4.1. GENERAL

4.1.1. Properties of fluid: mass, weight, specific weight, density, specific volume, specific gravity, viscosity

1.1.Continuum:

In dealing with fluid flow relations on a mathematical or analytical basis, it is necessary to consider that the actual molecular structure is replaced by a hypothetical continuous medium, called the continuum.

1.2.Fluid:

A fluid is a substance that deforms continuously when subjected to a shear stress, no matter how small the shear stress may be.

Shear force is the force component tangent to the surface, and this force divided by area of the surface is the average shear stress over the area. Shear stress at a point is the limiting value of shear force to area as area is reduced to the point.

1.3.Newton's Experiment:

A substance is placed between two closely spaced parallel plates so large that conditions at their edges may be neglected.

The lower plate is kept fixed.

Force F is applied to upper plate, if the force causes the upper plate to move with steady velocity, no matter how small the magnitude of F, the substance between two plates is a fluid.

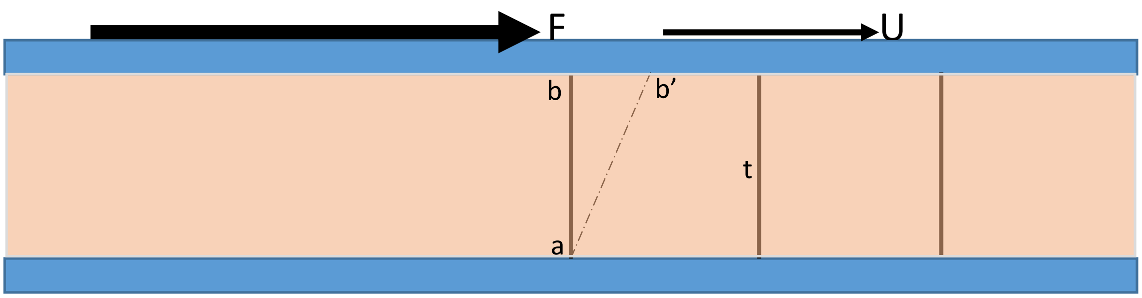


Figure 1 Newtons Experiment

F is directly proportional to area of contact A, steady velocity U and inversely proportional to the thickness between the plate t.

$F \propto \frac{AU}{t}$; $F = \mu\frac{AU}{t}$

μ is proportionality factor and includes the effect of particular fluid.

Shear Stress $(\tau) = \frac{F}{A} = \mu\frac{U}{t}$

The velocity at upper layer (adjacent to upper plate) of the fluid will be same as the velocity of upper plate, i.e. U. Similarly, velocity at lower layer (adjacent to lower plate) of the fluid will be same as velocity of lower plate, i.e. 0 (as lower plate is fixed). This phenomenon at the boundary of the fluid is known as **NO SLIP CONDITION**. This variation in velocity creates a velocity gradient in fluid.

The ratio U/t is the angular velocity of line ab, or it is the rate of angular deformation of the fluid. Angular velocity can also be written as; du/dy

du/dy is also known as velocity gradient.

$$\tau = \mu\frac{du}{dy}$$

It is the relation between shear stress and rate of angular deformation for the one-dimensional fluid flow. Proportionality factor μ is called the viscosity of fluid.

Fluids are classified as Newtonian and non-Newtonian. In Newtonian fluid, there is a linear relation between the magnitude of applied shear stress and the resulting rate of deformation.

$$\tau = \mu\frac{du}{dy}$$

In non-Newtonian fluid, there is a nonlinear relation between the magnitude of applied shear stress and the rate of angular deformation.

$\tau = \mu(\frac{du}{dy})^n$; Where n ≠1

An ideal plastic has definite yield stress and a constant linear relation of τ *and* $\frac{du}{dy}$

$$\tau = \tau_0 + \mu\frac{du}{dy}$$

A thixotropic substance such as printers ink, has viscosity that is dependent upon the immediately prior angular deformation of the substance and has tendency to solidify at rest

Gases and most common liquid tend to be Newtonian fluids; long chained hydrocarbons may be non-Newtonian.

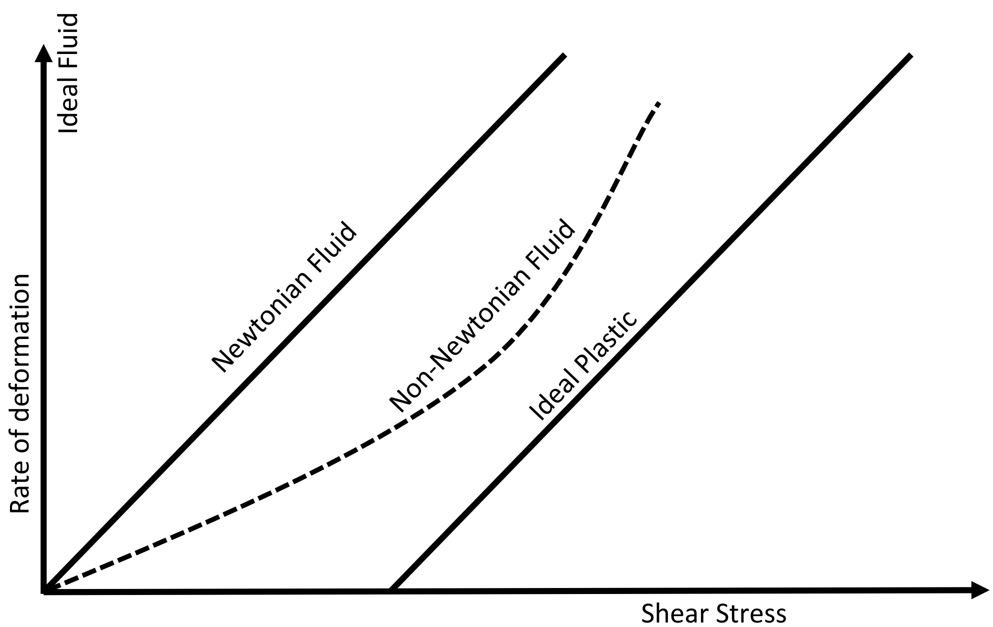


Figure 2 Newtonian and Non-Newtonian Fluids

1.4.Dimensions and Units

System	Mass	Length	Time	Force	Temperature
SI	kg	m	s	N	K
Metric, cgs	g	cm	s	dyn	K
Metric, mks	kg	m	s	kgf	K

1.5. Prefixes for powers of 10 SI units

Multiple	SI Prefix	Abbreviation
10^9	Giga	G
10^6	Mega	M
10^3	Kilo	k
10^{-2}	Centi	c
10^{-3}	Milli	m
10^{-6}	Micro	µ
10^{-9}	nano	n
10^{-12}	pico	p

1.6. Viscosity:

Viscosity is the property of fluid by virtue of which it offers resistance to shear. Newton's law of viscosity states that for a given rate of angular deformation of fluid the shear stress is directly proportional to viscosity.

Highly viscous: Molasses and Tar

Low Viscous: Water and air

Viscosity depends on the cohesion and rate of transfer of molecular momentum of fluid.

Cohesion appears to be the predominant cause of viscosity in liquid. So with increase in temperature, viscosity of liquid decreases.

Transfer of molecular momentum appears to be the predominant cause of viscosity in gases. So with increase in temperature, viscosity of gases also increases.

1.7. Absolute Viscosity

Also known as 'dynamic viscosity' of 'coefficient of viscosity' or simply as 'viscosity'.

$$\mu = \frac{\tau}{du/dy}$$

Unit of shear stress: N/m^2

Unit of velocity gradient: $\frac{m/s}{m} = \frac{1}{s}$

Unit of absolute viscosity = $N.s/m^2$

Parameter	Unit (SI)	IN cgs	IN FLT Dimension	IN MLT Dimension
Force (F)	N	dyn	F	MLT^{-2}
Area (A)	m^2	cm^2	L^2	L^2
Shear Stress (τ = F/A)	N/m^2	dyn/cm^2	FL^{-2}	$ML^{-1}T^{-2}$
Velocity gradient (du/dy)	1/s	1/s	T^{-1}	T^{-1}
Viscosity $\mu = \frac{\tau}{du/dy}$	$N.s/m^2$	$dyn.s/cm^2$	$FL^{-2}T$	$ML^{-1}T^{-1}$

A common unit of viscosity in cgs unit is called poise (P) = 1 $dyn.s/cm^2$. The SI unit is 10 times larger than the poise unit.

The dynamic viscosity of water is 8.90×10^{-4} Pa.s or 8.90×10^{-3} dyn. s/cm^2 or 0.890 cP at about 25 °C.Water has a viscosity of 0.0091 poise at 25 °C, or 1 centipoise at 20 °C.

1.8. Kinematic Viscosity:

Ratio of absolute viscosity to density of the fluid.

The equation is written as;

$\nu = \mu / \rho$

We have:
ν: Kinematic viscosity
ρ: fluid density
μ: Dynamic viscosity

Parameter	Unit (SI)	IN cgs	IN FLT Dimension	IN MLT Dimension
Absolute Viscosity **μ**	$N.s/m^2$	$dyn.s/cm^2$	$FL^{-2}T$	$ML^{-1}T^{-1}$
Density (ρ)	kg/m^3	g/cm^3	ML^{-3}	ML^{-3}
Kinematic Viscosity (ν)	m^2/s	cm^2/s	L^2T^{-1}	L^2T^{-1}

CGS unit for kinematic viscosity is the stokes (St), named after Sir George Gabriel Stokes.
The kinematic viscosity of water is 1.004×10^{-6} m^2/s at 20 °C.

1.9.Density:

The density ρ of a fluid is defined as its mass per unit volume.
For water at standard pressure (760 mm of Hg) and 4 degree Celsius of temperature, density is 1000 kg/m^3.

1.10. Specific Volume:

The specific volume is the reciprocal of the density, It is the volume occupied by unit mass of the fluid.
$v_s = 1/\rho$
Unit is m^3/kg.

1.11. Specific Weight:

The Specific weight γ of the fluid is its weight per unit volume. It changes with locations, depending on gravity.
$\gamma = \rho g$
The unit of specific gravity is N/m^3. The specific weight of water is generally taken as 9810 N/m^3 OR 9.81 KN/m^3

1.12. Specific Gravity (S of G)

The specific gravity of substance is the ratio of its weight to the weight of an equal volume of water at standard condition.

$G\ or\ S = \left(\frac{weight\ of\ substance}{\text{weight of an equal volume of water}}\right)$ at standard condition.

$$G\ or\ S = \left(\frac{density\ of\ substance}{\text{density of water}}\right)$$

$$G\ or\ S = \left(\frac{specific\ weight\ of\ substance}{\text{specific weight of water}}\right)$$

Specific gravity is the dimensionless quantity.

4.1.2. Pressure and Pascal's law

1.13. Fluid Statics

Considering Newton's second law, that is, d(mv)/dt = F.
Static is the case where d(mv)/dt = 0.
This can be achieved when either the fluid is at rest or velocity is constant.
There will be no relative motion of adjacent fluid layers, and consequently the shear stresses are zero
Therefore, only normal or pressure forces are considered acting on the fluid surfaces.

1.14. Fluid Pressure at a Point

Pressure or intensity of pressure may be defined as the force exerted on a unit area. If F represents the total force uniformly distributed over the area A, the pressure at any point in p = F/A.
Magnitude of pressure at any point, $p = \frac{dF}{dA}$
Unit of Pressure is N/m^2 (OR Pa) in SI units.
In metric gravitational units it is expressed in $kg(f)/cm^2$ of $kg(f)/m^2$

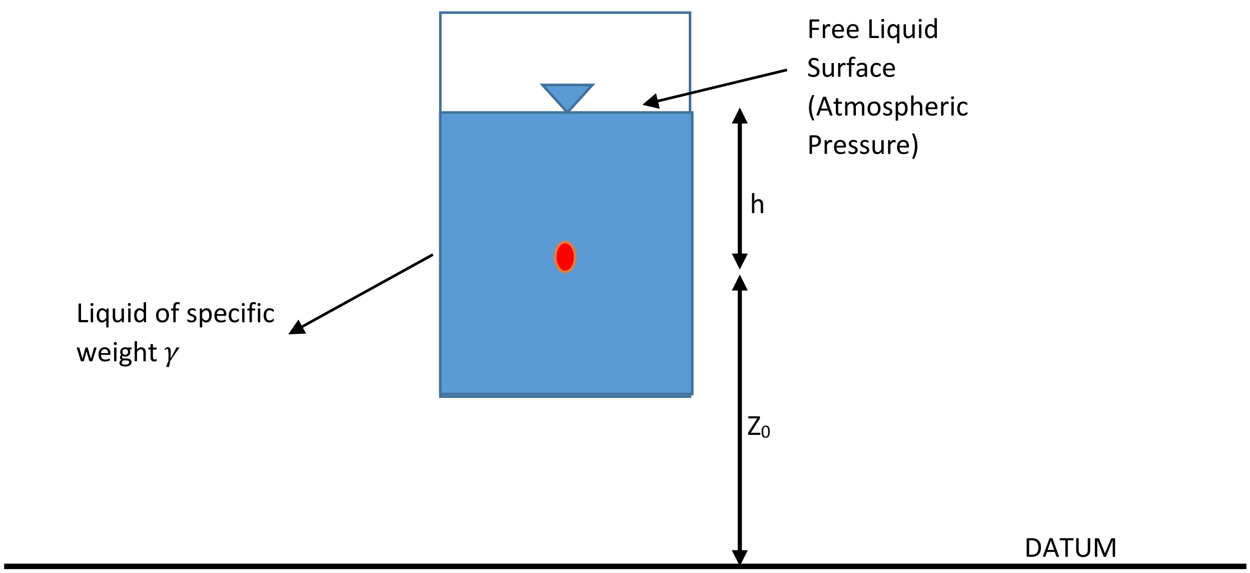


Figure 3 Concept of Pressure

1.15. Variation of pressure in a fluid

$$p = p_a + \gamma h$$

Where p_a is atmospheric pressure, Since the atmospheric pressure at a place is constant, at any point in a static mass of liquid, often only the pressure in excess of the atmospheric pressure is considered,
So,

$$p = \gamma h$$

1.16. Pressure Head

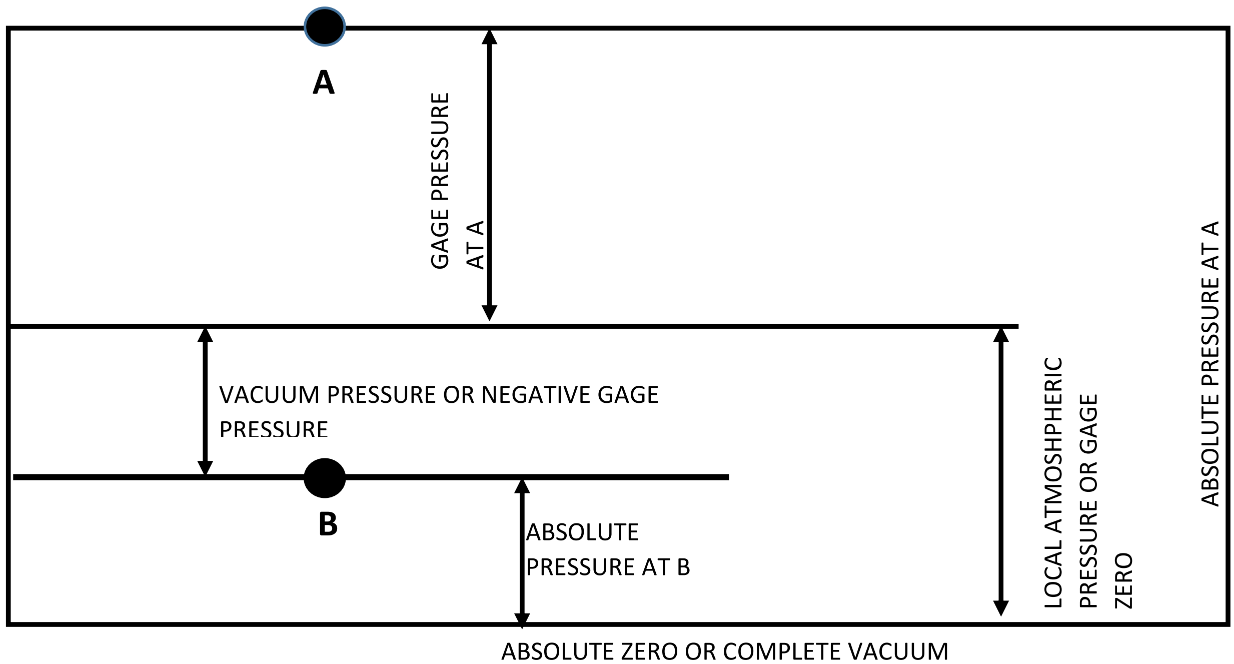
The vertical height of the free surface above any point in a liquid at rest is known as pressure head.

$$h = \frac{p}{\gamma}$$

1.17. Pascal Law

The pressure at any point in a fluid at rest has same magnitude in all directions.

1.18. Atmospheric, Absolute and Gage/Vacuum Pressure



$$Absolute\ Pressure = Atmospheric\ Pressure + Gage\ Pressure$$
$$Absolute\ Pressure = Atmospheric\ Pressure - Vacuum\ Pressure$$

Atmospheric Pressure = $10.1043 * 10^4$ N/m^2 OR 1.03 kg(f)/cm^2 OR 10.3 m of water OR 76 cm of Mercury.

1.19. Measurement of Pressure:

1. Manometers
2. Mechanical Gages

Manometers are those pressure measuring devices which are based on the principle of balancing column of liquid (which pressure is to be found) by the same or another column of liquid.

- Simple Manometers
  - ✓ Piezometer
  - ✓ U tube Manometer
  - ✓ Single Column Manometer

Simple manometer consists of a glass tube having one of its ends connected to the gage point where pressure is to be measured and the other remains open to atmosphere.

- Differential Manometers
  - ✓ Two Piezometer Manometer
  - ✓ Inverted U tube Manometer
  - ✓ U tube differential Manometer
  - ✓ Micromanometer

For measuring difference of pressure between any two points in a pipeline or in two pipes or containers.

Micromanometer is used to measure very small pressure differences, or for the measurement of pressure differences with very high precision.

Some Important Points

- ✓ The manomeric liquid should have high density and low vapor pressure.
- ✓ A simple U tube manometer can measure both negative and positive pressures.

