

3 marks () 1×2

NEPAL ENGINEERING COUNCIL LICENSE EXAM PREPARATION COURSE

FOR

CIVIL ENGINEERS



1. Basic Civil Engineering

1.4 Geometric properties of sections

Sub topics



- Axes of symmetry
 - Centre of gravity of different sections

1,2D,3D

- Moment of inertia (エ)
- Radius of gyration

$$R = \int \frac{1}{A}$$

Axes of symmetry



Axis of symmetry is a line that divides an object into two equal halves, two sides are mirror image to each other 6 ٦- ٢ M 41

Axes of symmetry (controind) (Shear centre)



Axis of symmetry is a line that divides an object into two equal halves





Centre of gravity of different sections

Centre of gravity: Imaginary point where weight of body is thought to be concentrated.

Centroid: $\langle \zeta \circ A \rangle$

Imaginary point where area is concentrated. Also, the centre of mass of an object of uniform density, especially of a geometric figure



Centroid of lines (1)



$$\overline{y} = \frac{\int y dL}{L} = \frac{\int y dL}{\int JL} = \frac{\int y dL}{\int JL} = \frac{\int x \int JL}{\int JL} = \frac{\int x \int JL}{\int JL} = \frac{\int y \int JL}{\int JL} = \frac{\int y \int JL}{\int JL} = \frac{\int x \int JL}{\int JL}$$

$$\overline{x} = \frac{\int x dL}{L} = \frac{\int x dL}{\int JL} = \frac{\int x \int JL}{\int JL} = \frac{\int x$$

Centroid of lines





$$y = x^2$$
$$x \in [0,2]$$

$$\frac{dy}{dx} = \frac{dx^2}{dx} = 2x,$$
$$dL = \left(\sqrt{1 + \left(\frac{dy}{dx}\right)^2}\right) dx$$
$$= \sqrt{1 + (2x)^2} dx$$
$$= \sqrt{1 + 4x^2} dx$$

Centroid of lines





$$\bar{y} = \frac{\int y\sqrt{1+4x^2} \, dx}{\int \sqrt{1+4x^2} \, dx} = \frac{\int x^2\sqrt{1+4x^2} \, dx}{\int \sqrt{1+4x^2} \, dx} = 1.82$$

$$\bar{x} = \frac{\int x\sqrt{1+4x^2} \, dx}{\int \sqrt{1+4x^2} \, dx} = 1.23$$

$$= \int x \, dL$$

$$\bar{x} = \frac{\int x \, dL}{\int \sqrt{1}}$$

$$\bar{y} = 1.82$$





The centroid of semi circular arch with center in origin is

(a)
$$x = 0; y = 2r/\pi$$

(b) $x = 2r/\pi; y = 2r/\pi$
(c) $x = 2r/\pi; y = 0$
(d) $x = 0; y = 0$



The centroid of semi circular arch with center in origin is

(b)
$$x = r; y = 2r/\pi$$

b) $x = 2r/\pi; y = 2r/\pi$
c) $x = 2r/\pi; y = 0$
d) $x = 2r/\pi; y = 0$





The centroid of semi circular arch with center in origin is

a)
$$x = 0; y = 2r/\pi$$

b) $x = 2r/\pi; y = 2r/\pi$
c) $x = 2r/\pi; y = 0$
d) $x = 0; y = 0$



The centroid of semi circular arch with center at (0, $2r/\pi$) is



Centroid of lines







The centroid path formed by semi circular arch (r=5) with center at origin and its diameter is

a)
$$x = 0; y = \frac{2r}{\pi + 2}$$
.
b) $x = 0; y = 2r/\pi$
c) $x = \frac{2r}{\pi + 2}; y = 0$
d) $x = \frac{2r}{\pi + 2}; y = 0$





The centroid path formed by semi circular arch with center in origin and its diameter is at







Centre of gravity of different sections

$$\bar{y} = \frac{\int \bar{y} dA}{A}$$
$$\bar{x} = \frac{\int \bar{x} dA}{A}$$

$$\bar{y} = \frac{\bar{y}_1 A_1 + \bar{y}_2 A_2}{A_1 + A_2}$$

$$\bar{x} = \frac{\bar{x}_1 A_1 + \bar{x}_2 A_2}{A_1 + A_2}$$

$$A = \int dA$$
$$dA = xdy = ydx$$





Centre of gravity of different sections

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$$\bar{y} = \frac{\int \bar{y} dA}{A}, \bar{x} = \frac{\int \bar{x} dA}{A} \qquad A = \int dA \\ dA = x dy = y dx$$

