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NEPAL ENGINEERING COUNCIL
LICENSE EXAM PREPARATION COURSE

FOR

CIVIL ENGINEERS

3. Basic Water Resources Engineering

3.3 Hydro-kinematics and hydro-dynamics

Sub topics

- Classification of fluid flow;
- conservation of mass (continuity equation)
- momentum equations and their applications
- Bernoulli’s equation and its application
- flow measurement.

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Classification of fluid flow

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Viscous or Non-viscous Flow

Viscous flow has relative motion between fluid layers.
Flow of ideal fluid are non viscous

Classification of fluid flow

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Steady or Unsteady Flow

Steady: A steady flow is one in which the conditions (velocity, pressure and crosssection) may differ from point to point but DO NOT change with time.

Unsteady: If at any point in the fluid, the conditions change with time, the flow is described as unsteady.

$$\frac{\partial(fluid\ property)}{\partial t} = 0, at\ any\ particular\ point$$

Classification of fluid flow

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Uniform or Non Uniform Flow

Uniform flow: If the flow velocity is the same magnitude and direction at every point in the fluid it is said to be uniform.

Non-uniform: If at a given instant, the velocity is not the same at every point the flow is non-uniform.

$$\frac{\partial(fluid\ property)}{\partial x} = 0, at\ any\ particular\ instant$$

Classification of fluid flow

Combining the above we can classify any flow in to one of four type:

1. Steady uniform flow. Conditions do not change with position in the stream or with time. An example is the flow of water in a pipe of constant diameter at constant velocity.
2. Steady non-uniform flow. Conditions change from point to point in the stream but do not change with time. An example is flow in a tapering pipe with constant velocity at the inlet - velocity will change as you move along the length of the pipe toward the exit.

Classification of fluid flow

3. Unsteady uniform flow. At a given instant in time the conditions at every point are the same, but will change with time. An example is a pipe of constant diameter connected to a pump pumping at a constant rate which is then switched off.
4. Unsteady non-uniform flow. Every condition of the flow may change from point to point and with time at every point. For example waves in a channel.

Classification of fluid flow

Reynolds number (Re) is a dimensionless quantity that helps predict fluid flow patterns.

Reynolds number =inertial force/ viscous forces

Mach number is the ratio of the speed of a body to the speed of sound in the surrounding medium. At 20°C velocity of sound is 343 m/s.

Classification of fluid flow

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Compressible or incompressible flow

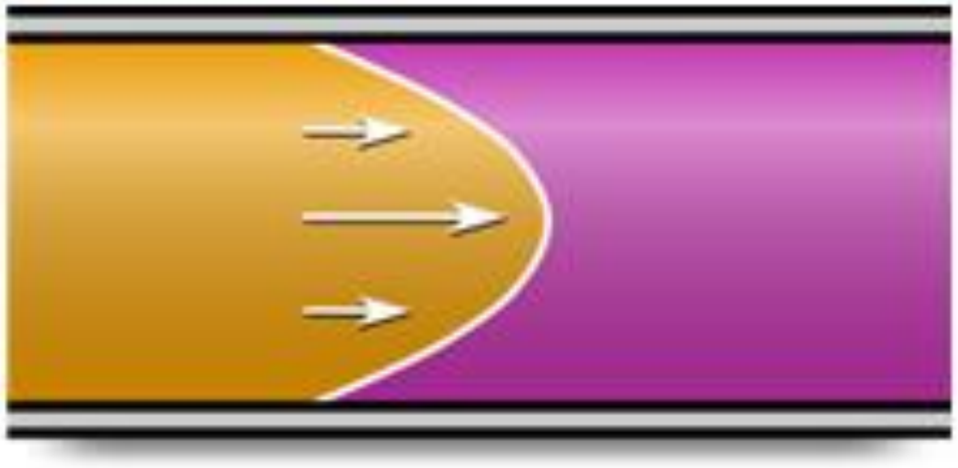
Compressible flow: Density changes

Incompressible flow:
Density remains same
Flow with mach number less than 0.3 is considered Incompressible flow.

Classification of fluid flow

Laminar flow or streamline flow in pipes (or tubes) occurs when a fluid flows in parallel layers, with no disruption between the layers.

Reynolds numbers smaller than 2300



Classification of fluid flow

Turbulent flow is a flow regime characterized by chaotic property changes. This includes a rapid variation of pressure and flows velocity in space and time. In contrast to laminar flow, the fluid no longer travels in layers, and mixing across the tube is highly efficient.