1.20. Forces on Plane Areas:

$$Pressure\ Force\ (P)\ = \gamma A\bar{X}$$

Where,

$\gamma = Specific\ weight\ of\ fluid$
$A = Plane\ surface\ in\ horizontal\ position$
$\bar{X} = vertical\ height\ of\ CG\ from\ fluid\ surface$
$\bar{h} = vertical\ height\ of\ CP\ from\ fluid\ surface$

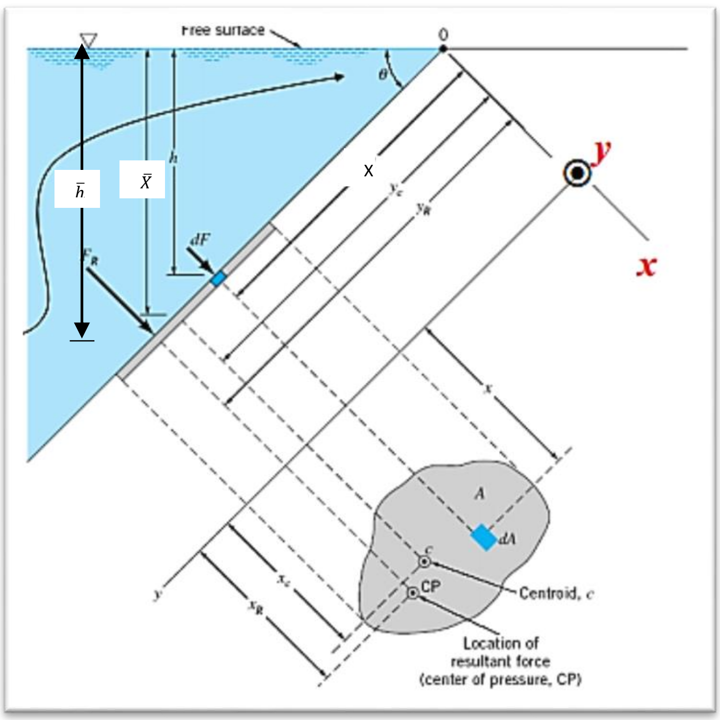
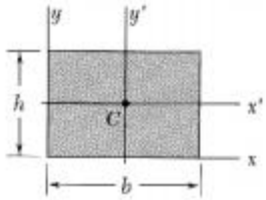
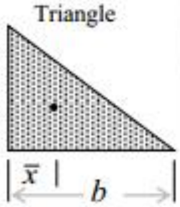
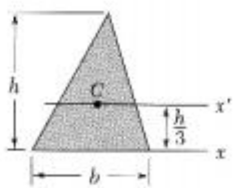
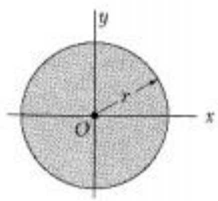
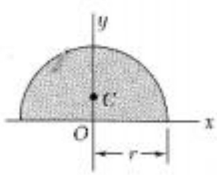
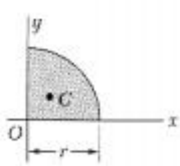


Figure 4 Hydrostatic Force on Submerged Body

1.21. Center of Pressure

The line of action of resultant force has its piercing point in the surface at a point called center of pressure.

$$\bar{h} = \bar{X} + \frac{I_G * sin^2\theta}{A\bar{X}}$$

Rectangle		$\bar{I}_{x'} = \frac{1}{12}bh^3$ $\bar{I}_{y'} = \frac{1}{12}b^3h$ $I_x = \frac{1}{3}bh^3$ about bottom left $I_y = \frac{1}{3}b^3h$ $J_C = \frac{1}{12}bh(b^2 + h^2)$	Area = bh $\bar{x}$ = b/2 $\bar{y}$ = h/2
Triangle 		$\bar{I}_{x'} = \frac{1}{36}bh^3$ $I_x = \frac{1}{12}bh^3$ about bottom $\bar{I}_{y'} = \frac{1}{36}b^3h$	Area = $bh/2$ $\bar{x} = b/3$ $\bar{y} = h/3$
Circle		$\bar{I}_x = \bar{I}_y = \frac{1}{4}\pi r^4$ $J_O = \frac{1}{2}\pi r^4$	Area = $\pi r^2 = \pi d^2/4$ $\bar{x}$ = 0 $\bar{y}$ = 0
Semicircle		$\bar{I}_x = 0.1098r^4$ $\bar{I}_y = \pi r^4/8$	Area = $\pi r^2/2 = \pi d^2/8$ $\bar{x}$ = 0 $\bar{y} = 4r/3\pi$
Quarter circle		$\bar{I}_x = 0.0549r^4$ $\bar{I}_y = 0.0549r^4$	Area = $\pi r^2/4 = \pi d^2/16$ $\bar{x} = 4r/3\pi$ $\bar{y} = 4r/3\pi$

1.22. Buoyancy

1.23. Buoyant Force

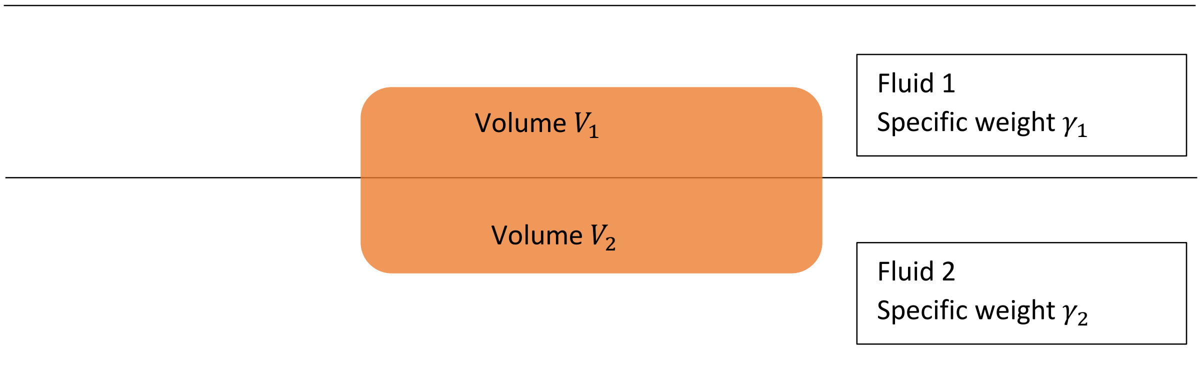
The resultant force exerted on a body by a static fluid in which it is submerged or floating.

The buoyant force always acts vertically upward, there can be no horizontal component of the resultant because the projection of the submerged body or submerged portion of the floating body on the vertical plane is always zero.

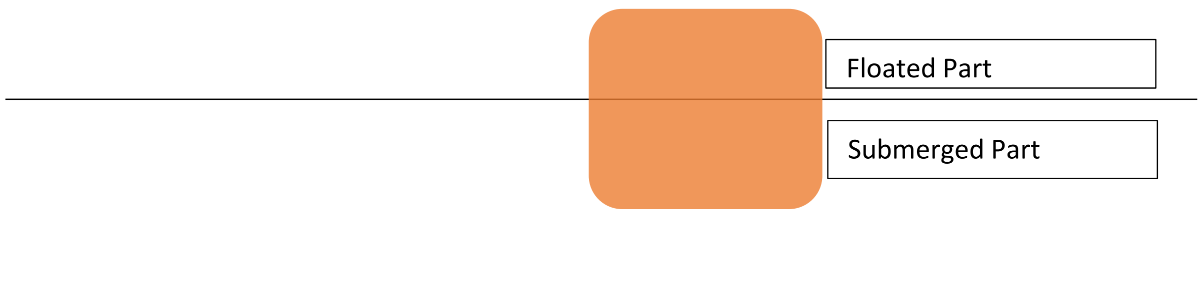
The buoyant force on ta submerged body is the difference between the vertical component of pressure force on its underside and the vertical component of pressure on its upper side.

1.24. Archimedes' Principle:

When a body is immersed in a fluid either wholly or partially, it is buoyed or lifted up by a force which is equal to the weight of the fluid displaced by the body.



Buoyant Force (F_B) = $\gamma_1 * V_1 + \gamma_2 * V_2$



Buoyant Force (F_B) = Weight

A body floating in a static liquid has vertical stability. A small upward displacement decreases the volume of liquid displaced, resulting in an unbalanced downward force, which tends to return the body to its original position.
A body has linear stability when a small liner displacement is any direction sets up restoring forces tending to return it to its original position. It has rotational stability when a restoring couple is set up by any small angular displacement.

1.25. Metacenter

It is the point of intersection between an imaginary line drawn vertically through the center of buoyancy of a floating vessel and a corresponding line through the new center of buoyancy when the vessel is tilted.

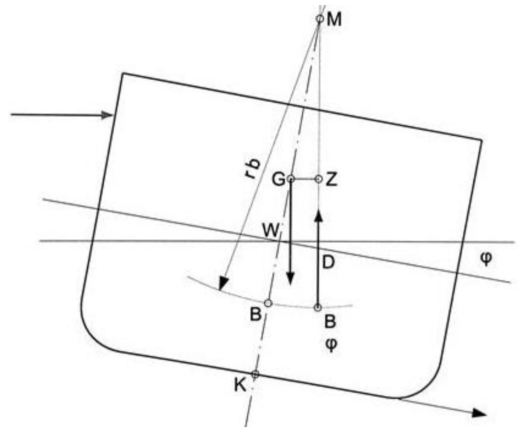


Figure 5 Metacenter

M- Metacenter
G- Center of Gravity
B- Center of Buoyancy

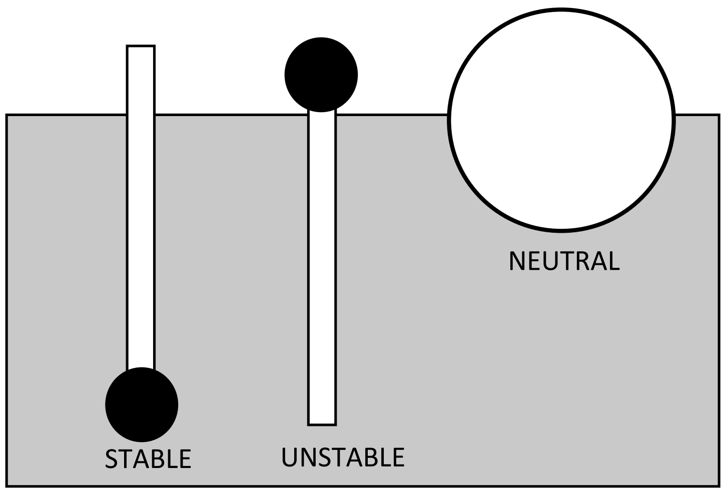


Figure 6 Types of Equilibrium

A completely submerged object is rotationally stable only when its center of gravity is below center of. When the object is rotated counterclockwise, the buoyant force and weight produce a couple in the clockwise direction.
A floating object with center of gravity below center of buoyancy floats in stable equilibrium. Certain floating objects however floats in stable equilibrium when their CG is above center of buoyancy.
$\overline{BM} > \overline{BG}$ $STABLE\ EQUILIBRIUM$
$\overline{BM} < \overline{BG}$ $UNSTABLE\ EQUILIBRIUM$
$\overline{BM} = \overline{BG}$ $NEUTRAL\ EQUILIBRIUM$
$\overline{GM} = \pm(\overline{BM} - \overline{BG}) = \pm(\frac{I}{V} - \overline{BG})$
I is Moment of inertia of cross sectional area at the liquid surface about longitudinal axis.
V is Volume of the liquid displaced.

4.2. Hydro-Kinematics and Hydro-Dynamics

4.2.1. Energy of flowing liquid: elevation energy, Kinetic energy, potential energy, internal energy

2.1. Fluid Kinematics

2.2. Kinematics

The nature of flow of a real fluid is very complex. The science, which deals with the geometry of motion of fluids without reference to the forces causing the motion, is kinematics.

2.3. Kinetics

Science, which deals with the action of the forces in producing or changing motion of fluids, is known as Kinetics.

2.4. Lagrangian Approach of Fluid Motion

In this method any individual fluid particle is selected, which is pursued throughout its course of motion and the observation is made about behavior of this particle during its course of motion through space.

2.5. Eulerian Approach of Fluid Motion

In this method, any point in the space occupied by the fluid is selected and observation is made of whatever changes of velocity, density and pressure, which take place at that point.
Eulerian Method is commonly adopted.

2.6. Velocity:

In case of solid, it is generally sufficient to measure the velocity of the body as a whole, but in case of fluids, the motion of fluid may be quite different at different points of observation. Therefore the velocity V at any point of fluid mass is expressed as the ratio between the displacement of a fluid element along its path and the corresponding increment of time as the later approaches zero.

$$V = \lim_{dt\to 0} \frac{ds}{dt}$$

As velocity is the vector quantity, It has magnitude as well as direction. Therefore the velocity V at any point in the fluid can be resolved into three components u, v and w along three mutually perpendicular directions s, y and z.

$$u = \lim_{dt\to 0} \frac{dx}{dt}$$

$$v = \lim_{dt\to 0} \frac{dy}{dt}$$

$$w = \lim_{dt\to 0} \frac{dz}{dt}$$

2.7. Types of Fluid flow:

2.7.1. Steady and Unsteady flow.

Fluid flow is said to be steady if at any point in the flowing fluid various characteristics such as velocity, pressure, density, temperature etc. which describe the behavior of the fluid motion, do not change with time.
Fluid flow characteristics remains independent of time.

$$\frac{\partial u}{\partial t} = 0, \quad \frac{\partial v}{\partial t} = 0, \quad \frac{\partial w}{\partial t} = 0$$

$$\frac{\partial p}{\partial t} = 0$$

Fluid flow is said to be unsteady if at any point in the flowing fluid any one or all which describe the behavior of the fluid motion, change with time.

$$\frac{\partial V}{\partial t} \neq 0, \quad \frac{\partial p}{\partial t} \neq 0$$

Steady flow is simpler to analyze than unsteady flow. Moreover, most of the practical problems in engineering involve only steady flow conditions.

2.7.2. *Uniform and Non-Uniform Flow*

When the velocity of flow of fluid does not change, both in magnitude and direction, from point to point in the flowing fluid, for any given instant of time, the flow is said to be uniform.

$$\frac{\partial V}{\partial s} = 0$$

Ex: Flow of liquids under pressure through long pipelines of constant diameter is uniform.
If the velocity of flow of fluid changes from point to point in the flowing fluid at any instant, the flow is said to be non- uniform.

$$\frac{\partial V}{\partial s} \neq 0$$

Ex: Flow of liquids under pressure through long pipelines of varying diameter is non-uniform.

Flow of liquid through pipe of constant diameter at constant rate - Steady - uniform
Flow of liquid through pipe of constant diameter at either increasing or decreasing rate – Unsteady - uniform
Flow of liquid through a tapering pipe at a constant rate – Steady –non- uniform
Flow through a tapering pipe at either increasing or decreasing rate – Unsteady-non-uniform

2.7.3. *One dimensional, two-dimensional and Three-dimensional Flows:*

The various characteristics of flowing fluid such as velocity, pressure, density, temperature etc.; are in general the functions of space and time, i.e. f(x,y,z,t). Flow is three-dimensional.

When various characteristics of flowing fluid are the function of only any two of the three coordinate directions, and time, Flow is two-dimensional.
When various characteristics of flowing fluid are the function of only any one of the three coordinate directions, and time, Flow is one-dimensional.

Types of Flow	**Unsteady**	**Steady**
3D Flow	V = f(x,y,z,t)	V = f(x,y,z)
2D Flow	V = f(x,y,t) OR f(x,z,t) OR f(y,z,t)	V = f(x,y) OR f(x,z) OR f(y,z)
1D Flow	V = f(x,t) OR f(y,t) OR f(z,t)	V = f(x) OR f(y) OR f(z)

2.7.4. *Rotational and Irrotational Flow:*

A flow is said to be rotational if the fluid particles while moving in the direction of flow rotate about their mass centers.
A flow is said to be irrotational if the fluid particles while moving in the direction of flow do not rotate about their mass centers.

2.7.5. Laminar and Turbulent Flow

A flow is said to be laminar when the various fluid particles move in layers (or laminae) with one layer of fluid sliding smoothly over an adjacent layer.
A fluid motion is said to be turbulent when the fluid particles move in an entirely haphazard and disorderly manner that results in a rapid and continuous mixing of the fluid leading to momentum transfer as flow occurs.

2.8. Basic Principles of Fluid Flow

Principle of conservation of mass: Mass can neither be created nor destroyed – Continuity Equation
Principle of conservation of energy: Energy can neither be created nor destroyed – Energy Equation
Principle of conservation of momentum: The impulse of the resultant force, or the product of the force and time increment during which it acts, is equal to the change of momentum of body. – Momentum Equation.

2.9. Continuity Equation:

It is mathematical statement of the principle of conservation of mass.

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = q$$

Where A is area, Q is discharge, q is net lateral flow
This equation is for unsteady flow
For steady flow, $\frac{\partial A}{\partial t}$ term of the equation becomes zero
So, $\frac{\partial Q}{\partial x} = q$
If there is no lateral flow, then q= 0
So, $\frac{\partial Q}{\partial x} = 0$ Gives Q= Constant OR A*V = Constant.
$A_1V_1 = A_2V_2 = A_3V_3 =$……………..=Constant

2.10. Energy Equation:

A fluid in motion is subjected to several forces which results in the variation of the acceleration and the energies involved in the flow phenomenon of the fluid. As such in the study of the fluid motion the forces and energy that are involved in the flow are required to be considered. This aspect of fluid motion is known as dynamics of fluid flow.
Forces acting on a fluid mass
Body or Volume forces
Proportional to volume – Weight, Centrifugal Force, Magnetic Force, Electromotive Force etc.
Surface forces
Proportional to surface area – Pressure Force, Shear Force, Force of Compressibility, Force due to turbulence etc.
Line forces
Proportional to length – Surface Tension.

$$\sum \mathrm{F} = \mathrm{Ma}$$

2.11. Bernoulli Equation:

Is applicable for steady irroatational flow for incompressible fluids;

$$\frac{p}{\gamma} + \frac{v^2}{2g} + z = Constant$$

$\frac{p}{\gamma}$ *is pressure head or static head*
$\frac{v^2}{2g}$ *is velocity head or kinetic head*

z is potential head or datum head.
The sum of $\frac{p}{\gamma}$ and z is known as piezometric head.
For flow of real fluid since there is always some energy of the flowing fluid converted into heat due to viscous and turbulent shear and consequently there is a certain amount of energy loss.

$$(\frac{p}{\gamma})_1 + (\frac{v^2}{2g})_1 + z_1 = \left(\frac{p}{\gamma}\right)_2 + (\frac{v^2}{2g})_2 + z_2 + h_L$$

h_L *is the loss of energy between points under consideration.*

Application of Bernoulli equation
a) VENTURI METER
b) ORIFICE METER

2.12. Venturi Meter
The venturimeter is used to measure the rate of flow of a fluid flowing through the pipes.

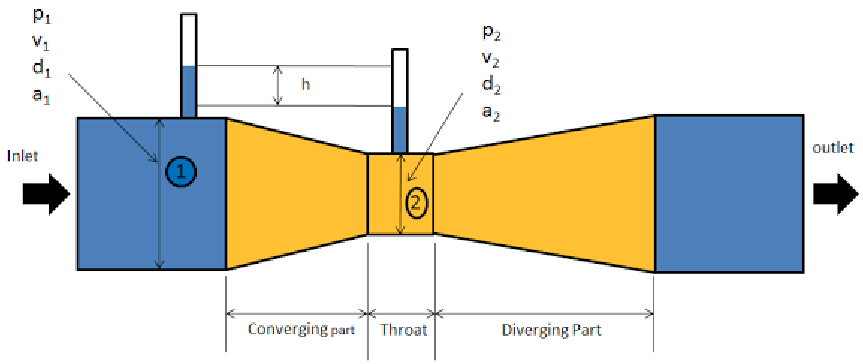


Figure 7 Venturimeter

Theoretical Discharge:

$$Q = \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}}\sqrt{2gh}$$

Actual Discharge:

$$Q_{act} = C_d \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}}\sqrt{2gh}$$

2.13. Orifice Meter :
Is a device used for measuring the rate of flow of a fluid flowing through a pipe.
• It is a cheaper device as compared to venturimeter. This also work on the same principle as that of venturimeter.
The diameter of orifice is generally 0.5 times the diameter of the pipe (D), although it may vary from 0.4 to 0.8 times the pipe diameter.
The coefficient of discharge of the orifice meter is much smaller than that of a venturimeter.

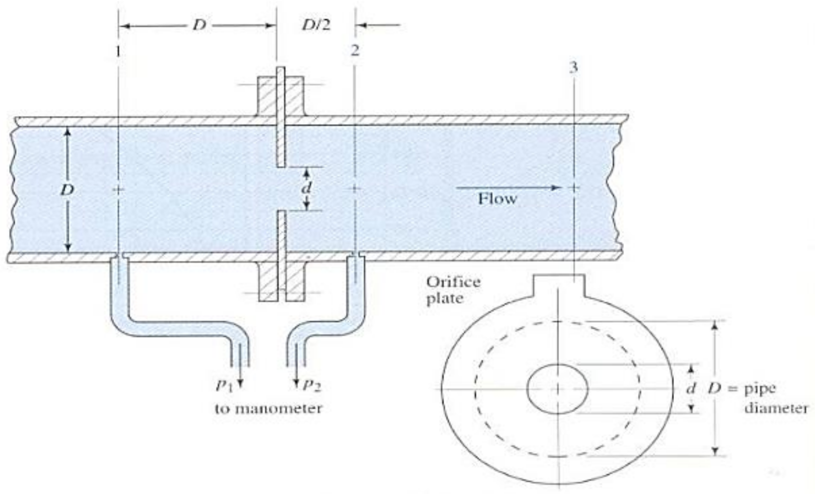


Figure 8 Orificemeter

$$Q = C_d \frac{A_0 A_1 \sqrt{2gh}}{\sqrt{A_1^2 - A_0^2}}$$

$$C_d = C_c \frac{\sqrt{1 - \frac{A_0^2}{A_1^2}}}{\sqrt{1 - \frac{A_0^2}{A_1^2} C_c^2}}$$

$$\Rightarrow C_c = C_d \frac{\sqrt{1 - \frac{A_0^2}{A_1^2} C_c^2}}{\sqrt{1 - \frac{A_0^2}{A_1^2}}}$$

A_1 is Area of section 1
A_0 is Area of Opening
C_c is Coefficient of Contraction
C_d is Coefficient of discharge
C_v is Coefficient of velocity
$C_d = C_c * C_v$

2.14. Hydraulic gradient line (H.G.L):

It is defined as the line which gives the sum of pressure head $(\frac{p}{\gamma})$ and datum head (z) of a flowing fluid in pipe with respect to some reference or it is line which is obtained by joining the top of all vertical ordinates, showing the pressure head $(\frac{p}{\gamma})$ of a flowing fluid in a pipe from the centre of the pipe. The line so obtain is called the H.G.L.