$$dA = y dx \\ A = \int dA = \int_0^b kx^n dx = k \frac{b^{n+1}}{n+1} = \frac{bh}{n+1} \qquad \int x \sqrt[n]{d} y$$

$$\int \bar{y} dA = \int \frac{y}{2} \cdot y dx = \int \frac{y^2 dx}{2} = \int_0^b \frac{k^2 x^{2n} dx}{2} \\ = \frac{1}{2} k^2 \frac{b^{2n+1}}{2n+1} = \frac{k^2 b^{2n} \cdot b}{4n+2} = \frac{h^2 b}{4n+2}$$



$$\overline{y} = \frac{\int \overline{y} dA}{A} = \frac{\frac{h^2 b}{4n+2}}{\frac{bh}{n+1}} = \frac{n+1}{4n+2}h$$





$$\overline{y} = \frac{\overline{y}_{1}A_{1} + \overline{y}_{2}A_{2}}{A_{1} + A_{2}}, \quad \overline{x} = \frac{\overline{x}_{1}A_{1} + \overline{x}_{2}A_{2}}{A_{1} + A_{2}}$$

$$a_{1} = \frac{\overline{y}_{1}A_{1} + \overline{y}_{2}A_{2}}{A_{1} + A_{2}}, \quad \overline{x} = \frac{\overline{x}_{1}A_{1} + \overline{x}_{2}A_{2}}{A_{1} + A_{2}}$$

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$$a_{1} = \frac{\overline{y}_{1}A_{2} + \overline{y}_{2}A_{2}}{A_{1} + A_{2}}, \quad \overline{x} = \frac{\overline{y}_{1}A_{2} + \overline{y}_{2}A_{2}}{A_{1} + A_{2}}$$

$$a_{1} = \frac{\overline{y}_{1}A_{2} + \overline{y}_{2}A_{2}}{A_{1} + A_{2}}, \quad \overline{y} = \frac{\overline{y}_{1}A_{2} + \overline{y}_{2}A_{2}}{A_{1} + A_{2}}$$

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$$a_{1} = \frac{\overline{y}_{1}A_{2} + \overline{y}_{2}A_{2}}{A_{1} + A_{2}}, \quad \overline{y} = \frac{\overline{y}_{1}A_{2} + \overline{y}_{2}}{A_{1} + A_{2}}, \quad \overline{y} = \frac{\overline{$$

а





The centroid of given shape

$$\bar{y} = \frac{\bar{y}_1 A_1 + \bar{y}_2 A_2}{A_1 + A_2} \quad \int -367.5$$

 \overline{y}_1 =250, A_1 =100*500 \overline{y}_2 =500+50, A_2 =300*100

362.5 from bottom





The centroid of given shape

$$\bar{y} = \frac{\bar{y}_1 A_1 = \bar{y}_2 A_2}{A_1 - A_2} = \frac{\zeta_6 2 \cdot \zeta_m}{\zeta_6 \cdot \zeta_m}$$

 \bar{y}_1 =600/2, A_1 =300*600 \bar{y}_2 =500/2, A_2 =200*500

362.5 from bottom







Centre of weight of different volumes





Er. Dilendra Raj Panta



Centre of weight of different volumes





Second moment of area.

$$I_{x} = \int y^{2} dA = \sum y^{2} dA$$
Moment of inertia is defined as the quantity expressed by
the body resisting angular acceleration which is the sum
of the product of the mass of every particle with its
square of a distance from the axis of rotation.

$$\int Always positive$$

Er. Dilendra Raj Panta

PANA A



Product moment of area is

$$\underline{I_{xy}} = \int xydA = \sum xydA$$

Principle axis – product moment of inertia is zero (I_{uv})

fxy2





Parallel axis theorem:

$$I_{xx} = I_{cg} + Ah^2$$

Perpendicular axis theorem:

$$I_{zz} = I_{xx} + I_{yy}$$

 I_{zz} is also termed as polar moment of inertia

Shape	Centroid	Moment of inertia	
	$\begin{array}{l} \bar{x} = b/2\\ \bar{y} = h/2 \end{array}$	$I_{\bar{x}\bar{x}} = \frac{bh^3}{12}$	PANA ACADEI
	$\bar{x} = b/2$ $\bar{y} = h/2$	$I_{xx} = \frac{bh^3}{12} + bh * \left(\frac{h}{2}\right)^2$ $= \frac{bh^3}{3}$	