Classification of fluid flow

Pipe flow:

Laminar: $\boldsymbol{R_e < 2000}$

Transitional: $\boldsymbol{2000 < R_e < 4000}$

Turbulent: $\boldsymbol{R_e > 4000}$

V is velocity, d is diameter of pipe

$$R_e = \frac{\rho V d}{\mu} = \frac{V d}{\nu}$$

Classification of fluid flow

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Open channel flow:

Laminar: $\boldsymbol{R_e < 500}$

Transitional: $\boldsymbol{500 < R_e < 2000}$

Turbulent: $\boldsymbol{R_e > 2000}$

$$R_e = \frac{\rho V l}{\mu} = \frac{V l}{\nu}$$

V is velocity, l is characteristic length or hydraulic radius in 2D flow

Classification of fluid flow

Rotational Flow vs Irrotational Flow:

In rotational flow, fluid particles move around their own axis leading to a non-zero vorticity. In contrast, irrotational flow is defined by zero vorticity, i.e., fluid particles do not rotate around their own axis.

$$vortosity = \text{curl of vector} = \nabla \times v = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{vmatrix}$$

$$vortosity = \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right) i + \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right) j + \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) k$$

Classification of fluid flow

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Rotational Flow vs Irrotational Flow:

$$angular\ velocity = \frac{1}{2}\nabla \times v = \frac{1}{2}\begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{vmatrix}$$

$$angular\ velocity = \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)i + \frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)j + \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)k$$

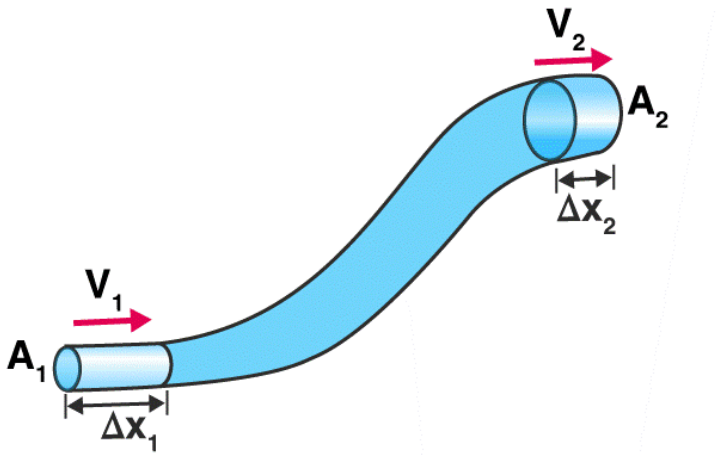
For $\omega = \omega_x i + \omega_y \mathrm{j} + \omega_z k = 0, flow\ is\ irrotational$

Conservation of mass

- The tube is having a single entry and single exit
- The fluid flowing in the tube is non-viscous
- The fluid flow is steady

mass inflow=mass outflow + change in mass in syst

$$\dot{m}_{in} = \dot{m}_{out} + \dot{m}_{ch}$$
$$\dot{m}_{ch} = 0$$
$$\dot{m}_{in} = \dot{m}_{out}$$
$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$$



If flow is incompressible, $\rho_1 = \rho_2$

$$A_1 v_1 = A_2 v_2$$

Conservation of mass

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mass inflow=mass outflow + change in mass in system

$$\dot{m}_{in} = \dot{m}_{out} + \dot{m}_{ch}$$

$$\frac{\partial \rho}{\partial t} + \nabla.(\rho v) = 0$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0$$

Conservation of mass

Incompressible flow

$$\rho = constant$$

$$\nabla . v = 0$$

$$\frac{\partial (u)}{\partial x} + \frac{\partial (v)}{\partial y} + \frac{\partial (w)}{\partial z} = 0$$

Conservation of mass

Steady flow and compressive

No change of fluid property with time

$$\nabla.(\rho v) = 0$$

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0$$

Conservation of momentum

$$\sum F = \frac{change\ in\ momentum}{time}$$

If flow is incompressible

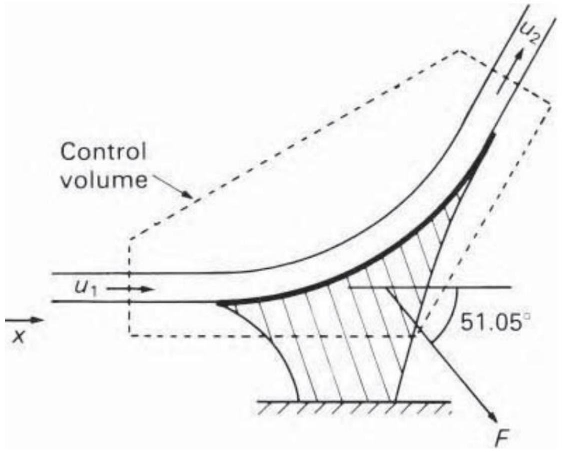
$$\sum F = \rho A_2 u_2^2 - \rho A_1 u_1^2$$

If flow is continuous and

$$\sum F = \rho A_1 u_1 (u_1 - u_2) = \rho Q (u_2 - u_1)$$

Conservation of momentum

A jet of water flows smoothly on to a stationary curved vane which turns it through 60°. The initial jet is 50 mm in diameter, and the velocity, which is uniform, is $36\ \mathrm{m \cdot s^{-1}}$. As a result of friction, the velocity of the water leaving the surface is $30\ \mathrm{m \cdot s^{-1}}$. Neglecting gravity effects, calculate the hydrodynamic force on the vane.