2.15. Total energy loss (TEL or EGL)

It is known that the total head (which is also total energy per unit weight) with respect to any arbitrary datum, is the sum of the elevation (potential) head, pressure head and velocity head.

$$Total\ Energy = \frac{p}{\gamma} + \frac{v^2}{2g} + z$$

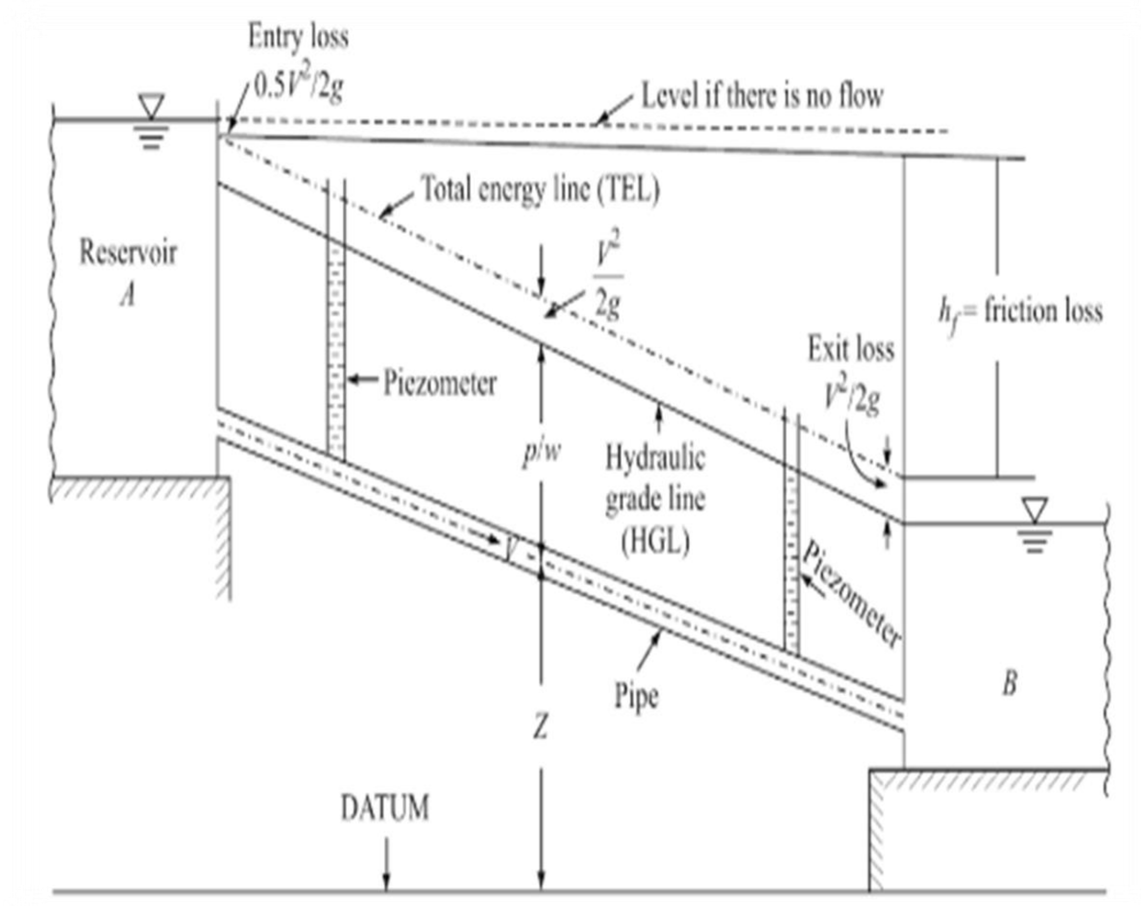


Figure 9 TEL and HGL

4.3. **Measurement of Discharge**

4.3.1. Weirs and notches

3.1. Weir and Notches:

3.2. Weir :

A concrete or masonry structure built across rivers in order to raise the level of water on the u/s side to allow the excess water to flow over its entire length to d/s side.

Similar to small dam constructed across river but in dam excess water flows d/s through small portion called spillway, in weir water flows in entire length.

Nappe is sheet of water flowing through weir or notch.

3.3. Notches:

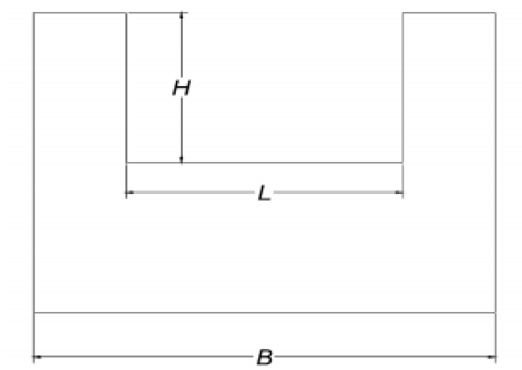
Opening provided in side of tank (or vessel) such that the liquid surface in the tank is below the top edge of the opening.

Notches are used to measure rate of flow of liquid from a tank or in channel.

Notches	**Weir**	**Weir According to shape of Crest**	**Weir According to discharging Behavior**
Rectangular	Rectangular	Sharp edge weir	Freely Discharging
Triangular	Triangular	Narrow Crested	Submerged
Trapezoidal	Trapezoidal	Broad Crested	
Parabolic		Ogee Shaped	
Stepped			

4.3.2. Discharge formulas

3.4. Rectangular Weir:



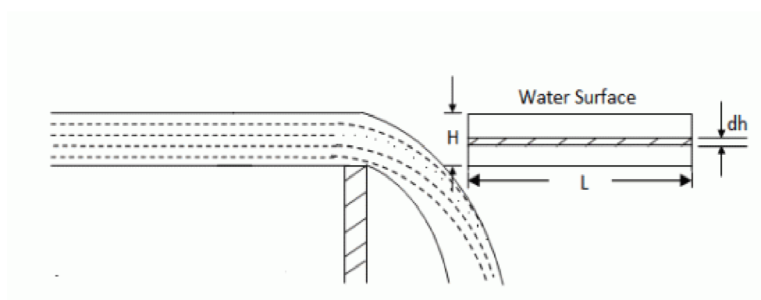


Figure 10 Rectangular Notch and Weir

Discharge Formula:

$Q = \frac{2}{3} C_d \sqrt{2g}\, LH^{3/2}$ – C_d is coefficient of discharge, L is length & H is head over crest

Considering End Contraction:

$Q = \frac{2}{3} C_d \sqrt{2g} * (L - 0.1nH)H^{3/2}$ – Where n is number of end contraction.

Considering Approach Velocity

$Q = \frac{2}{3} C_d \sqrt{2g} * (L)[(H + h_a)^{\frac{3}{2}} - h_a^{\frac{3}{2}}]$ – Where h_a is velocity head = $\frac{v_a^2}{2g}$

Considering Approach Velocity and end contraction both

$Q = \frac{2}{3} C_d \sqrt{2g} * (L - 0.1nH)[(H + h_a)^{\frac{3}{2}} - h_a^{\frac{3}{2}}]$

3.5. V notch Notch:

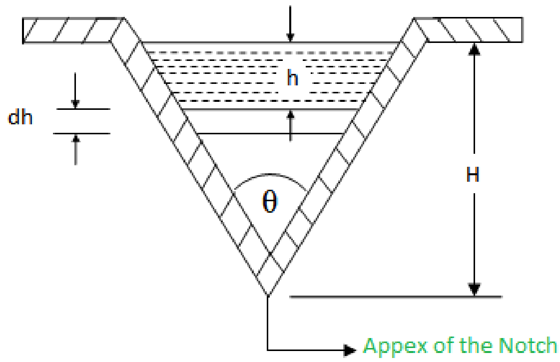


Fig : Triangular Notch

Figure 11 Triangular Notch

Discharge Formula

$Q = \frac{8}{15} C_d \sqrt{2g} \tan\frac{\theta}{2} H^{5/2}$ – C_d is coefficient of discharge

Considering Approach Velocity

$Q = \frac{8}{15} C_d \sqrt{2g} \tan\frac{\theta}{2} [(H + h_a)^{\frac{5}{2}} - h_a^{\frac{5}{2}}]$ – Where h_a is velocity head = $\frac{v_a^2}{2g}$

3.6. Trapezoidal Notch:

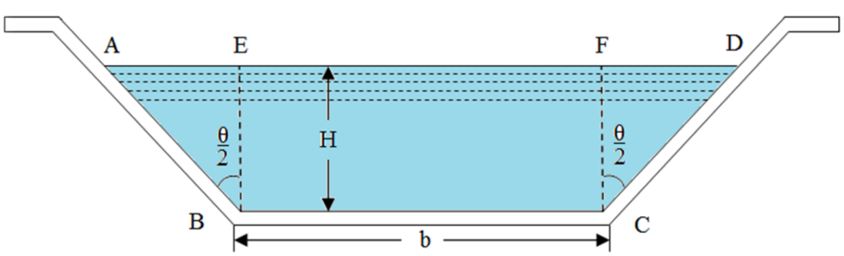


Fig : Trapezoidal Notch

Figure 12 Trapazoidal Notch

Discharge Formula

$$Q = \frac{2}{3} C_{d1} \sqrt{2g}\, L H^{3/2} + \frac{8}{15} C_{d2} \sqrt{2g} \tan\frac{\theta}{2} H^{5/2}$$

If $C_{d1} = C_{d2} = C_d$

$$Q = C_d \sqrt{2g}\, H^{\frac{3}{2}} \left(\frac{2}{3} L + \frac{8}{15} H \tan\frac{\theta}{2}\right)$$

3.7. Cippoletti Notch

$$\frac{\theta}{2} = 14$$

Special condition of trapezoidal notch where side slope is kept, 1H:4V

$$tan\frac{\theta}{2} = \frac{1}{4}$$

$Q_1 = \frac{2}{3}C_d\sqrt{2g} * (L - 0.1nH)H^{3/2}$

$Q_2 = \frac{8}{15}C_d\sqrt{2g}\, tan\frac{\theta}{2}H^{5/2}$

$Q = Q_1 + Q_2$

$Q = \frac{2}{3}C_d\sqrt{2g}\, LH^{3/2}, C_d = 0.632$

$Q = 1.86\, LH^{3/2}$

3.8. Submerged Weir

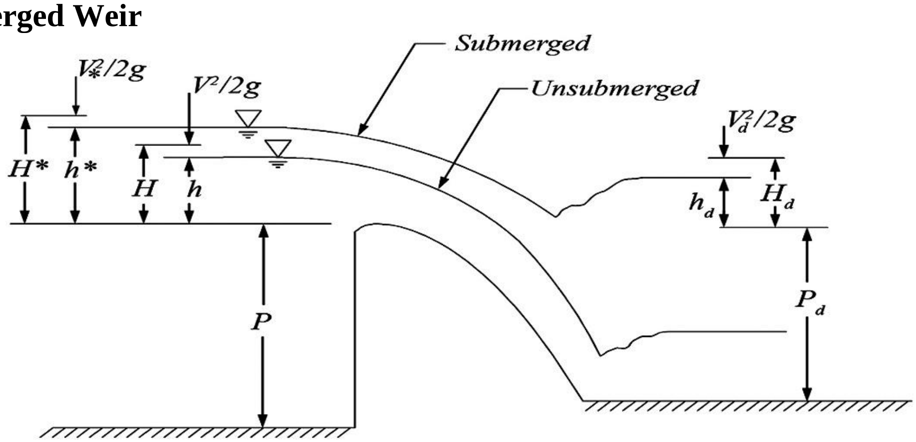
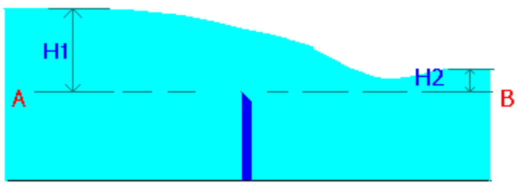


Figure 13 Submerged and Unsubmerged Weir

If d/s water table is below crest level weir is unsubmerged or free flowing weir
If d/s water table is above crest level weir is submerged of Drowned weir



$Q_1 = \frac{2}{3}C_{d1}\sqrt{2g}\, L(H_1 - H_2)^{3/2}$

$Q_2 = C_{d2}\sqrt{2g(H_1 - H_2)}\, LH_2$

$Q = Q_1 + Q_2$

4.4. Flows: Characteristics of pipe flow and open channel flow

4.4.1. Pipe Flow:

4.1. Reynolds Color Dye Experiment:

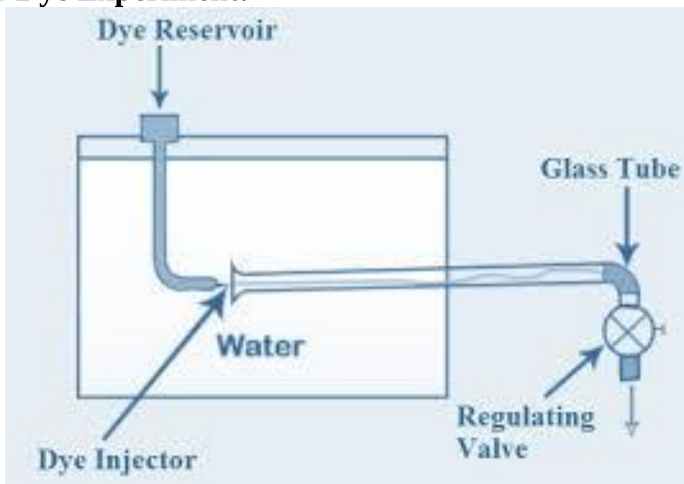


Figure 14 Reynolds Experiment

In this experiment, a horizontal pipe is immersed in a tank filled with water. Just outside the tank the pipe is bended downward in order to provide a sufficient velocity to the flow for the various experiments. For this reason, the tank is placed on an elevated platform, since the water is discharged at the level of the floor. At the bottom, a valve connected to the free atmosphere controls the water flow in order to generate the wanted pressure gradients along the horizontal section of the pipe. The valve is regulated by a long lever operated by a technician from the elevated platform. Pipes of different sections are used to see the dependence of the phenomenon on the radius of the pipe and the viscosity of the water is modified by changing its temperature. Accurate readings of the water level permit to measure the flow rate in the pipe. The intake of the pipe is fitted with a trumpet mouthpiece, in order to avoid the formation of vortices along its edges. The regimes of the flow are made visible by introducing a colored tracer, able to provide an image of the velocity field. When the flow is laminar, the tracer appears as a straight colored line. As soon as the regime becomes turbulent, the tracer spreads over the whole cross-section of the pipe, so that the fluid changes its color everywhere.

4.2. Reynolds Number (Re)

It is the ratio of Inertia force to viscous force.

$$Re = \frac{Inertia\, force}{Viscous\, force} = \frac{F_i}{F_v}$$

$$Re = \frac{\rho V L}{\mu}$$

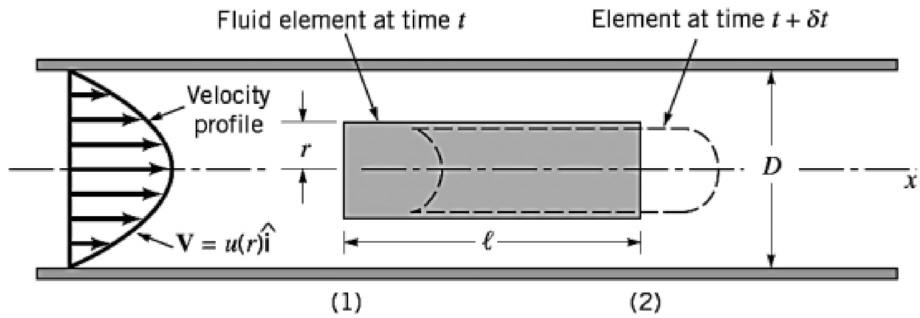
ρ *and* μ are respectively the mass density and viscosity of the flowing fluid, V is the characteristic velocity of flow and L is the characteristic linear dimension. In the case of flow through pipes the characteristic linear dimension L is taken as diameter D of the pipe and characteristic velocity is taken as average velocity v.

$\frac{\mu}{\rho} = \vartheta$, kinematic viscosity of the flowing fluid.

Reynolds number is therefore a very useful parameter in predicting whether the flow is laminar or turbulent.

Flow	Reynolds Number
Pipe flow	Re<2000 - Laminar flow Re>4000 - Turbulent flow Re between 2000 and 4000 – Transition
Flow between parallel plates	Re<1000 – Laminar flow
Open channel flow	Re<500 – Laminar flow
Flow around sphere	Re<1 – Laminar flow

4.3. Laws of fluid friction
4.4. Laminar flow:
In laminar flow the fluid particles move along parallel path in layers or laminae., such that the paths of the individual fluid particles do not cross those of the neighboring particles. It occurs at low velocity so that forces due to viscosity predominate over the inertial forces.



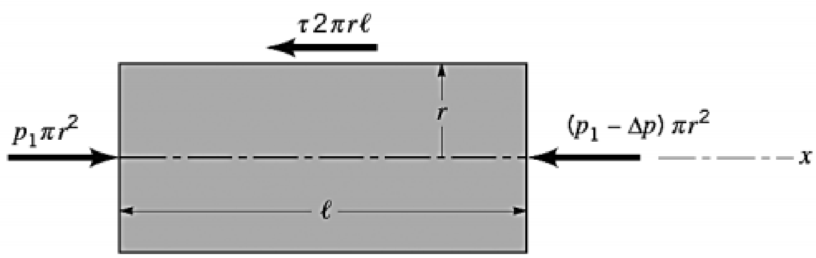


Figure 15 Motion of Cylindrical Fluid Element within a Pipe

Differential Equation of Laminar flow is;

$$\mu \frac{\delta^2 v}{\delta y^2} = \frac{\partial p}{\partial x}$$

This equation is valid for steady uniform laminar flow.

- Shear stress in laminar flow varies linearly along the radius of pipe. At center, shear stress is zero, and at the wall of the pipe, shear stress is maximum.

  $$\tau = -\frac{\partial p}{\partial x}\frac{r}{2}$$

- In laminar flow through circular pipes, velocity of flow varies parabolically. The maximum magnitude of velocity v_{max} occurs at the axis of the pipe and has magnitude of ,

  $$v_{max} = \frac{1}{4\mu}\left(-\frac{\partial p}{\partial x}\right)R^2$$

  Velocity distribution:

  $$v = v_{max}\left[1 - \left(\frac{r}{R}\right)^2\right]$$

Frictional Resistance in a laminar flow is
Hagen – Poiseuille Equitation,
Head Loss in Laminar Pipe flow, $h_f = \frac{32\mu vL}{\gamma D^2} = \frac{128\mu QL}{\pi\gamma D^4}$

i) Proportional to the velocity of flow.
ii) Independent of pressure.
iii) Proportional to the area of surface in contact.
iv) Independent of nature of the surface in contact.
v) Greatly affected by the variation of the temperature of the flowing fluid.