$$\frac{1}{I_{b}} = \frac{1}{I_{b}} = \frac{1}{I_{b}}$$

Shape	Centroid	Moment of inertia
	$\frac{\bar{x} = b/2}{\bar{y} = h/3}$	$I_{\bar{x}\bar{x}} = \frac{bh^3}{36}$
$\int d = h/$	$\bar{x} = b/2$ $\bar{y} = h/3$	$I_{xx} = \frac{bh^3}{36} + \frac{1}{2}bh * \left(\frac{h}{3}\right)^2 \\ = \frac{bh^3}{12}$
Aper 1 1 1 1 1 1 1 1 1 1 1 1 1	$ \bar{x} = b/2 \bar{y} = h/3 $	$I_{xx} = \frac{bh^3}{36} + \frac{1}{2}bh * \left(\frac{2h}{3}\right)^2 \\ = \frac{bh^3}{4}$
	Apex: 694 = 3:1	















Section modulus of hollow circular section



Moment of inertia π^{α} , $\pi^{\alpha'}$, $\pi^{\alpha'}$



$$I_{2x} = \frac{1}{12}bh^3 + (bh)d^2$$

= $\frac{1}{12}(30)(65)^3 + (30 \times 65)(42.5)^2 = 4.209 \times 10^6 \text{ mm}^4$

$$I_{3x} = \frac{1}{36}bh^3 + (\frac{1}{2}bh)d^2$$

= $\frac{1}{36}(30)(42)^3 + (\frac{1}{2}30 \times 42)(47)^2 = 1.453 \times 10^6 \text{ mm}^4$

$$I_x = I_{1x} + I_{2x} + I_{3x} = 7.626 \times 10^6 \text{ mm}^4$$







MOI of complex shape











About Base:

$$I_{1} = I_{CG_{1}} + A_{1}d_{1}^{2}$$
$$I_{CG_{1}} = \frac{100 \times 40^{3}}{12}$$
$$A_{1}d_{1}^{2} = 100 \times 40 \times \left(150 + \frac{40}{2}\right)$$

2

$$I_{2} = I_{CG_{2}} + A_{2}d_{2}^{2}$$

$$I_{CG_{2}} = \frac{40 \times 150^{3}}{12}$$

$$A_{1}d_{1}^{2} = 40 \times 1500 \times \left(\frac{150}{2}\right)^{2}$$

 $I = I_1 + I_2 \quad \textbf{z}$

Er. Dilendra Raj Panta



Radius of gyration



Radius where entire mass is thought to be concentrated

such that ,moment of inertia is same. MULTO $I = AR^2$ MLT $R = \sqrt{\frac{I}{A}}$ As, $I_z = I_x + I_y$, $AR_z = AR_z + AR_y$, $R_z = R_z + AR_y$, $R_z = R_z + AR_y$, $R_z^2 = R_x^2 + R_v^2$

Mass moment of inertia







A circular section consists of

- a) No line of symmetry
- b) One line of symmetry
- c) Two line of symmetry
- Infinite line of symmetry



The center of gravity of a circle of radius 20 cm will be

a. At the center of the radius
b. At its center of the diameter
c. Anywhere on the circumference
d. Anywhere in its area



In case of an area, the figure is assumed to be a lamina of negligible thickness so that its centre of gravity will be practically on the surface. As the area has no weight, this point is also called the.

a. moment of inertia of areas . b. neutral axis centroid d. None



The point through which the whole weight of the body acts is called

a. inertial point
center of gravity
c. central point
d. centroid



The centre of _____ is the ratio of the product of centroid and volume to the total volume. $\int \frac{\sqrt{y}}{\sqrt{y}} = \int \frac{\sqrt{y}}{\sqrt{y}} dy$ a) Centroid axis b) Density c) Mass d) Volume





Point, where the total volume of the body assumed to be concentrated is.

a. center of area b. centroid of volume c. centroid of mass d. all of the mentioned



The Moment of inertia of a circular section having diameter 10 mm is a. 490.6 mm⁴ b. 480.6 mm⁴ c. 981.2 mm⁴ d 961.2 mm⁴



The composite section of area 250 mm² has centroidal axis 8mm from the bottom. The moment of inertia about centroidal x-axis is 6250 mm⁴. The moment of inertia about the base is

a. 9750 mm⁴ min \rightarrow (entroidal axis b. 22250 mm⁴ c c. 8250 mm⁴ Jene d. 16000 mm⁴



The radius of gyration along x and y axis are 30 mm and 40 mm respectively, the radius of gyration along the axis perpendicular to given axis is $\gamma T = \alpha \gamma T$

a. 70 mm b. 60 mm c. 50 mm d. 10 mm





Thank YOU !!!