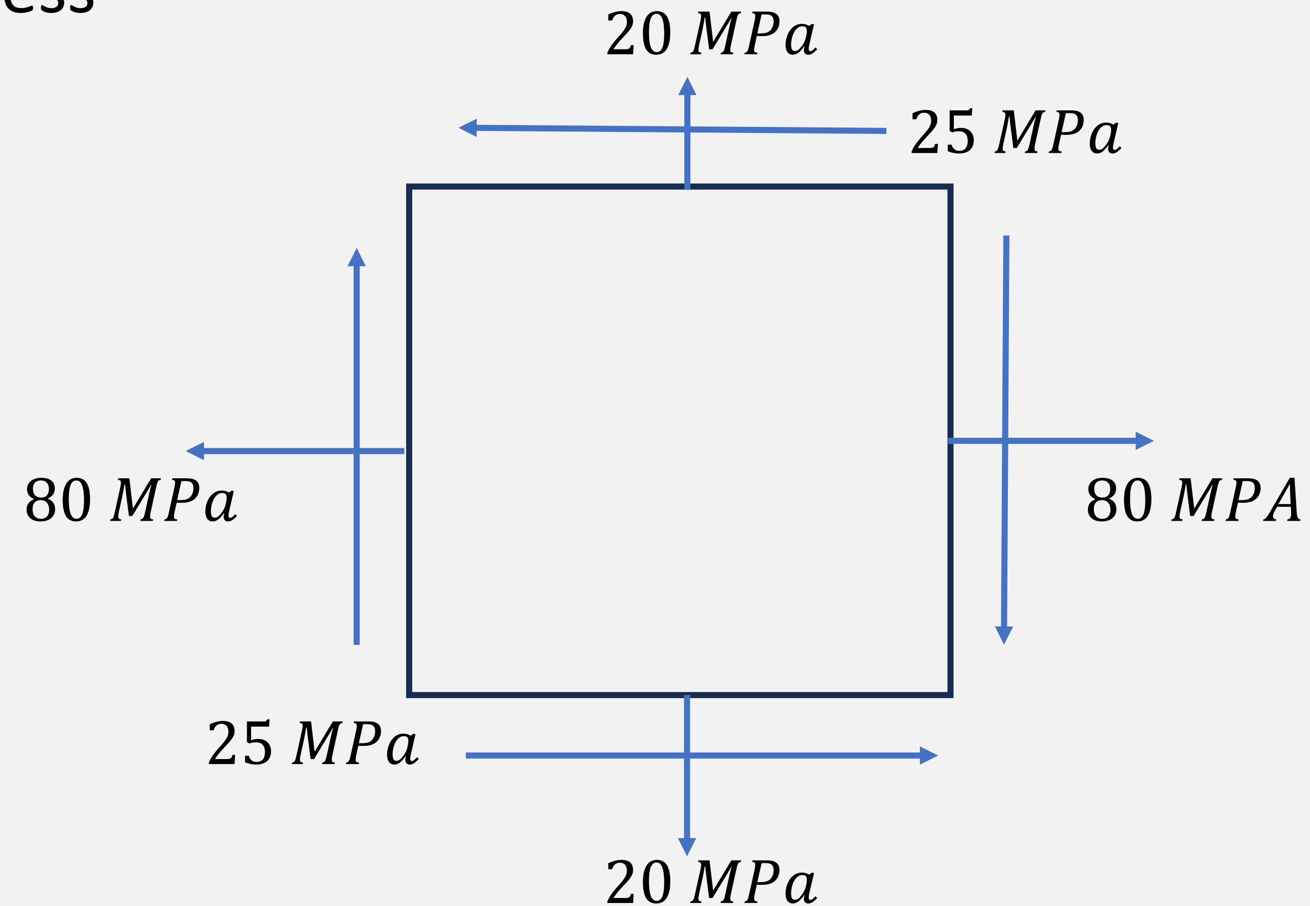
For a plane stress problem, how much angle in degree the stress element should be rotated in counter clockwise to get plane of minimum normal stress

a) -19.9

b) 25.1

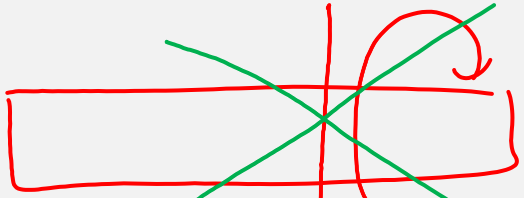
c) 31.7

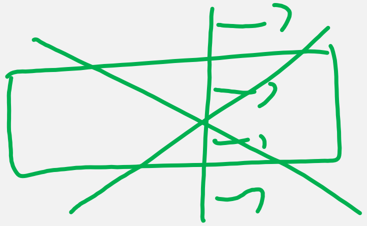
d) 70.1



MCQs

For a brittle material subjected to torque, failure surface occurs at





a) *perpendicular to axis*

b) *along the axis*

c) *at* 45^o *to axis* ✓

d) *none*

PANA ACADEMY

MCQs

PANA ACADEMY

For a brittle material subjected to tension, failure surface occurs at

a) $perpendicular\ to\ axis$

b) $along\ the\ axis$

c) $at\ 45^o\ to\ axis$

d) $none$

MCQs

For a ductile material subjected to tension, failure surface occurs at

a) $perpendicular\ to\ axis$

b) $along\ the\ axis$

c) $at\ 45^o\ to\ axis$

d) $none$

PANA ACADEMY

Thank YOU !!!