4.5. Turbulent Flow

Mostly the flow in pipes is turbulent. Velocity distribution in turbulent flow is relatively uniform.
In the case of turbulent flow the velocity fluctuations influence the mean motion in such a way that an additional shear resistance is caused.
Frictional Resistance in a turbulent flow is
Head Loss in Laminar Pipe flow, $h_f = \frac{fLv^2}{2gD} = \frac{8fLQ^2}{\pi^2 gD^5}$; This equation is known as Darcy Weisbach equation
Here, f is friction factor, which is a dimensionless quantity.

i) Proportional to the (velocity) ^{n}, where the index n varies from 1.72 to 2.0.
ii) Independent of pressure.
iii) Proportional to the density of flowing fluid.
iv) Proportional to area of surface in contact.
v) Dependent of nature of the surface in contact.
vi) Slightly affected by the variation of the temperature of the flowing fluid.

4.6. Hydrodynamically rough and smooth boundary:

In general a boundary with irregularities of large average height k, on its surface is considered to be a rough and the one with smaller k values is considered as smooth boundary. However for proper classification of smooth and rough boundaries, the flow and fluid characteristics are required to be considered in addition to the boundary characteristics.

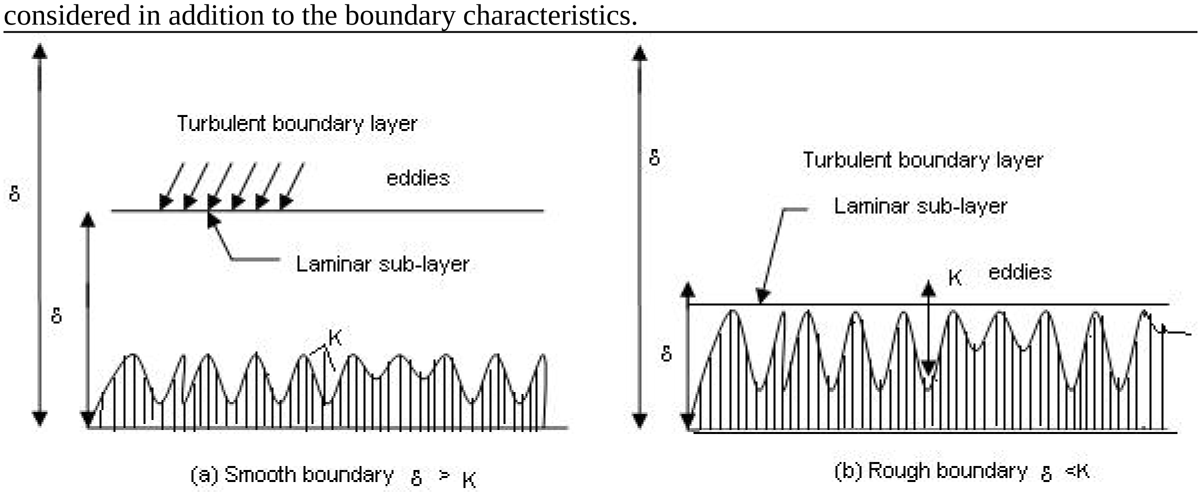


Figure 16 Smooth and rough pipe boundary

$$\frac{k}{\delta'} = \frac{v^*k}{\vartheta}\frac{1}{11.6}$$

$$v^* = shear\ velocity = \sqrt{(\frac{\tau_0}{\rho})}$$

$$\tau_0 = average\ shear\ stess$$

$$\rho = density$$

$$\delta' = sublimanar\ thickness$$

$$\vartheta = kinematic\ viscosity$$

$$k = average\ height\ of\ boundary\ irregularities$$

Smooth:

$$\frac{k}{\delta'} \leq 0.25\ OR\ \frac{v^*k}{\vartheta} \leq 3$$

Rough

$$\frac{k}{\delta'} \geq 6\ OR\ \frac{v^*k}{\vartheta} \geq 70$$

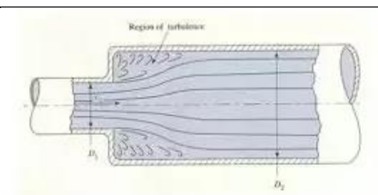
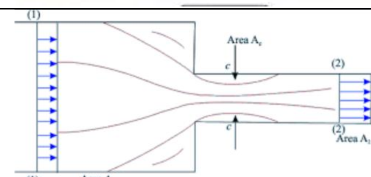
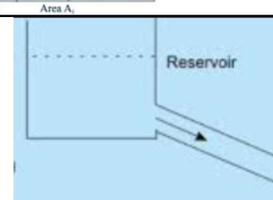
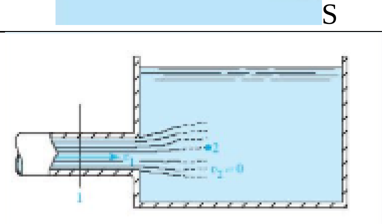
Transitional Flow

$$3 < \frac{v^*k}{\vartheta} < 70\ OR\ 0.25 < \frac{k}{\delta'} < 6.0$$

Velocity distribution for both smooth and rough boundary,

$$\frac{v_{max} - V}{V^*} = 3.75$$

4.7. Head Loss

Major	Minor		
$h_f = \frac{fLv^2}{2gD}$	Due to Sudden Enlargement	$h_L = \frac{(v_1 - v_2)^2}{2g}$	
	Due to Sudden Contraction	$h_L = 0.5\frac{(v)^2}{2g}$	
	Entrance to pipe from reservoir	$h_L = 0.5\frac{(v)^2}{2g}$	 S
	Exit from pipe to reservoir	$h_L = \frac{(v)^2}{2g}$	
	Bend/ Tee/Valve etc	$h_L = k\frac{(v)^2}{2g}$	K is the cofficient.

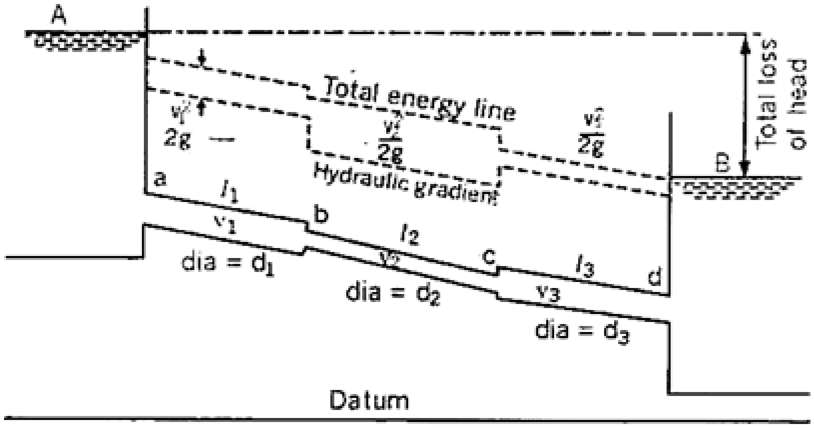


Figure 17 TEL and HGL

4.8. Pipe in Series:

Pipes are said to be in series if they are connected end to end (in continuation with each other) so that the fluid flows in a continuous line without any branching. The volume rate of flow through the pipes in series is the same throughout.

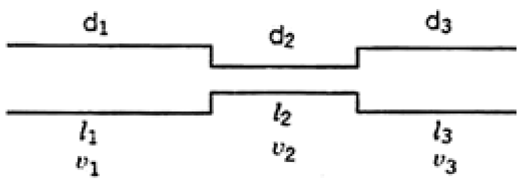


Figure 18 Series Connection

If h_{f1}, h_{f2}, h_{f3} be the losses of head in the individual pipes the total loss of head h_f is given by

$$h_f = \frac{8fLQ^2}{\pi^2 g D^5} = h_{f1} + h_{f2} + h_{f3} = \frac{8f_1L_1Q^2}{\pi^2 g {D_1}^5} + \frac{8f_2L_2Q^2}{\pi^2 g {D_2}^5} + \frac{8f_3L_3Q^2}{\pi^2 g {D_3}^5} + \cdots$$

$$\frac{fL}{D^5} = \frac{f_1L_1}{{D_1}^5} + \frac{f_2L_2}{{D_2}^5} + \frac{f_3L_3}{{D_3}^5} + \cdots$$

Equivalent Pipe Corresponding to a Given Set of Pipes in Series:

Let d_1, d_2, d_3 be the diameters, and l_1, l_2, l_3 be the lengths of the various pipes in a series connection. Let Q be the discharge. Let h_f be the total loss of head.

Let d be the diameter of an equivalent pipe of length l to replace the compound pipe to pass the same discharge at the same loss of head.

If

$f = f_1 = f_2 = f_3 = \cdots$ is assumed , then

$$\frac{L}{D^5} = \frac{L_1}{{D_1}^5} + \frac{L_2}{{D_2}^5} + \frac{L_3}{{D_3}^5} + \cdots$$

This formula is known as Dupits Formula.

4.9. Pipes Connected in Parallel:

Pipes are said to be in parallel when they are so connected that the flow from a pipe branches or divides into two or more separate pipes and then reunite into a single pipe.

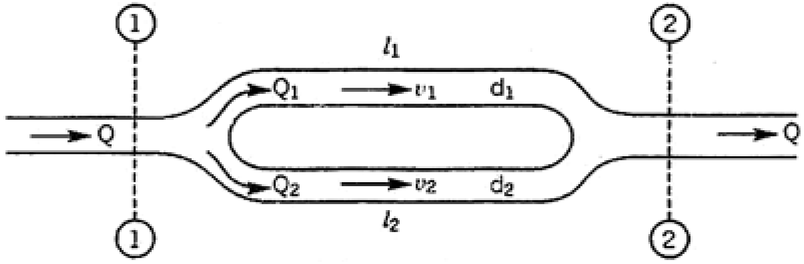


Figure 19 Parallel Connection

In this arrangement the total discharge Q divides into components Q_1 and Q_2 along the branch pipes such that –

$Q = Q_1 + Q_2$

In this arrangement, the loss of head from section 1-1 to section 2-2 is equal to the loss of head in any one of the branch pipes.

$h_f = h_{f1} = h_{f2}$

$$\frac{8f_1L_1{Q_1}^2}{\pi^2 g{D_1}^5} = \frac{8f_2L_2{Q_2}^2}{\pi^2 g{D_2}^5}$$

$$\frac{f_1L_1{Q_1}^2}{{D_1}^5} = \frac{f_2L_2{Q_2}^2}{{D_2}^5}$$

If $f_1 = f_2$ is assumed,

$$\frac{L_1{Q_1}^2}{{D_1}^5} = \frac{L_2{Q_2}^2}{{D_2}^5}$$

$$\frac{{Q_1}^2}{{Q_2}^2} = \frac{L_2{D_1}^5}{L_1{D_2}^5}$$

4.10. Siphon:

Siphon is a long bent which is used to carry water from a reservoir at a higher elevation to another reservoir at a lower elevationwhen the two reservoirs are seperated by a hill of high level ground in between as shown in figure.

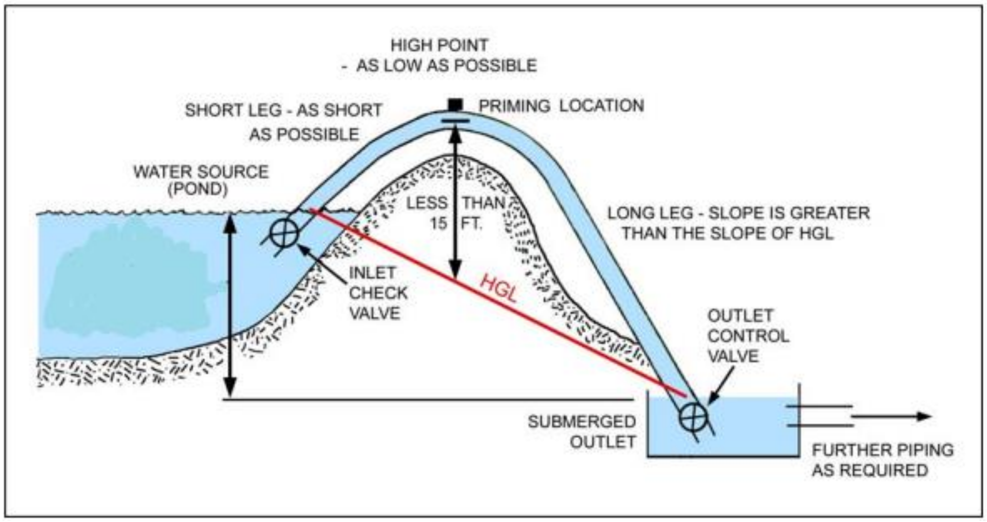


Figure 20 SIPHON

4.11. Unsteady Flow In Pipes:

When the water flowing in a long pipe is suddenly brought to rest by closing the valve or by any similar cause, there will be a sudden rise in pressure due to the momentum of the moving water being destroyed. This causes a wave of high pressure to be transmitted along the pipe which creates noise known as knocking. This phenomenon of sudden rise in pressure in the pipe is known as water hammer. The rise in pressure in some cases may be so large that the pipe may even brust and therefore it is essential to take into account this rise in the design of the pipes.

Allievi Formula for maximum pressure rise for instantenious closure of valve,

$$h = \frac{cv}{g}$$

Where c = pressure wave velocity & v = velocity of flow.

$$c = \sqrt{\frac{\frac{k}{\rho}}{\left(1 + \frac{kD}{eE}\right)}}$$

Where,
K = Bulk Modulus of Water
E = Young Modulus of Elasticity of Pipe material
D = Diameter of Pipe
e = Thickness of Pipe
$\rho = Density\ of\ water$

Water hammer is the case of unsteady flow.

4.4.2. Open Channel Flow :

Flow of water takes place with a free surface which is subjected to atmoshpheric pressure.
The HGL of Open channel flow is its water surface.

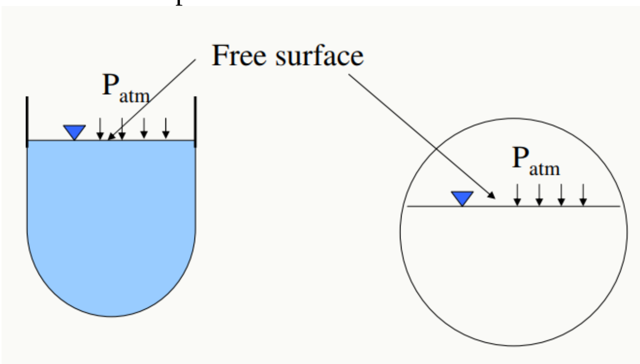


Figure 21Open Channel Flow

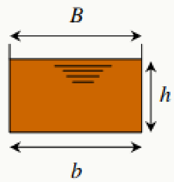
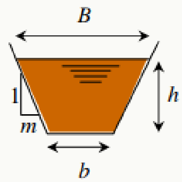
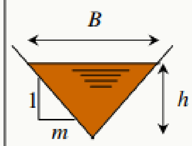
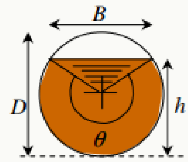
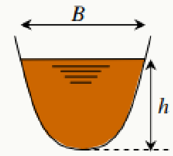
4.12. Properties of OCF vs Pipe Flow

OCF	PIPE FLOW
• OCF must have a free surface. • A free surface is subject to atmospheric pressure • The driving force is mainly the component of gravity along the flow direction. • HGL is coincident with the free surface. • Flow area is determined by the geometry of the channel plus the level of free surface, which is likely to change along the flow direction and with as well as time. • The cross section may be of any from circular to irregular forms of natural streams, which may change along the flow direction and as well as with time. • Relative roughness changes with the level of free surface • The depth of flow, discharge and the slopes of channel bottom and of the free surface are interdependent.	• No free surface in pipe flow • No direct atmospheric pressure, hydraulic pressure only. • The driving force is mainly the pressure force along the flow direction. • HGL is (usually) above the conduit (In case of Siphon it is below center line) • Flow area is fixed by the pipe dimensions the cross section of a pipe is usually circular. • The cross section may be of any from circular to irregular forms of natural streams, which may change along the flow direction and as well as with time. • The relative roughness is a fixed quantity. • No such dependence.

4.13. Geometrical Properties of Channel Section :

y : Flow Depth.
T : Top width.
A : Wetted Area.
P : Wetted Perimeter.
R = A/P : Hydraulic Radius.
D = A/T : Hydraulic Depth.

Z = A√T : Section Factor

	rectangular	trapezoidal	triangular	circular	parabolic
					
flow area A	bh	$(b+mh)h$	mh^2	$\frac{1}{8}(\theta-\sin\theta)D^2$	$\frac{2}{3}Bh$
wetted perimeter P	$b+2h$	$b+2h\sqrt{1+m^2}$	$2h\sqrt{1+m^2}$	$\frac{1}{2}\theta D$	$B+\frac{8}{3}\frac{h^2}{B}$ *
hydraulic radius R_h	$\frac{bh}{b+2h}$	$\frac{(b+mh)h}{b+2h\sqrt{1+m^2}}$	$\frac{mh}{2\sqrt{1+m^2}}$	$\frac{1}{4}\left[1-\frac{\sin\theta}{\theta}\right]D$	$\frac{2B^2h}{3B^2+8h^2}$ *
top width B	b	$b+2mh$	$2mh$	$(\sin\theta/2)D$ or $2\sqrt{h(D-h)}$	$\frac{3}{2}\frac{A}{h}$
hydraulic depth D_h	h	$\frac{(b+mh)h}{b+2mh}$	$\frac{1}{2}h$	$\left[\frac{\theta-\sin\theta}{\sin\theta/2}\right]\frac{D}{8}$	$\frac{2}{3}h$

4.14. Shear Stress in Open channel flow

$$\tau = \gamma R S_o$$

$$S_o = bed\ slope\ of\ the\ canal$$

4.15. Some velocity formula used in OCF

Mannings Equation:

$$V = \frac{1}{n} R^{\frac{2}{3}} S_o^{\frac{1}{2}}$$

n is Mannings Constant

Chezys Formula:

$$V = C\sqrt{RS_o}$$

C is Chezys Constant

$$C = \frac{1}{n} R^{\frac{1}{6}}$$

Kutters Formula

$$v = \frac{23 + \frac{0.00155}{S_o} + \frac{1}{n}}{1 + \left(23 + \frac{0.00155}{S_o}\right)\frac{n}{\sqrt{R}}} * \sqrt{RS}$$

Where n is kutter's n

4.16. Most Economical Section of Channels:

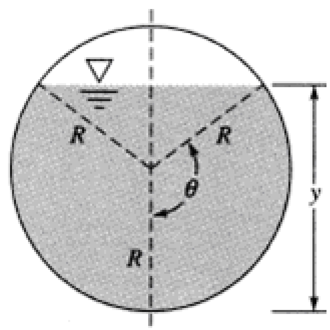
A section of a channel is said to be most economical when the cost of construction of the channel is minimum. However, the cost of construction of a channel depends on excavation and the lining. To keep the cost down or minimum, the wetted perimeter, for a given discharge, should be minimum. This condition is utilized for determining the dimensions of economical sections of different forms of channels.

4.17. Most Economical Rectangular Channel:
R= y/2 (Hydraulic radius is half the depth)

4.18. Most Economical Trapezoidal Channel:
R= y/2 (Hydraulic radius is half the depth)
"A trapezoidal section channel is most economical if when a semi-circle is drawn with its center, O, on the water surface and radius equal to the depth of flow, D, the three sides of the channel are tangential to the semi-circle".

Best side slope is at 60^o to the horizontal, i.e.; of all trapezoidal sections a half hexagon is most economical. However, because of constructional difficulties, it may not be practical to adopt the most economical side slopes.

4.19. Most Economical Circular Channel:



Condition for Maximum Discharge for Circular Section:
If Chezys Formula is used:
α = 154 Degree
y = 0.95d
If Mannings Formula is used:
α = 151 Degree
y = 0.94d
Condition for Maximum Velocity for Circular Section:
If Chezys or Mannings Formula is used:
α = 128.75 Degree
y = 0.81y

4.20. Froud Number (F):
The Froude number is defined as the ratio of gravitational forces to inertial forces.

$$F = \frac{Inertial\ Force}{Gravity\ Force} = \frac{v}{\sqrt{gD}}$$

v is velocity of flow, D is hydraulic depth.

Subcritical flow is deep, slow flow with a low energy state and has a Froude number less than one (F < 1).
Critical flow occurs when the Froude number equals one (F=1); there is a perfect balance between the gravitational and inertial forces.
Supercritical flow is shallow, fast flow with a high-energy state and has a Froude number greater than one (F>1).

4.21. Energy Principles in Open Channel Flow:

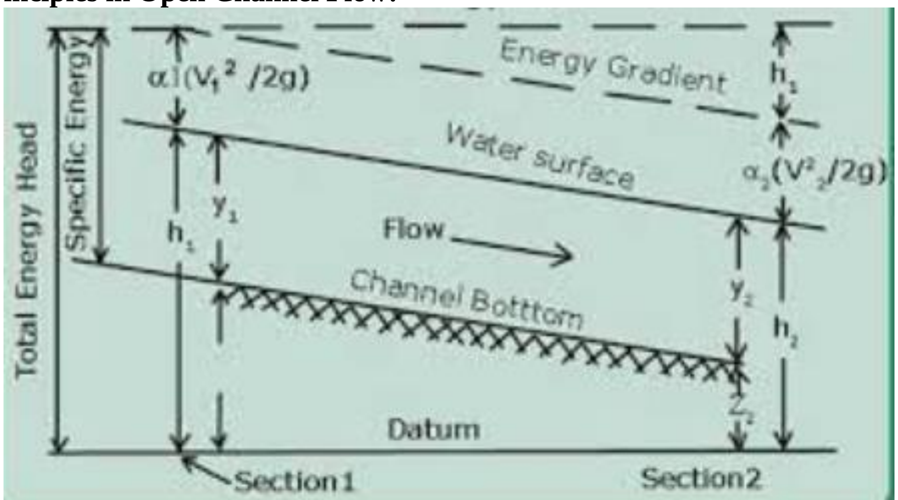


Figure 22 TEL and HGL in OCF

$$Total\ Energy = Z + y + \frac{v^2}{2g}$$

If the channel bed is taken as the datum (as shown), then the total energy per unit weight will be,

$$E_{Specific} = y + \frac{v^2}{2g} = y + \frac{Q^2}{2gA^2}$$

This energy is known as specific energy, Es. Specific energy of a flowing liquid in a channel is defined as energy per unit weight of the liquid measured from the channel bed as datum. It is a very useful concept in the study of open channel flow.

4.22. Specific Energy Curve:

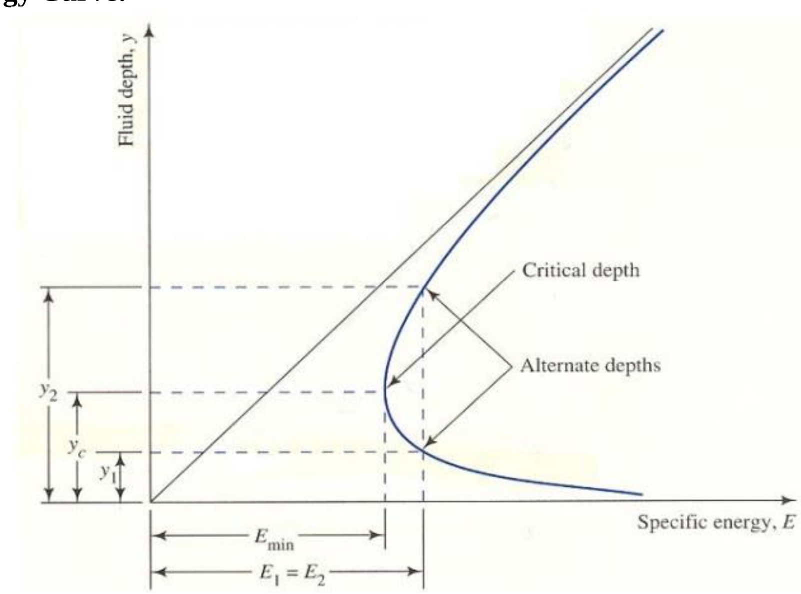


Figure 23 Specific Energy Curve

Critical Depth in Rectangular Canal Section (y_c)

$$y_c = \sqrt[3]{\frac{q^2}{g}}$$

Where q = Q/B

4.23. Hydraulic Jump:

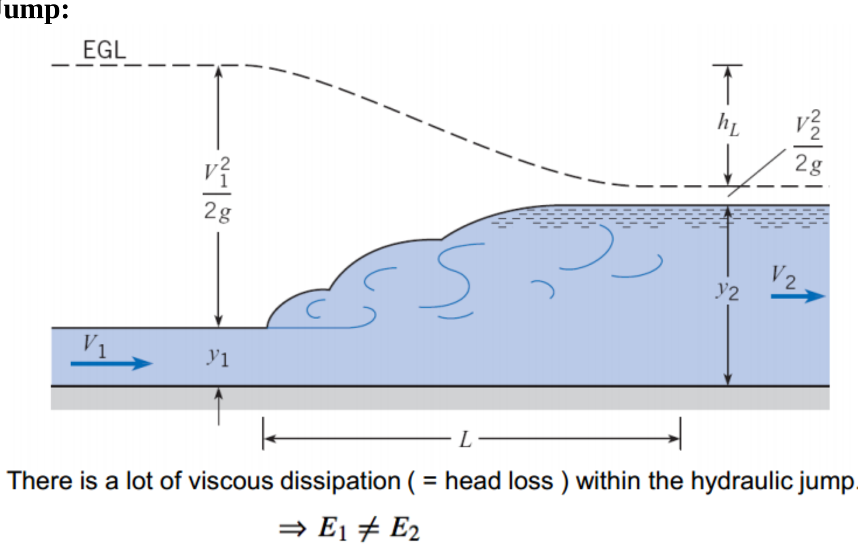


Figure 24 Hydraulic Jump

Energy Dissipation in Hydraulic Jump (HJ)

$$\Delta E = \frac{(y_2 - y_1)^3}{4y_1y_2}$$

SOME USEFUL SITES

https://www.sanfoundry.com/1000-fluid-mechanics-questions-answers/

https://www.indiabix.com/civil-engineering/hydraulics/

http://www.geekmcq.com/civil-engineering/hydraulicsfluid/

SOME OBJECTIVE QUESTIONS

1. **Fluid is a substance that**
    - A. cannot be subjected to shear forces
    - B. always expands until it fills any container
    - C. has the same shear stress.at a point regardless of its motion
    - D. cannot remain at rest under action of any shear force
    - E. Flows.
2. **Fluid is a substance which offers no resistance to change of**
    - A. pressure
    - B. flow
    - C. shape
    - D. volume
    - E. Temperature.
3. **Practical fluids**
    - A. are viscous
    - B. possess surface tension
    - C. are compressible
    - D. possess all the above properties
    - E. Possess none of the above properties.
4. **In a static fluid**
    - A. resistance to shear stress is small
    - B. fluid pressure is zero
    - C. linear deformation is small
    - D. only normal stresses can exist
    - E. Viscosity is nil.
5. **A fluid is said to be ideal, if it is**
    - A. incompressible
    - B. inviscous
    - C. viscous and incompressible
    - D. inviscous and compressible
    - E. inviscous and incompressible.
6. **If no resistance is encountered by displacement, such a substance is known as**
    - A. fluid
    - B. water
    - C. gas
    - D. perfect solid
    - E. Ideal fluid.
7. **Liquids**
    - A. cannot be compressed
    - B. are not affected by change in pressure and temperature
    - C. are not viscous
    - D. None of the above.
8. **Which of the following is dimensionless**
    - A. specific weight
    - B. specific volume
    - C. specific speed
    - D. specific gravity
    - E. Specific viscosity.
9. **The normal stress in a fluid will be constant in all directions at a point only if**
    - A. it is incompressible
    - B. it has uniform viscosity
    - C. it has zero viscosity
    - D. it is frictionless
    - E. It is at rest.
10. **An object is a mass of 2 kg and weighs 19N on the spring balance. The value of gravity at this location, in m/s^2 is**
    - A. 0.105
    - B. 2
    - C. 9.81
    - D. 9.5
    - E. 19
    - F. None of above
11. **A pressure intensity of 10^9 Pa can be written as**
    - A. gPa
    - B. Gpa
    - C. kMPa
    - D. μPa
    - E. None of above

12. Viscosity has the dimensions

A. $FL^{-1}T$
B. $FL^{-1}T^{-1}$
C. FLT^{-2}
D. $FL^{2}T$
E. FLT^{2}
F. None

13. Select the incorrect statement. Apparent Shear Force is

A. Can never occur when the fluid is at rest.
B. May occur owing cohesion when the liquid is at rest.
C. Depends upon molecular interchange of momentum.
D. Depend upon cohesive forces.
E. Can never occur in a frictionless fluid regardless of its motion.

14. $\mu = 0.06\frac{kg}{ms}$ and specific gravity =0.6, kinematic viscosity in stoke is

A. 2.78
B. 1.00
C. 0.60
D. 0.36
E. None of these answer

15. Choose the correct relationship

A. specific gravity = gravity x density
B. dynamic viscosity = kinematic viscosity x density
C. gravity = specific gravity x density
D. kinematic viscosity = dynamic viscosity x density
E. Hydrostatic force = surface tension x gravity.

16. If mercury in a barometer is replaced by water, the height of 3.75 cm of mercury will be following cm of water

A. 51 cm
B. 50 cm
C. 52 cm
D. 52.2 cm
E. 51.7 cm.

17. A one dimensional flow is one which

A. is uniform flow
B. is steady uniform flow
C. takes place in straight lines
D. involves zero transverse component of flow

18. Specific weight of sea water is more that of pure water because it contains

A. dissolved air
B. dissolved salt
C. suspended matter
D. all of the above
E. Heavy water.

19. The buoyancy depends on

A. mass of liquid displaced
B. viscosity of the liquid
C. pressure of the liquid displaced
D. depth of immersion
E. None of the above.

20. Bernoulli equation deals with the law of conservation of

A. mass
B. momentum
C. energy
D. work
E. Force.

21. **For pipes, turbulent flow occurs when Reynolds number is**

A. less than 2000
B. between 2000 and 4000
C. more than 4000
D. less than 4000
E. None of the above.

22. Two pipe systems can be said to be equivalent, when the following quantities are same

A. friction loss and flow
B. length and diameter
C. flow and length
D. friction factor and diameter
E. Velocity and diameter.

23. All the terms of energy in Bernoulli's equation have dimension of

A. energy
B. work
C. mass
D. length
E. Time.

24. According to Bernoulli's equation, for steady ideal fluid flow

A. principle of conservation of mass holds
B. velocity and pressure are inversely proportional
C. total energy is constant throughout
D. the energy is constant along a streamline but may vary across streamlines
E. None of the above.

25. Normal depth in open channel flow is the depth of flow corresponding to

A. steady flow
B. unsteady flow
C. laminar flow
D. uniform flow
E. Critical flow.

26. **A piece of metal of specific gravity 7 floats in mercury of specific gravity 13.6. What fraction of its volume is under mercury?**

A. 0.5
B. 0.4
C. 0.515
D. 0.5
E. None of the above.

27. Flow occurring in a pipeline when a valve is being opened is

A. steady
B. unsteady
C. laminar
D. vortex
E. Rotational.

28. The flow in which the velocity vector is identical in magnitude and direction at every point, for any given instant, is known as

A. one dimensional flow
B. uniform flow
C. steady flow
D. turbulent flow
E. Streamline flow.

29. The flow in which conditions do not change with time at any point, is known as

A. one dimensional flow
B. uniform flow
C. steady flow
D. turbulent flow
E. streamline flow

30. The flow, which neglects changes in a transverse direction, is known as

A. one dimensional flow
B. uniform flow
C. steady flow
D. turbulent flow
E. Streamline flow.

31. In the case of steady flow of a fluid, the acceleration of any fluid particle is

A. Constant
B. variable
C. zero
D. zero under limiting conditions
E. never zero

32. For measuring flow by a venturimeter, it should be installed in

A. vertical line
B. horizontal line
C. inclined line with flow downward
D. inclined line with upward flow
E. In any direction and in any location.

33. In an immersed body, center of pressure is

A. at the center of gravity
B. above the center of gravity
C. below be center of gravity
D. could be above or below e.g. depending on density of body and liquid
E. Unpredictable.

34. Differential monometer is used to measure

A. Pressure in pipes, channels etc.
B. atmospheric pressure
C. very low pressure
D. difference of pressure between two points
E. velocity in pipes

35. The center of gravity of the volume of the liquid displaced by an immersed body is called

A. center of gravity
B. center of pressure
C. metacenter
D. center of buoyancy
E. Centroid.

36. The line of action of the buoyant force acts through the centroid of the

A. submerged body
B. volume of the floating body
C. volume of the fluid vertically above the body
D. displaced volume of the fluid
E. None of the above.

37. The two important forces for a floating body are

A. buoyancy, gravity
B. buoyancy, pressure
C. buoyancy, inertial
D. inertial, gravity
E. Gravity, pressure.

38. According to the principle of buoyancy a body totally or partially immersed in a fluid will be lifted up by a force equal to

A. the weight of the body
B. more than the weight of the body
C. less than the weight of the body
D. weight of the fluid displaced by the body
E. Weight of body plus the weight of the fluid displaced by the body.

39. Metacenter is the point of intersection of

A. vertical upward force through CG of body and center line of body
B. buoyant force and the center line of body
C. midpoint between CG and center of buoyancy
D. all of the above
E. None of the above.

40. A body floats in stable equilibrium

A. when its metacentric height is zero
B. When the metacenter is above CG.
C. when its CG is below its center of buoyancy
D. metacenter has nothing to do with position of CG for determining stability
E. None of the above

41. The horizontal component of buoyant force is

A. negligible
B. same as buoyant force
C. zero

42. Which of the following instruments is used to measure flow on the application of Bernoulli's theorem

A. Venturimeter
B. Orifice plate
C. nozzle
D. pitot tube
E. All of the above.

43. Minor losses occur due to

A. sudden enlargement in pipe
B. sudden contraction in pipe
C. bends in pipe
D. all of the above

44. The highest point of syphon is called as

A. syphon top
B. summit
C. reservoir
D. none of the above

45. The friction factor in fluid flowing through pipe depends upon

A. Reynold's number
B. relative roughness of pipe surface
C. Both a. and b.
D. none of the above

46. **The increase of temperature results in**
    - A. increase in viscosity of gas
    - B. increase in viscosity of liquid
    - C. decrease in viscosity of gas
    - D. decrease in viscosity of liquid
    - E. (a) and (d) above.

47. The difference between total head line and piezo-metric head line represents :
   - A. the velocity head
   - B. the pressure head
   - C. the elevation of the bed of the channel
   - D. the depth of flow

48. **Normally velocity at …… depth from free surface in the OCF is very close to mean velocity of flow**
    - A. 0.2
    - B. 0.8
    - C. 0.6
    - D. Average of velocity at 0.2 depth and 0.8 depth
    - E. Both C and D

49. If layers of fluid have frictional force between them, then it is known as
   - A. viscous
   - B. non-viscous
   - C. incompressible
   - D. both a and b

50. Cavitation is caused by
   - A. high velocity
   - B. high pressure
   - C. weak material
   - D. low pressure
   - E. Low viscosity.

Answer Key

1	D	**11**	B	**21**	C	**31**	C	**41**	C
2	C	**12**	F	**22**	A	**32**	E	**42**	E
3	D	**13**	A	**23**	D	**33**	C	**43**	D
4	D	**14**	B	**24**	D	**34**	D	**44**	B
5	E	**15**	B	**25**	D	**35**	D	**45**	C
6	E	**16**	A	**26**	C	**36**	D	**46**	E
7	D	**17**	D	**27**	B	**37**	A	**47**	A
8	D	**18**	D	**28**	B	**38**	D	**48**	E
9	E	**19**	A	**29**	C	**39**	B	**49**	A
10	D	**20**	C	**30**	A	**40**	B	**50**	D

(Please feel free to contact at madhukhanal72@gmail.com for any suggestion or comment regarding this note. Your valuable suggestions will give this note a better shape)

15.10 MOMENTUM IN OPEN-CHANNEL FLOW-SPECIFIC FORCE

If V is the mean velocity of flow of discharge Q in a channel, the momentum of the flow passing a channel section per unit time may be expressed by (wQV/g), where w is the specific weight of water. According to Newton's second law of motion the rate of change of momentum in the body of water flowing in a channel is equal to the resultant of all the forces that are acting on the body. Thus applying this principle to a channel of large bed slope, the following expression for the rate of change of momentum in the body of water enclosed between sections 1 and 2 may be written

$$\frac{wQ}{g}(V_2 - V_1) = P_1 - P_2 + W \sin\theta - F_f \qquad ...(15.39)$$

where w, Q and V are previously defined with subscripts referring to sections 1 and 2; P_1 and P_2 are the resultant pressures acting on the two sections; W is the weight of water enclosed between the sections; θ is the angle of inclination of the channel bottom with the horizontal; and F_f is the total external force of frictional resistance acting in the direction opposite to the flow along the surface of contact between the water and the channel.

When the momentum equation is applied to a short horizontal reach of a prismatic channel, the external force of friction and the component of the weight of water can be ignored. Thus with $\theta = 0$ and $F_f = 0$ Eq. 15.39 becomes

$$\frac{wQ}{g}(V_2 - V_1) = P_1 - P_2 \qquad ...(15.40)$$

Assuming hydrostatic pressure distribution at sections 1 and 2

$$P_1 = wA_1\bar{z}_1 \text{; and } P_2 = wA_2\bar{z}_2$$

where A_1, $\bar{z}_1$ and A_2, $\bar{z}_2$ are cross-sectional area and the vertical depth of the centroid of the area below the surface of flow at sections 1 and 2 respectively. Also $V_1 = (Q/A_1)$ and $V_2 = (Q/A_2)$, thus Eq. 15.40 may be written as

$$\frac{Q^2}{gA_1} + A_1\bar{z}_1 = \frac{Q^2}{gA_2} + A_2\bar{z}_2 \qquad ...(15.41)$$

The terms on both sides of Eq. 15.41 are analogous, hence Eq. 15.41 may be expressed for any channel section by a general function

$$F = \frac{Q^2}{gA} + A\bar{z} \qquad ...(15.42)$$

This function consists of two terms. The first term is the momentum of the flow passing through the channel per unit time per unit weight of water, and the second term is the force per unit weight of water. Since both these terms are force per unit weight of water, their sum is known as *specific force*. As such Eq. 15.41 may be expressed as $F_1 = F_2$, which means that the specific forces of sections 1 and 2 are equal, provided that external force of friction and the component of the weight of water in the channel reach between the two sections can be ignored.

It is evident from Eq. 15.42 that for a given channel section and discharge Q the specific force depends only on the depth of flow. Thus by plotting the depth of flow against the specific force for a given channel section and discharge, a specific force curve is obtained as shown in Fig. 15.10. Alike specific energy curve this curve also has two limbs *AC* and *BC*. The limb *AC* approaches the horizontal axis asymptotically towards right. The limb *BC* rises upward and extends indefinitely to the right. It can be seen from the curve that for a given value of specific force there are two possible depths y_1 and y_2. It has been shown in the next chapter that these two depths y_1 and y_2 constitute the initial and sequent depths of a hydraulic jump. At point *C* on the curve the two depths become one and corresponding to this depth the specific force is minimum.

For a given discharge the condition for minimum specific force can be obtained by differentiating Eq. 15.42 with respect to *y* and then considering $(dF/dy) = 0$. Thus

$$\frac{dF}{dy} = -\left(\frac{Q^2}{gA^2}\right)\frac{dA}{dy} + \frac{d(A\bar{z})}{dy} = 0$$

since *Q* is constant and both *A* and $\bar{z}$ are the functions of *y*. As shown in Fig.15.10 for a change *dy* in the depth, the corresponding change $d(A\bar{z})$ in the moment of the cross-sectional area about the free surface may be expressed as

$$\left[A(\bar{z}+dy)+\frac{T(dy)^2}{2}\right]-(A\bar{z}) = d(A\bar{z})$$

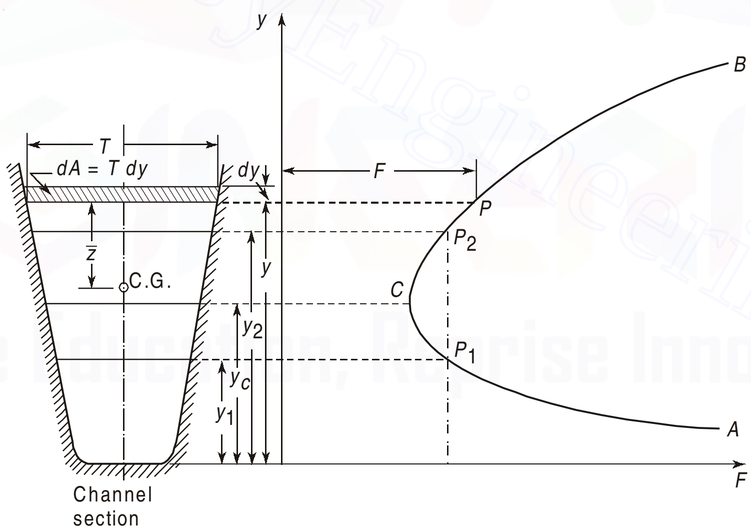


Figure 15.10 *Specific-force curve*

Assuming $(dy)^2 \approx 0$, the above expression becomes

$$d(A\bar{z}) = A(dy)$$

Substituting this value of $d(A\bar{z})$ the preceding equation becomes

$$\frac{dF}{dy} = -\left(\frac{Q^2}{gA^2}\right)\frac{dA}{dy} + \frac{A(dy)}{dy} = 0$$

Again since $(dA/dy) = T$, the above equation reduces to

$$\frac{Q^2}{g} = \frac{A^3}{T}$$

which is the condition for the critical state of flow as indicated earlier. It is thus seen that the depth of flow at the minimum value of the specific force is equal to the critical depth.

Further solving Eq. 15.42 for Q, it may be expressed as

$$Q = \sqrt{(F - A\bar{z})gA} \qquad \text{...(15.43)}$$

In a given channel section for a given value of specific force F, the condition for maximum discharge can be obtained by putting $(dQ/dy) = 0$. Thus differentiating Eq. 15.43 with respect to y

$$\frac{dQ}{dy} = \frac{g\left[F\left(\frac{dA}{dy}\right) - A\frac{d(A\bar{z})}{dy} - (A\bar{z})\frac{dA}{dy}\right]}{2\sqrt{(F - A\bar{z})(gA)}} = 0$$

or

$$F = \frac{A^2}{T} + A\,\bar{z}$$

Substituting the value of F obtained above in Eq. 15.43 it may be simplified and rearranged as

$$\frac{Q^2}{g} = \frac{A^3}{T}$$

which is again the criterion for the critical state of flow as indicated earlier. Therefore it may be stated that for a given specific force the discharge in a given channel section is maximum when the flow is in the critical state.

15.11 CRITICAL FLOW AND ITS COMPUTATION

When the depth of flow of water over a certain reach of a given channel is equal to the critical depth y_c, the flow is described as critical flow or in critical state. As indicated by the critical flow criterion, Eq. 15.36, the critical depth for a given discharge Q is the depth y_c corresponding to which the cross-sectional area A and top width T of the channel section are such that the value of (A^3/T) is given by the following expression,

$$\left(\frac{A^3}{T}\right)_c = \frac{Q^2}{g}$$

or

$$\left(A\sqrt{\frac{A}{T}}\right)_c = Z_c = \frac{Q}{\sqrt{g}} \qquad \text{...(15.44)}$$

Equation 15.44 indicates that the section factor for critical flow computation, $Z[= A\sqrt{A/T}]$ for a channel section at the critical state of flow is equal to $(Q/\sqrt{g})$. For a prismatic channel the section factor Z is a function of the depth of flow. Hence it is evident from Eq. 15.44 that in a prismatic channel there is only one possible depth y_c which makes the given discharge Q, in a channel to flow in critical

Non-uniform Flow in Channels

Chapter 16

16.1 INTRODUCTION

As explained in Chapter 15 the non-uniform or varied flow in a channel is the one in which depth of flow changes from section to section along the length of the channel. It may be further classified as *gradually varied flow* (G.V.F.) and *rapidly varied flow* (R.V.F.). The gradually varied flow is a steady non-uniform flow in which the depth of flow varies gradually. Many cases of gradually varied flow are of practical interest to engineers such as flow upstream of a weir or a dam, flow downstream of a sluice gate, flow in channels with break in bottom slopes etc., wherein study of back water and the location of hydraulic jump is of major importance. In a rapidly varied flow the depth of flow changes abruptly over a comparatively short distance. Typical examples of rapidly varied flow are hydraulic jump and hydraulic drop. In this chapter both the types of non-uniform flows have been discussed.

16.2 GRADUALLY VARIED FLOW

The problem of gradually varied flow is that of predicting overall flow pattern, or in other words prediction of the water surface profile to be expected in a given channel with given steady discharge. Such problems can be solved by writing the differential equation for the water surface profile and then integrating it.

1. Dynamic Equation of Gradually Varied Flow. The dynamic equation for gradually varied flow can be derived from the basic energy equation with the following assumptions:

(a) The uniform flow formulae (such as Manning's or Chezy's) may be used to evaluate the energy slope of a gradually varied flow and the corresponding coefficients of roughness developed primarily for uniform flow are applicable to the gradually varied flow also.

Thus,

$$(S_f)_{\text{G.V.F.}} = \left(\frac{Vn}{R^{2/3}}\right)^2 \quad \text{...(Manning's)}$$

$$(Sf)_{\text{G.V.F.}} = \left(\frac{V}{C\sqrt{R}}\right)^2 \quad \text{...(Chezy's)}$$

(b) The bottom slope of the channel is very small.

(c) The channel is prismatic.

(d) The energy correction factor α is unity.

(e) The pressure distribution in any vertical is hydrostatic.

(f) The roughness coefficient is independent of the depth of flow and it is constant throughout the channel reach considered.

Considering a short reach of channel having gradually varied flow as shown in Fig. 16.1, the energy equation at any section may be written as

$$H = \frac{V^2}{2g} + y + z$$

or
$$H = \frac{Q^2}{2gA^2} + y + z \qquad \left(\text{since } V = \frac{Q}{A}\right)$$

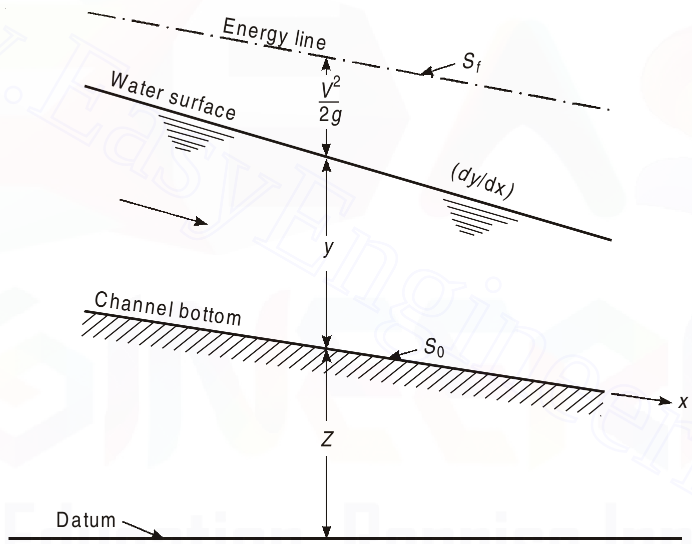


Figure 16.1 *Gradually varied flow*

Differentiating each term of the above equation with respect to x, where x is measured along the channel bottom, the following differential equation can be obtained

$$\frac{dH}{dx} = \frac{d}{dx}\left(\frac{Q^2}{2gA^2}\right) + \frac{dy}{dx} + \frac{dz}{dx}$$

or
$$\frac{dH}{dx} = -\frac{Q^2}{gA^3}\frac{dA}{dx} + \frac{dy}{dx} + \frac{dz}{dx}$$

In the above differential equation, $\frac{dH}{dx}$ is the slope of the energy line and hence $\frac{dH}{dx} = -S_f$;

$\frac{dz}{dx}$ is the slope of the channel bed and hence $\frac{dz}{dx} = -S_0$ (the negative signs for S_f and S_0 indicate that as x increases H and z decreases ; and $\frac{dy}{dx}$ is the slope of the water surface with respect to the channel bottom. Further the term $\frac{dA}{dx}$ can be written as $\frac{dA}{dx} = \frac{dA}{dy} \times \frac{dy}{dx} = T\frac{dy}{dx}$, since $\frac{dA}{dy}$ is equal to the surface width T.

The substitution of these terms in the above differential equation yields

$$-S_f = -\frac{Q^2 T}{gA^3}\frac{dy}{dx} + \frac{dy}{dx} - S_0$$

Solving for $\frac{dy}{dx}$, the following differential equation for the water surface slope can be obtained,

$$\frac{dy}{dx} = \frac{S_0 - S_f}{1 - \frac{Q^2 T}{gA^3}} = \frac{S_0 - S_f}{1 - Fr^2} \qquad \text{...(16.1)}$$

Since $$\frac{Q^2 T}{gA^3} = Fr^2$$

Equation 16.1 is the basic differential equation for the gradually varied flow. It may be observed from Eq. 16.1 that when $\left(\frac{dy}{dx}\right) = 0$; $S_0 = S_f$ and the water surface is parallel to channel bottom thus representing a uniform flow as explained in Chapter 15. When $\left(\frac{dy}{dx}\right)$ is positive the water surface is rising and when $\left(\frac{dy}{dx}\right)$ is negative the water surface is falling.

2. Alternative Derivation for Gradually Varied Flow Equation. Equation 16.1 can also be derived by considering the total energy (or total head) at sections 1 and 2, dx apart along the bottom of the channel. Thus

$$z_1 + y_1 + \frac{V_1^2}{2g} = z_2 + y_2 + \frac{V_2^2}{2g} + S_f(dx)$$

or $$(z_1 - z_2) - S_f\,(dx) = (y_2 - y_1) + \frac{V_2^2}{2g} - \frac{V_1^2}{2g}$$

or $$S_0(dx) - S_f\ (dx) = dy + d\left(\frac{V^2}{2g}\right)$$

or $$S_0 - S_f = \frac{dy}{dx} + \frac{d}{dx}\left(\frac{V^2}{2g}\right)$$

or $$S_0 - S_f = \frac{dy}{dx} + \frac{d}{dy}\left(\frac{V^2}{2g}\right)\frac{dy}{dx}$$

or $$\frac{dy}{dx} = \frac{S_0 - S_f}{1 + \frac{d}{dy}\left(\frac{V^2}{2g}\right)}$$

In the above equation the term $\frac{d}{dy}\left(\frac{V^2}{2g}\right)$ represents the change in velocity head which may also be expressed as

$$\frac{d}{dy}\left(\frac{V^2}{2g}\right) = \frac{d}{dy}\left(\frac{Q^2}{2gA^2}\right) = \frac{Q^2}{gA^3}\left(\frac{dA}{dy}\right) = -\frac{Q^2T}{gA^3}$$

Thus by substitution the value of $\frac{d}{dy}\left(\frac{v^2}{2g}\right)$ in the above equation, it becomes

$$\frac{dy}{dx} = \frac{S_0 - S_f}{1 - \frac{Q^2T}{gA^3}}$$

which is same as Eq. 16.1 derived earlier.

Equation 16.1 may also be derived by adopting another approach which is based on the concept of the specific energy as described below. For a given discharge Q flowing in a channel the specific energy at a channel section may be expressed as

$$E = y + \frac{Q^2}{2gA^2}$$

Obviously E is function of the depth of flow. However, in uniform flow since the depth of flow remains constant from section to section, $(dE/dx) = 0$. On the other hand in varied flow since the depth of flow varies from section to section, the specific energy will also vary. Thus differentiating both the sides of the above noted expression for E with respect to the direction of flow x, we have

$$\frac{dE}{dx} = \frac{dy}{dx} + \frac{Q^2}{2g}\left(\frac{2}{A^3}\right)\frac{dA}{dx}$$

or
$$\frac{dE}{dx} = \frac{dy}{dx} - \frac{Q^2}{gA^3}\left(\frac{dA}{dy}\times\frac{dy}{dx}\right)$$

or
$$\frac{dE}{dx} = \frac{dy}{dx}\left(1-\frac{Q^2T}{gA^3}\right)$$

The above expression represents the change of specific energy in a small reach dx of a channel having the nonuniform flow. Further if S_0 is the bottom slope of the channel then $(S_0\, dx)$ is the work done by gravity in a small reach dx. Similarly if S_f is the slope of the energy line then $(S_f dx)$ is the loss of energy which is spent in overcoming the resistance in a small reach dx. Thus in a small reach dx of the channel the net change of specific energy dE becomes

$$dE = (S_0\, dx - S_f\, dx)$$

or
$$\frac{dE}{dx} = S_0 - S_f$$

By substituting this value of (dE/dx) and solving for (dy/dx)

$$\frac{dy}{dx} = \frac{S_0 - S_f}{1-\dfrac{Q^2T}{gA^3}}$$

which is again same as Eq. 16.1 derived earlier.

3. Dynamic Equation for G.V.F. in Wide Rectangular Channel. Equation 16.1 can be expressed in simplified form for a wide rectangular channel as indicated below.

For a wide rectangular channel section of width B the hydraulic radius can be replaced by the depth of flow y since

$$R = \frac{By}{B+2y} \approx \frac{By}{B} \approx y$$

Further according to Manning's formula

$$Q = \frac{1}{n}(By)\, y^{2/3} S_f^{1/2}$$

$$= \frac{1}{n}(By_n)\,(y_n)^{2/3} S_0^{1/2}$$

It may be noted that the hydraulic radius has been replaced by the depth of flow and n is assumed to be same for uniform and non-uniform flows. Therefore

$$\frac{S_f}{S_0} = \left(\frac{y_n}{y}\right)^{10/3}$$

However, if Chezy's formula is used instead of the Manning's, the value of

$$\frac{S_f}{S_0} = \left(\frac{y_n}{y}\right)^3$$

In the same manner for a channel of rectangular section

$$\frac{Q^2T}{gA^3} = \frac{Q^2B}{g(B^3y^3)} = \frac{q^2}{gy^3} = \left(\frac{y_c}{y}\right)^3$$

By substituting in Eq. 16.1 it becomes

$$\frac{dy}{dx} = S_0 \frac{1-\left(\frac{y_n}{y}\right)^{10/3}}{1-\left(\frac{y_c}{y}\right)^3} \qquad ...(16.2)$$

and

$$\frac{dy}{dx} = S_0 \frac{1-\left(\frac{y_n}{y}\right)^3}{1-\left(\frac{y_c}{y}\right)^3} \qquad ...(16.3)$$

4. Relation between Water Surface Slopes and Channel Bottom Slope. The term (dy/dx) represents the slope of the water surface with respect to the channel bottom. But often the water surface slope S_w with respect to horizontal may be required to be determined. As such a relation between the water surface slope S_w, , the channel bottom slope S_0 and the slope (dy/dx) may be developed which facilitates the determination of S_w when S_0 and (dy/dx) are known. For rising water surface as shown in Fig. 16.2 (a), from triangle *abd*,

$$S_w = \sin \alpha = \frac{bd}{ba} = \frac{cd - cb}{ba}$$

But

$$\frac{cd}{ba} = \sin \theta = S_0, \text{ and } \frac{cb}{ba} = \frac{dy}{dx}$$

$$\therefore \quad S_w = S_0 - \left(\frac{dy}{dx}\right) \qquad ...(16.4)$$

However, in this case if the water surface is such that point *b* lies above point *d*, then $(dy/dx) > S_0$, and hence Eq. 16.4 becomes

$$S_w = \left(\frac{dy}{dx}\right) - S_0 \qquad \text{...(16.4 a)}$$

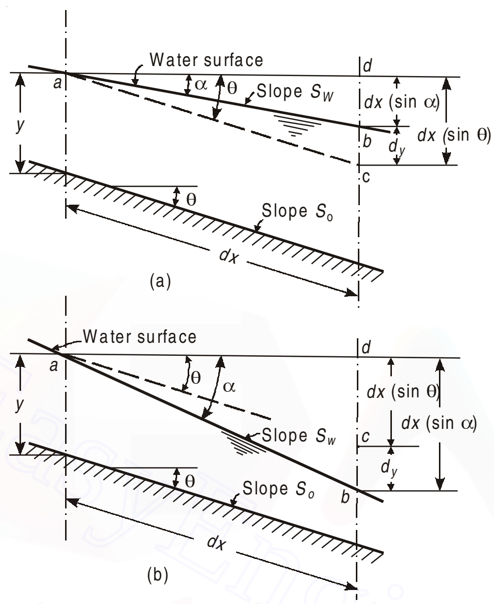


Figure16.2 *Relation between water surface and channel bottom slopes*

For falling water surface as shown in Fig. 16.2 (b), from triangle *abd*,

$$S_w = \sin\alpha = \frac{bd}{ba} = \frac{cd + cb}{ba}$$

Again $\frac{cd}{ba} = \sin\theta = S_0$, and $\frac{cb}{ba} = \frac{dy}{dx}$

$$\therefore \quad S_w = S_0 + \left(\frac{dy}{dx}\right) \qquad \text{...(16.5)}$$

16.3 CLASSIFICATION OF CHANNEL BOTTOM SLOPES

The channel bottom slopes are classified in the various categories as mentioned below:

(i) Critical Slope. The channel bottom slope is designated as critical when the bottom slope S_0 is equal to the critical slope S_c, i.e., $S_0 = S_c$. Thus in this case the normal depth of flow will be equal to the critical depth, i.e., $y_n = y_c$.

(ii) Mild Slope. The channel bottom slope is designated as mild when the bottom slope S_0 is less than the critical slope S_c, i.e., $S_0 < S_c$. The application of Manning's or Chezy's formula will then indicate that when the bottom slope is mild, the normal depth of flow is greater than the critical depth i.e., $y_n > y_c$.

(iii) Steep Slope. The channel bottom slope is designated as steep when the bottom slope S_0 is greater than the critical slope, i.e., $S_0 > S_c$. Again the application of Manning's or Chezy's formula will indicate that when the bottom slope is steep, the normal depth of flow is less than the critical depth i.e., $y_n < y_c$.

(iv) Horizontal Slope. When the channel bottom slope is equal to zero i.e., $S_0 = 0$, the bottom slope is designated as horizontal. Obviously for a channel with horizontal bottom the normal depth of flow $y_n = \infty$ (infinity).

(v) **Adverse Slope.** When the channel bottom slope instead of falling rises in the direction of flow it is designated as an adverse slope. Thus in a channel with adverse bottom slope, S_0 is less than zero [i.e., $(S_0 < 0)$] or it is negative. Obviously for an adverse-slopped channel the normal depth of flow y_n is imaginary or it is non-existent.

16.4 CLASSIFICATION OF SURFACE PROFILES

The various water surface profiles occurring in the channels are designated with reference to the bottom slopes of the channels. Thus surface profiles which occur in mild-sloped channels are known

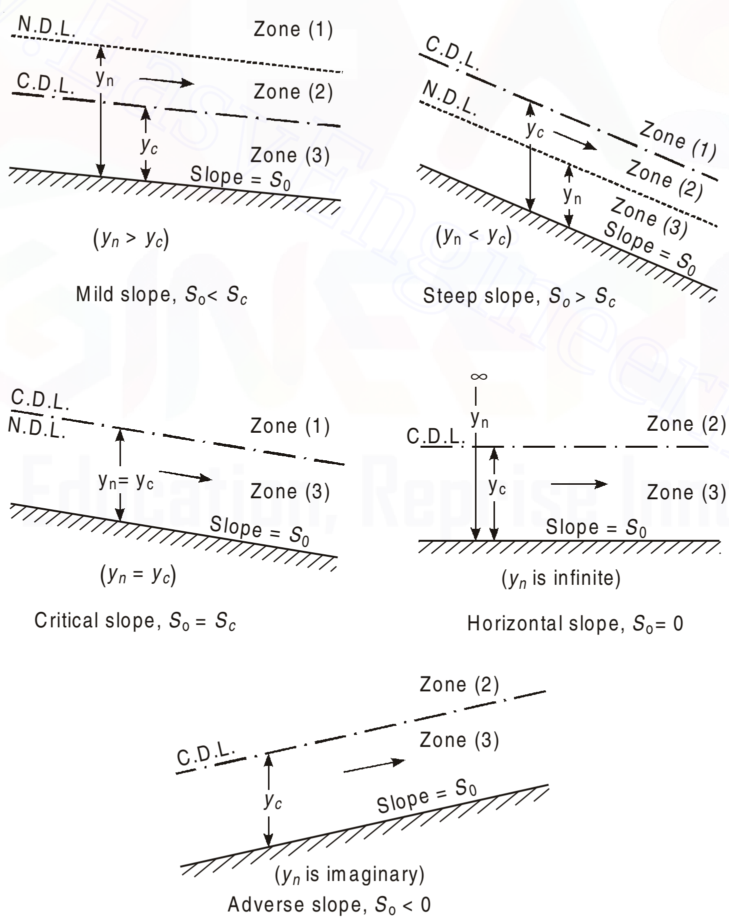


Figure 16.3 *Different zones for water surface profiles in channels of different bottom slopes*

as *M*-curves; those which occur in steep-slopped channels are known as *S*-curves; those which occur in critical-slopped channels are known as *C*-curves; those which occur in horizontal channels are known as *H*-curves, and those which occur in adverse-slopped channels are known as *A*-curves. These water surface profiles may be further classified depending upon the position of water surface relative to critical depth y_c and normal depth y_n. For the given discharge and channel section the normal depth line (N.D.L.) and the critical depth line (C.D.L.) divide the entire space above the channel bottom into three zones (Fig. 16.3). In zone 1 the given depth y lies above y_n and y_c ; in zone 2 the given depth y lies between y_n and y_c and in zone 3, y lies below y_n and y_c. For mild and steep slopes three such zones are possible and the corresponding three surface profiles are designated as M_1, M_2, M_3 and S_1, S_2, S_3 on the mild and steep slopes respectively. On the critical slope the normal depth y_n and the critical depth y_c being the same, zone 2 vanishes and only two zones 1 and 3 exist, so that the given depth, y lies either above the critical depth or below the critical depth resulting in either C_1 or C_2 curves. For horizontal-slopped channels normal depth is infinite and for adverse-slopped channels it is imaginary, as such only two zones 2 and 3 are possible, resulting in only two types of profiles H_2, H_3 and A_2, A_3 on these two slopes. Thus it can be seen that in all twelve surface profiles are possible as shown in Fig. 16.4.

16.5 CHARACTERISTICS OF SURFACE PROFILES

Figure 16.4 shows the various surface profiles. The surface profiles can be classified as back water curves and drawdown curves depending on whether the depth of flow increases or decreases in the direction of flow (or in other words, dy/dx is positive or negative). The study of surface profiles shown in Fig. 16.4 will indicate that all the surface profiles with subscript 1 and 3, that is M_1, M_3, S_1, S_3, C_1, C_3, H_3 and A_3, are back water (or rising) curves while those with subscript 2, that is M_2, S_2, H_2 and A_2 are drawdown (or falling) curves. In order to plot a certain type of profile, it is necessary to know the characteristics of each profile, which may be determined from Eq. 16.2 or 16.3 as discussed below.

1. Surface Profile in Mild-Sloped Channels. In a mild sloped channel there will be three zones viz., $y > y_n > y_c$, $y_n > y > y_c$, and $y_n > y_c > y$, in which respectively M_1, M_2 and M_3 curves will be formed. In the first zone, that is, when $y > y_n > y_c$, the given depth y can have the limiting values as $y \to y_n$ on the upstream side and $y \to \infty$ on the downstream side. Equation 16.2 or 16.3 shows that when $y > y_n > y_c$, $\frac{dy}{dx}$ is positive and as $y \to y_n$, $\frac{dy}{dx} \to 0$ and as $y \to \infty$, $\frac{dy}{dx} \to S_0$. This indicates that M_1 curve meets the normal depth (or y_n) line asymptotically on the upstream side and it tends to be horizontal on the downstream side.

In the second zone, that is, when $y_n > y > y_c$, $\frac{dy}{dx}$ is negative and as $y \to y_n$, $\frac{dy}{dx} \to 0$ and as $y \to y_c$, $\frac{dy}{dx} \to -\infty$, thereby indicating that M_2 curve meets the normal depth (or y_n) line asymptotically on the upstream side and it meets the critical depth (or y_c) line normally on the downstream side.

In the third zone, that is, when $y_n > y_c > y$, $\frac{dy}{dx}$ is positive, and as $y \to y_c$, $\frac{dy}{dx} \to -\infty$ and as $y \to 0$, $\frac{dy}{dx} \to \infty$, thereby indicating that M_3 curve meets the critical depth (or y_c) line and the channel bottom line normally.

2. Surface Profiles in Steep-Sloped Channels. In a steep-sloped channel also there will be three zones viz., $y > y_c > y_n$, $y_c > y > y_n$, and $y_c > y_n > y$, in which respectively S_1, S_2 and S_3 curves will be formed. In the first zone, that is when $y > y_c > y_n$, $\frac{dy}{dx}$ is positive and as $y \to y_c$, $\frac{dy}{dx} \to \infty$ and when $y \to \infty$, $\frac{dy}{dx} \to S_0$. That is on the upstream side S_1 curve meets the critical depth (or y_c) line normally and on the downstream side it tends to be horizontal.

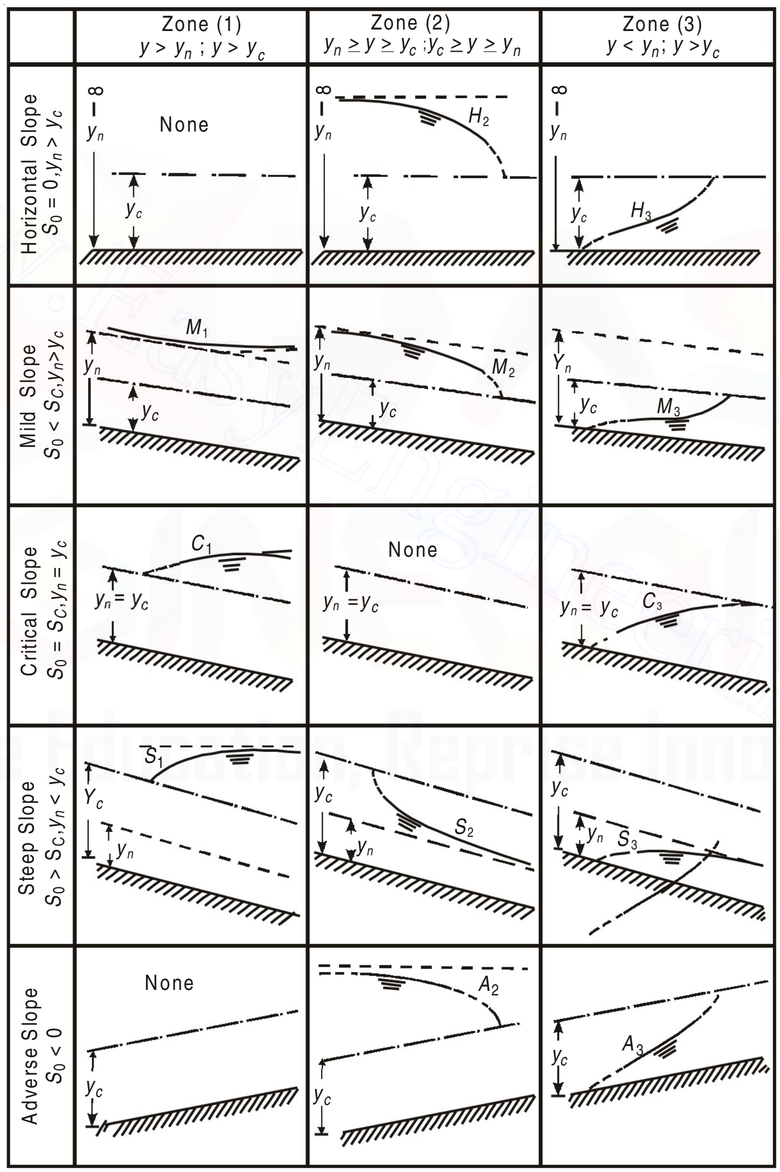


Figure 16.4. *Flow profiles of gradually varied flow*

In the second zone, that is, when $y_c > y > y_n$, $\frac{dy}{dx}$ is negative and as $y \to y_c$, $\frac{dy}{dx} \to \infty$ and as $y \to y_n$, $\frac{dy}{dx} \to 0$, thereby indicating that the S_2 curve meets the critical depth (or y_c) line normally on the upstream side and on the downstream side it meets the normal depth (or y_n) line asymptotically.

In the third zone, that is, when $y_c > y_n > y$, $\frac{dy}{dx}$ is positive and as $y \to y_n$, $\frac{dy}{dx} \to 0$, and as $y \to 0$, $\frac{dy}{dx} \to \infty$, thereby indicating that the S_3 curve meets the channel bed normally and it is asymptotic to the normal depth (or y_n) line.

3. Surface Profiles in Critical-Sloped Channels. In a critical-slopped channel as discussed earlier only two zones 1 and 3 exist and hence only C_1 and C_3 curves will be formed. In the first zone, that is, when $y > y_c = y_n$, $\frac{dy}{dx}$ is positive and as $y \to y_c$, $\frac{dy}{dx} \to S_0 = S_c$, and as $y \to \infty$, $S_0 = S_c$, thereby indicating that C_1 curve will be more or less a horizontal line.

In the third zone, that is, when $y < y_c = y_n$, $\frac{dy}{dx}$ is positive and as $y \to y_c$, $\frac{dy}{dx} \to S_0 = S_c$, and as $y \to 0$, $\frac{dy}{dx} \to S_0 = S_c$, thereby indicating that C_3 curve will also be more or less a horizontal line.

It may however be stated that if Chezy's formula is used and accordingly Eq. 16.3 is considered then the C_1 and C_3 curves will be horizontal straight lines, but if Manning's formula is used and accordingly Eq. 16.2 is considered then the C_1 and C_3 curves will be slightly curved.

4. Surface Profiles in Horizontal Channels. In a channel with horizontal bottom $S_0 = 0$ and hence Eq. 16.1 may be expressed as

$$\frac{dy}{dx} = \frac{-S_f}{1-\frac{Q^2T}{gA^3}} = \frac{-\frac{n^2V^2}{y^{4/3}}}{1-\left(\frac{y_c}{y}\right)^3} \quad \ldots(16.6)$$

if Manning's formula is used, and

$$\frac{dy}{dx} = \frac{-\frac{V^2}{C^2y}}{1-\left(\frac{y_c}{y}\right)^3} \quad \ldots(16.7)$$

if Chezy's formula is used.

Further in a channel with horizontal bottom since $y_n = \infty$ only two zones 2 and 3 exist and accordingly only H_2 and H_3 curves will be formed.

In the second zone, that is, when $y > y_c$, $\frac{dy}{dx}$ is negative and as $y \to y_c$, $\frac{dy}{dx} \to -\infty$, thereby indicating that H_2 curve meets the critical depth (or y_c) line normally at the downstream end and at the upstream end it tends to approach horizontal line tangentially.

In the third zone, that is, when $y < y_c$, $\frac{dy}{dx}$ is positive and as $y \to y_c$, $\frac{dy}{dx} \to -\infty$ and as $y \to 0$, $\frac{dy}{dx} \to \infty$, thereby indicating that H_3 curve meets the critical depth (or y_c) line and the channel bottom line normally (i.e., vertically).

TABLE 16.1 *Types of Flow Profiles in Prismatic Channels*

Channel Slope	*Symbol*	*Depth Relations*	$\frac{dy}{dx}$	*Type of profile*	*Type of flow*
Horizontal $[S_0 = 0]$	None	$y > y_n > y_c$		None	None
	H_2	$y_n > y > y_c$	–	Drawdown	Subcritical
	H_3	$y_n > y_c > y$	+	Backwater	Supercritical
Mild $[0 < S_0 < S_c]$	M_1	$y > y_n > y_c$	+	Backwater	Subcritical
	M_2	$y_n > y > y_c$	–	Drawdown	Subcritical
	M_3	$y_n > y_c > y$	+	Backwater	Supercritical
Critical $[S_0 = S_c > 0]$	C_1	$y > y_c = y_n$	+	Backwater	Subcritical
	None	$y_c = y = y_n$		None	None
	C_3	$y_c = y_n > y$	+	Backwater	Supercritical
Steep $[S_0 > S_c > 0]$	S_1	$y > y_c > y_n$	+	Backwater	Subcritical
	S_2	$y_c > y > y_n$	–	Drawdown	Supercritical
	S_3	$y_c > y_n > y$	+	Backwater	Supercritical
Adverse $[S_0 < 0]$	None			None	None
	A_2	$y > y_c$	–	Drawdown	Subcritical
	A_3	$y_c > y$	+	Backwater	Supercritical

5. Surface Profiles in Adverse-Slopped Channels. In a channel with adverse bottom slope since $S_0 < 0$ (i.e., S_0 is negative) the normal depth is imaginary and hence only two zones 2 and 3 exist in which A_2 and A_3 curves will be formed.

In the second zone, that is, when $y > y_c$, Eq. 16.1 indicates that $\frac{dy}{dx}$ is negative. Again when $y \to y_c$, $\frac{dy}{dx} \to -\infty$ and when $y \to \infty$, $\frac{dy}{dx} \to S_0$ thereby indicating that A_2 curve tends to be horizontal at the upstream end and at the downstream end it meets the critical depth (or y_c) line normally.

In the third zone, that is, when $y < y_c$, $\frac{dy}{dx}$ is positive, and as $y \to y_c$, $\frac{dy}{dx} \to -\infty$, and as $y \to 0$, $\frac{dy}{dx} \to \infty$ thereby indicating that A_3 curve meets the critical depth (or y_c) line and the channel bottom line normally.

For the sake of convenience the above discussed characteristics of the various surface profiles have been tabulated in Table 16.1.

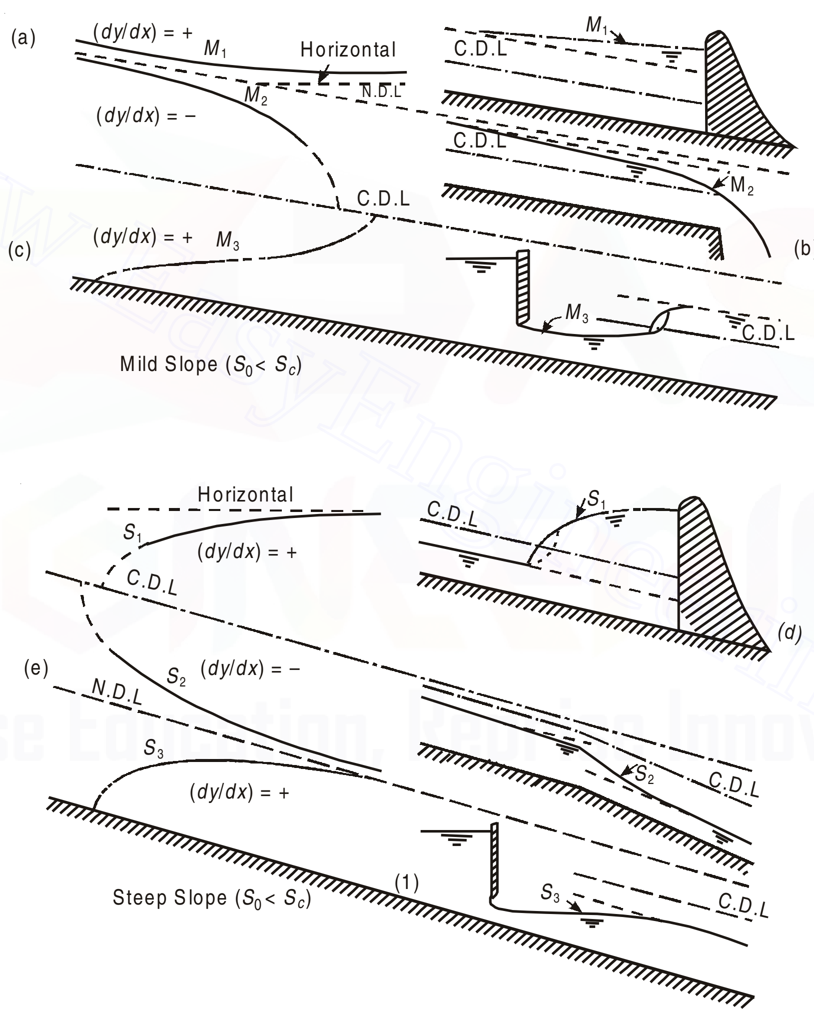


Figure 16.5 *Practical examples of M and S surface profiles*

Some of the practical examples of the various surface profiles are shown in Figs. 16.5 and 16.6. Since the flow profiles near the critical depth line and the channel bottom cannot be accurately defined by the theory of gradually varied flow, they are shown with dotted lines.

In order to determine the class of surface profile in a given channel the following procedure may be adopted :

(i) Compute y_n and y_c for the given discharge and plot the lines representing channel bed and the lines of normal and critical depths.

(ii) By comparing the normal depth and the critical depth determine whether the channel slope is mild, critical, steep, adverse or horizontal.

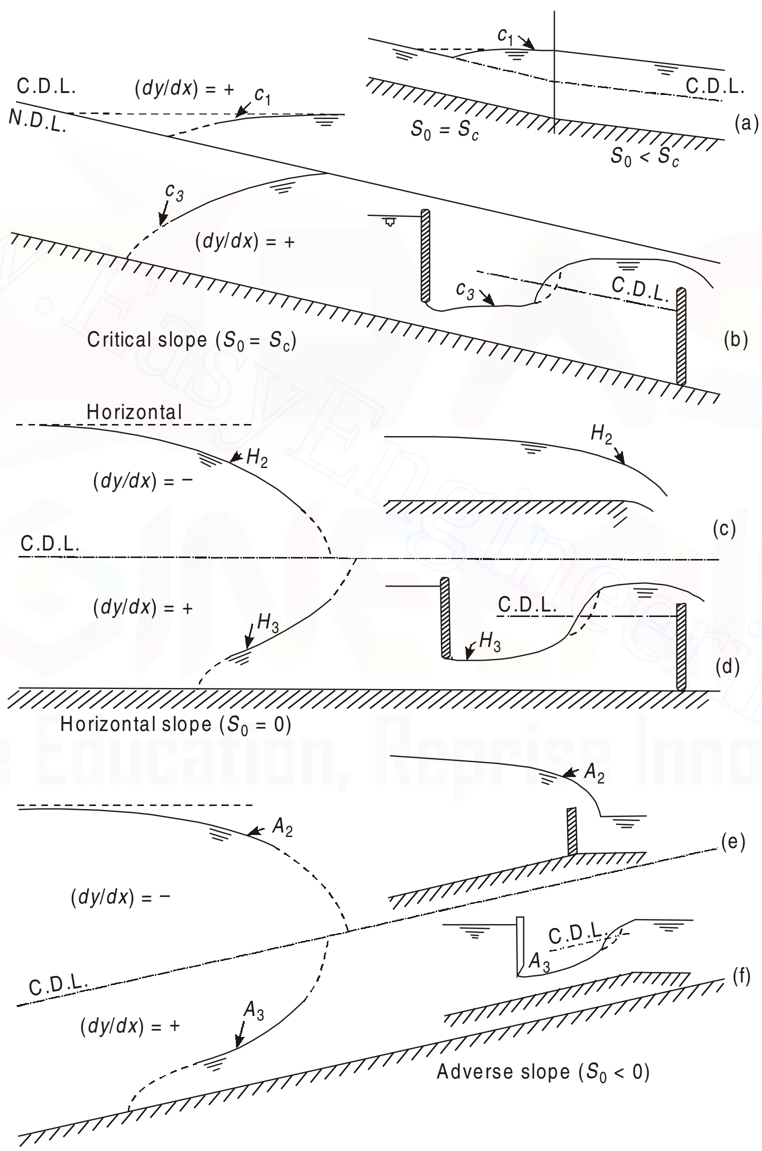


Figure 16.6 *Practical examples of C, H and A surface profiles.*

(iii) Knowing the normal depth and the depth at the control section, determine the type of the surface profile.

A *control section* is that at which for a given discharge the depth of flow is known or it can be controlled to a required value. If the depth of non-uniform flow y is above the critical depth it is governed by a downstream control and if it is less than the critical depth it is governed by an upstream control.

Thus knowing the values of the given depth y, normal depth y_n and the critical depth y_c the appropriate surface profile can be sketched.

(iv) If in any reach the supercritical depth has to meet the subcritical depth, i.e., when the stream of water has to cross the critical depth line then there will be a hydraulic jump developed in between, as shown in Figs. 16.5 and 16.6.

16.6 INTEGRATION OF THE VARIED FLOW EQUATION

In practice it is often required to determine the distance upto which the surface profile of gradually varied flow extends. For instance, if a weir is constructed across a river having a mild slope then it may be required to estimate the distance on the upstream side upto which the effect of the resulting M_1 profile exists. In order to solve the problems of this type it is necessary to integrate the dynamic equation of gradually varied flow. The various methods developed for integrating the varied flow equation may be broadly classified as follows:

(a) The step method.

(b) The graphical integration method.

(c) The direct integration method.

(a) The Step Method. In the step method of integration the entire length of the channel is divided into short reaches and the computation is carried out step by step from one end of the reach to the other. Figure 16.7 illustrates a channel reach of length dx, which is sufficiently small so that in this reach the

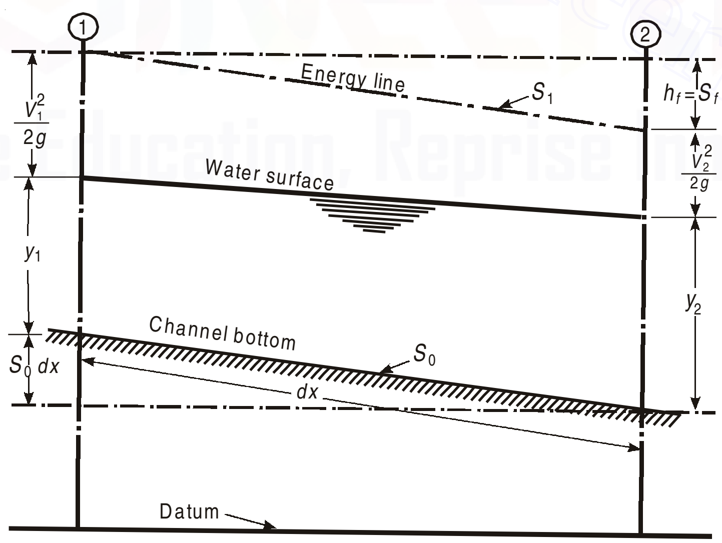


Figure 16.7 *Short reach of channel*

16.7 HYDRAULIC JUMP

The hydraulic jump is defined as the sudden and turbulent passage of water from a supercritical state to subcritical state. It has been classified as rapidly varied flow, since the change in depth of flow from rapid to tranquil state is in an abrupt manner over a relatively short distance. The flow in a hydraulic jump is accompanied by the formation of extremely turbulent rollers and there is a considerable dissipation of energy.

A hydraulic jump will form when water moving at a supercritical velocity in a relatively shallow stream strikes water having a relatively large depth and subcritical velocity. It occurs frequently in a canal below a regulating sluice, at the foot of a spillway, or at the place where a steep channel bottom slope suddenly changes to a flat slope.

In order to study the conditions of flow before and after the hydraulic jump the application of the energy equation does not provide an adequate means of analysis, because hydraulic jump is associated with an appreciable loss of energy which is initially unknown. As such in the analysis of hydraulic jump the momentum equation is used by considering the portion of the hydraulic jump as the control volume. The following assumptions are, however, made in this analysis:

(1) It is assumed that before and after jump formation the flow is uniform and the pressure distribution is hydrostatic.

(2) The length of the jump is small so that the losses due to friction on the channel floor are small and hence neglected.

(3) The channel floor is horizontal or the slope is so gentle that the weight component of the water mass comprising the jump is negligibly small.

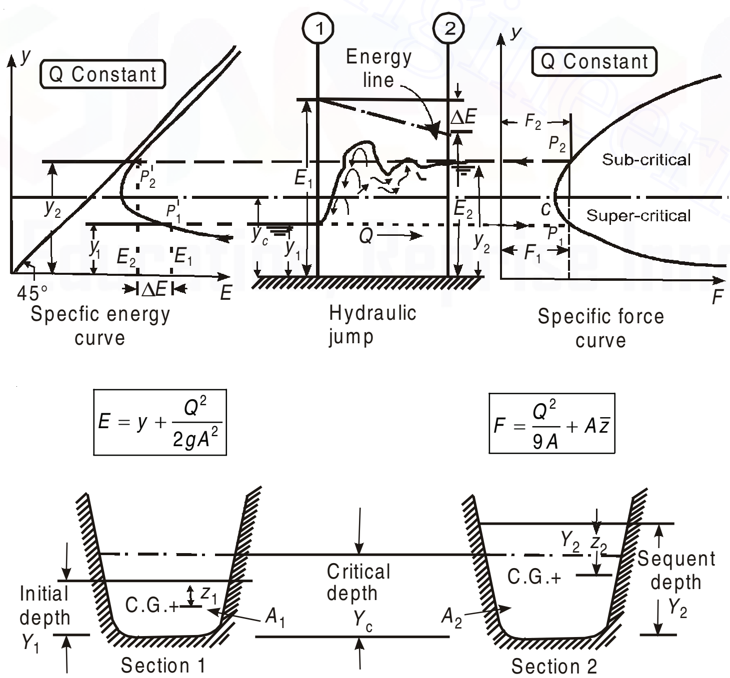


Figure 16.8 *Hydraulic jump in a prismatic channel*

Consider a hydraulic jump formed in a prismatic channel with horizontal floor carrying a discharge Q as shown in Fig. 16.8. Let the depth of flow before the jump at section 1 be y_1 and the depth of flow after the jump at section 2 be y_2. The depth y_1 is known as *initial depth* and y_2 is known as *sequent depth.* The symbols A_1, V_1 and $\bar{z}_1$ represent the area of cross-section, mean velocity of flow and the depth of the centroid of area A_1 below the free surface respectively at section 1 before the jump and A_2, V_2 and $\bar{z}_2$ are the corresponding quantities at section 2 after the jump.

The only external forces acting on the mass of water between the sections 1 and 2 are the hydrostatic pressures P_1 and P_2 at sections 1 and 2 respectively, as the frictional loss has been assumed to be negligible. Hence in accordance with the momentum equation

$$(P_2 - P_1) = \rho Q (V_1 - V_2)$$

or

$$wA_2\,\bar{z}_2 - wA_1\,\bar{z}_1 = \frac{w}{g}\,Q\left(\frac{Q}{A_1} - \frac{Q}{A_2}\right)$$

or

$$\frac{Q^2}{gA_1} + A_1\,\bar{z}_1 = \frac{Q^2}{gA_2} + A_2\,\bar{z}_2 \quad ...(16.14)$$

As already explained in Chapter 15, the sum $\left(\frac{Q^2}{gA} + A\bar{z}\right)$ is called the *specific force,* designated by F. Thus if F_1 and F_2 represent the specific force at sections 1 and 2 respectively then Eq. 16.14 may be written as

$$F_1 = F_2 \quad ...(16.15)$$

The specific force F is a function of the depth of flow y and hence it can be plotted against the depth of flow y to obtain specific force curve as shown in Fig. 16.8. It may be noted from the specific force curve that alike specific energy the specific force also attains a minimum value at critical depth y_c.

It is obvious from Eq. 16.15 that the specific force F_1 corresponding to y_1 and the specific force F_2 corresponding to y_2 are same which may also be seen from the specific force curve shown in Fig. 16.8. The initial depth y_1 and the sequent depth y_2 are commonly known as the *conjugate depths,* which indicate the same specific force (in order to distinguish them from the alternate depths which indicate the same specific energy).

Equation 16.14 enables the determination of y_2, if y_1 is given or vice versa for a known discharge Q flowing in a given channel section. Alternatively from the specific force curve also, knowing y_1 and hence F_1, y_2 can be determined or vice versa, since both y_1 and y_2 correspond to same specific force.

Knowing the conjugate depths y_1 and y_2 for a known discharge Q in a given channel section the specific energies E_1 and E_2 at the sections 1 and 2 respectively may be computed from which the loss of energy in the jump can be determined. Alternatively the loss of energy in the jump may also be determined by using the specific energy curve in combination with specific force curve as indicated below.

Knowing y_1 the corresponding specific force F_1 can be found from specific force curve as indicated by the point P_1. A vertical through point P_1 will cut the curve at point P_2. Since $F_1 = F_2$, the ordinate of point P_2 will indicate the depth y_2. Horizontal lines drawn through points P_1 and P_2 will cut the specific energy curve at points P'_1 and P'_2 respectively, indicating the values of the specific energies before and after the jump, i.e., E_1 and E_2 respectively. The horizontal distance between points P'_1 and P'_2 is the energy loss ΔE, due to hydraulic jump.

(a) Hydraulic Jump in Rectangular Channels. For rectangular channels Eq. 16.14 can be further simplified and a relation between the conjugate depths y_1 and y_2 can be obtained. Thus if B is the width of the rectangular channel, then $A_1 = By_1$; $A_2 = By_2$, $\bar{z}_1 = (y_1/2)$, $\bar{z}_2 = (y_2/2)$ and $q = (Q/B)$. By substituting these values in Eq. 16.14, it becomes

$$\frac{Q^2}{g(By_1)} + By_1\left(\frac{y_1}{2}\right) = \frac{Q^2}{g(By_2)} + By_2\left(\frac{y_2}{2}\right)$$

or $$\frac{Q^2}{gB^2}\left(\frac{1}{y_1} - \frac{1}{y_2}\right) = \frac{1}{2}(y_2^2 - y_1^2)$$

or $$\frac{q^2}{g}\left(\frac{y_2 - y_1}{y_1 y_2}\right) = \frac{1}{2}(y_2 - y_1)(y_2 + y_1)$$

or $$\frac{2q^2}{g} = y_1 y_2 (y_1 + y_2) \quad \text{...(16.16)}$$

Equation 16.16 is the momentum equation for hydraulic jump in rectangular channels.

Equation 16.16 can be considered as quadratic equation in terms of y_1 or y_2 and its solution gives the relations between the conjugate depths y_1 and y_2 as

$$y_2 = -\frac{y_1}{2} + \sqrt{\left(\frac{y_1}{2}\right)^2 + \frac{2q^2}{gy_1}} \quad \text{...(16.17)}$$

$$y_1 = -\frac{y_2}{2} + \sqrt{\left(\frac{y_2}{2}\right)^2 + \frac{2q^2}{gy_2}} \quad \text{...(16.18)}$$

In Eqs 16.17 and 16.18 the negative sign before the square root has not been used since it gives negative values.

Equations 16.17 and 16.18 may also be written as

$$\frac{y_2}{y_1} = \frac{1}{2}\left[-1 + \sqrt{1 + \frac{8q^2}{gy_1^3}}\right] \quad \text{...(16.19)}$$

$$\frac{y_1}{y_2} = \frac{1}{2}\left[-1 + \sqrt{1 + \frac{8q^2}{gy_2^3}}\right] \quad \text{...(16.20)}$$

Since for a rectangular channel $\left(\frac{q^2}{g}\right) = y_c^3$, where y_c is the critical depth for discharge q. The Eqs 16.19 and 16.20 may also be written as

$$\frac{y_2}{y_1} = \frac{1}{2}\left[-1+\sqrt{1+8\left(\frac{y_c}{y_1}\right)^3}\right] \qquad \text{...(16.21)}$$

$$\frac{y_1}{y_2} = \frac{1}{2}\left[-1+\sqrt{1+8\left(\frac{y_c}{y_2}\right)^3}\right] \qquad \text{...(16.22)}$$

Since the Froude numbers Fr_1 and Fr_2 before and after the hydraulic jump respectively are given as

$$Fr_1 = \frac{q}{\sqrt{gy_1^3}} \quad \text{and } Fr_2 = \frac{q}{\sqrt{gy_2^3}}$$

Equations 16.21 and 16.22 may be written as

$$\frac{y_2}{y_1} = \frac{1}{2}\left[-1+\sqrt{1+8Fr_1^2}\right] \qquad \text{...(16.23)}$$

and

$$\frac{y_1}{y_2} = \frac{1}{2}\left[-1+\sqrt{1+8Fr_2^2}\right] \qquad \text{...(16.24)}$$

When the conjugate depths are known the energy loss ΔE in a hydraulic jump may be computed as

$$\Delta E = \left(y_1 + \frac{V_1^2}{2g}\right) - \left(y_2 + \frac{V_2^2}{2g}\right)$$

or

$$\Delta E = \left[y_1 + \frac{Q^2}{2g(By_1)^2}\right] - \left[y_2 + \frac{Q^2}{2g(By_2)^2}\right]$$

or

$$\Delta E = \frac{q^2}{2g}\left(\frac{1}{y_1^2} - \frac{1}{y_2^2}\right) - (y_2 - y_1)$$

or

$$\Delta E = \frac{q^2}{2g}\frac{y_2^2 - y_1^2}{(y_1 y_2)^2} - (y_2 - y_1)$$

By substituting for $\left(\frac{q^2}{g}\right)$ from Eq. 16.16 the above expression becomes

$$\Delta E = \frac{1}{4}\frac{y_1 y_2 (y_1 + y_2)(y_2^2 - y_1^2)}{(y_1 y_2)^2} - (y_2 - y_1)$$

which on simplification becomes

$$\Delta E = \frac{(y_2 - y_1)^3}{4y_1y_2} \qquad ...(16.25)$$

However, as indicated in Illustrative Example 16.12, the energy loss ΔE in a hydraulic jump in a rectangular channel may also be expressed as

$$\Delta E = \frac{(V_1 - V_2)^3}{2g(V_1 + V_2)} \qquad ...(16.25\text{ a})$$

where V_1 and V_2 are the mean velocities of flow before and after the jump respectively.

The *height of the jump* h_j may be defined as the difference between the depths after and before the jump, i.e., $h_j = (y_2 - y_1)$.

The *length of the jump* L_j may be defined as the distance measured from the front face of the jump to a point on the surface immediately downstream from the roller. However, the length of the jump cannot be determined analytically. In addition, practical complications arise from the general instability of the phenomenon and the difficulty of defining the beginning and the end sections of the jump. The length of the jump has been investigated experimentally by many hydraulicians and as a general statement it may be said that for a rectangular channel the length of the jump L_j varies between 5 and 7 times the height of the jump, that is,

$$L_j = (5 \text{ to } 7)\, h_j = (5 \text{ to } 7)\,(y_2 - y_1) \qquad ...(16.26)$$

(b) Types of Hydraulic Jump. Equation 16.23 emphasizes the importance of the Froude number Fr_1 of the incoming supercritical flow, as a parameter describing the phenomenon of hydraulic jump. As such according to the studies of U.S. Bureau of Reclamation, depending upon the value of Froude number Fr_1 of the incoming flow, there are five distinct types of the hydraulic jump which may occur on a horizontal floor. These different types of hydraulic jump are shown in Fig. 16.9 and are described below.

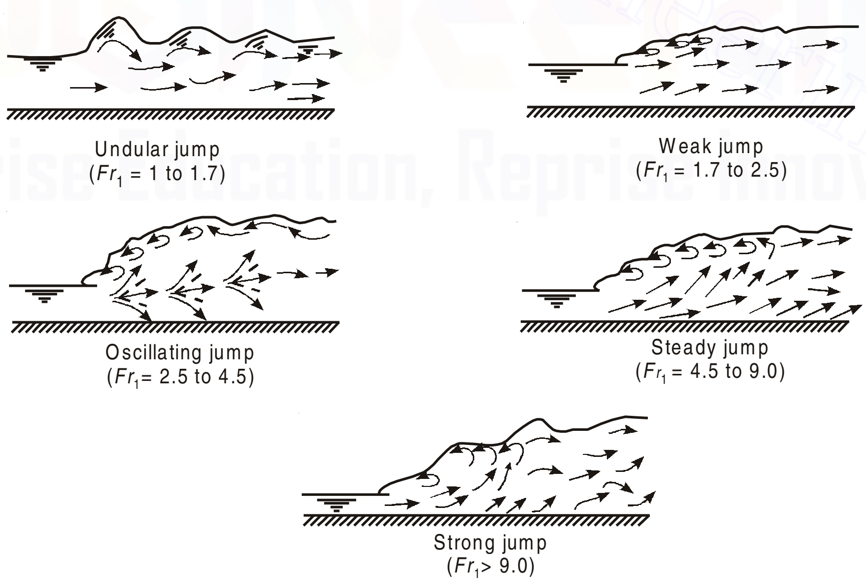


Figure 16.9 *Types of hydraulic jump*

(1) For Fr_1 = 1.0 to 1.7, the water surface shows undulations and the jump is called an *undular jump.*

(2) For Fr_1 = 1.7 to 2.5, the jump formed is called *weak jump,* as the velocity throughout is fairly uniform and only a small amount of energy is dissipated. In this case a series of small rollers form on the jump surface, but the downstream water surface remains quite smooth.

(3) For Fr_1 = 2.5 to 4.5, jump formed is known as an *oscillating jump.* In this case the entering jet of water oscillates back and forth from the bottom to the surface and back again.

(4) For Fr_1 = 4.5 to 9.0 the jump formed is well stabilized and is called a *steady jump.* For this jump the energy dissipation ranges from 45 to 70 per cent.

(5) For Fr_1 = 9.0 and larger the jump formed is called a *strong jump.* In this case a rough surface prevails which continues downstream for a long distance. The jump action is quite rough but is effective since the energy dissipation may reach 85 per cent.

The above described different types of hydraulic jump refer only to channels of rectangular section. In other channel sections the shape of the jump is often complicated additionally by cross currents.

In the above paragraphs only the hydraulic jump in rectangular channel has been discussed, but it may however be mentioned that by using Eq. 16.14 the hydraulic jump in prismatic channel of any shape can be analysed.

(c) Applications of Hydraulic Jump. The phenomenon of hydraulic jump has many practical applications as listed below.

(1) It is a useful means of dissipating excess energy of water flowing over spillways and other hydraulic structures or through sluices and thus preventing possible erosion on the downstream side of these structures.

(2) It raises the water level in the channels for irrigation etc.

(3) It increases the weight on an apron of a hydraulic structure due to increased depth of flow and hence the uplift pressure acting on the apron is considerably counterbalanced.

(4) It increases the discharge through a sluice by holding back the tail water.

(5) It may be used for mixing chemicals in water and other liquids, since it facilitates thorough mixing due to turbulence created in it.

16.8 LOCATION OF HYDRAULIC JUMP

Often it is required to locate the exact position of the hydraulic jump in a channel under different conditions of flow. As such the following three typical cases for the location of the exact position of the hydraulic jump are described below.

Case (1)

In this case a jump forms below a regulating sluice in a mild sloped channel, see Fig. 16.10 (a). The jet of water issuing from the sluice will contract upto vena contracta section at a distance L_e from the sluice which is taken approximately equal to the sluice opening *h.* Thereafter it will follow M_3 profile as indicated by *DE* in Fig.16.10 (a). The location of the jump in this case will be considerably affected by the length of the channel reach on the downstream side of the sluice. Thus if there exists a long reach of channel with same slope on the downstream side of the sluice, then after the formation of the jump, uniform flow with depth of flow equal to the normal depth of flow will be developed. Hence the depth of flow after the jump or the sequent depth will be equal to the normal depth of flow. As such from the vena contracta section the depth of flow will gradually increase, following M_3 profile, upto a certain section on the downstream side, where the depth of flow will be equal to the initial depth required for the formation of the jump corresponding to the sequent depth equal to the normal depth