Conservation of momentum

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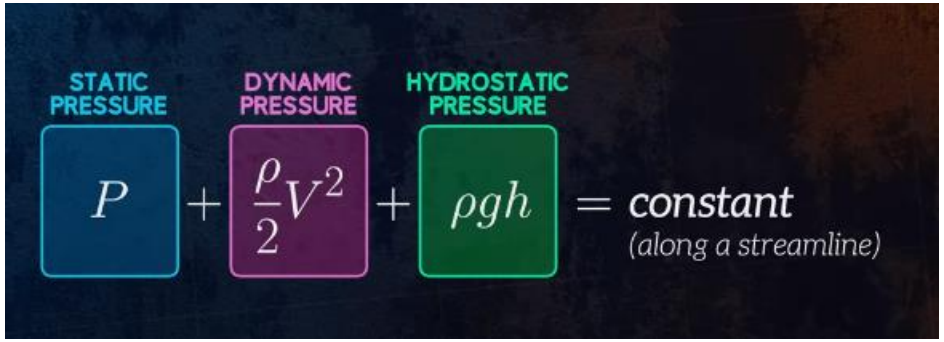
Force *on fluid* in x direction

$$= \text{Rate of increase of } x\text{-momentum}$$
$$= \varrho Q u_2 \cos 60^\circ - \varrho Q u_1$$
$$= (1000\ \text{kg} \cdot \text{m}^{-3}) \left\{ \frac{\pi}{4}(0.05)^2\ \text{m}^2 \times 36\ \text{m} \cdot \text{s}^{-1} \right\}$$
$$\times (30 \cos 60^\circ\ \text{m} \cdot \text{s}^{-1} - 36\ \text{m} \cdot \text{s}^{-1})$$
$$= -1484\ \text{N}$$

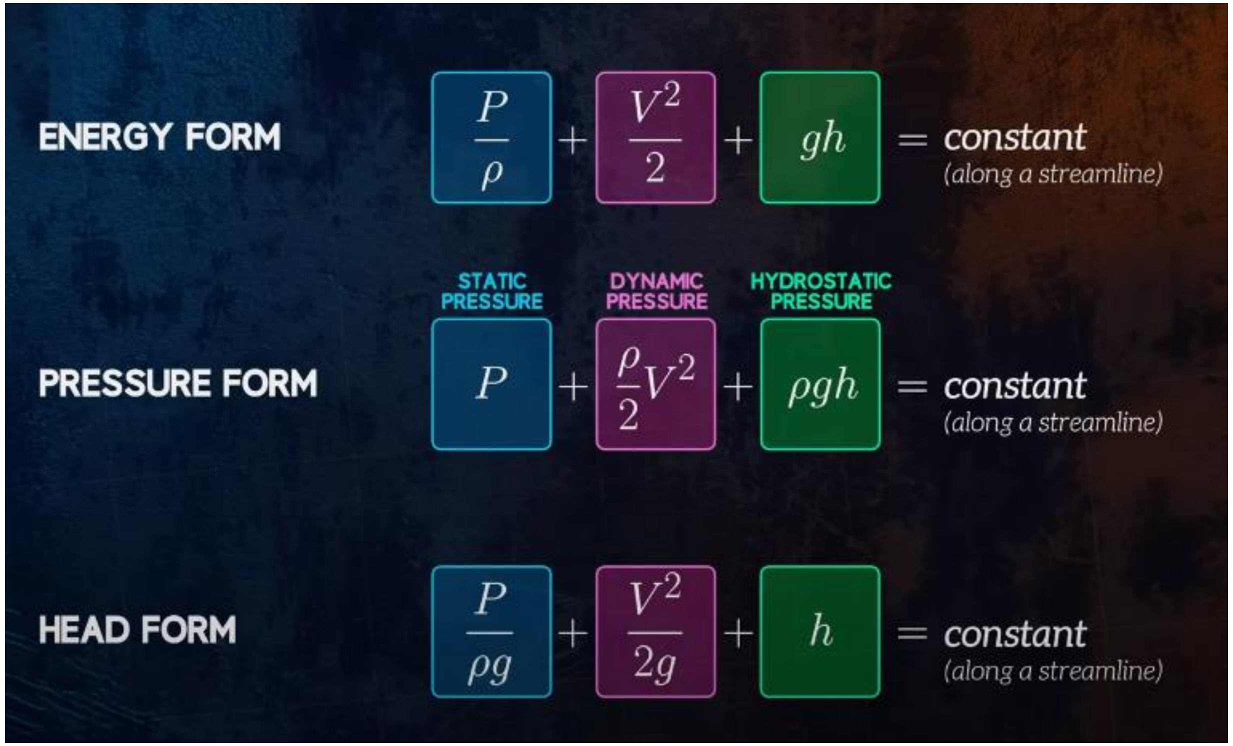
Similarly, force *on fluid* in y direction

$$= \varrho Q u_2 \sin 60^\circ - 0$$
$$= \left\{ 1000 \frac{\pi}{4}(0.05)^2 36\ \text{kg} \cdot \text{s}^{-1} \right\} (30 \sin 60^\circ \text{m} \cdot \text{s}^{-1})$$
$$= 1836\ \text{N}$$

Bernoulli's Equation



Bernoulli's Equation



Venturimeter

$$\frac{P_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{P_2}{\gamma}+\frac{V_2^2}{2g}+z_2$$

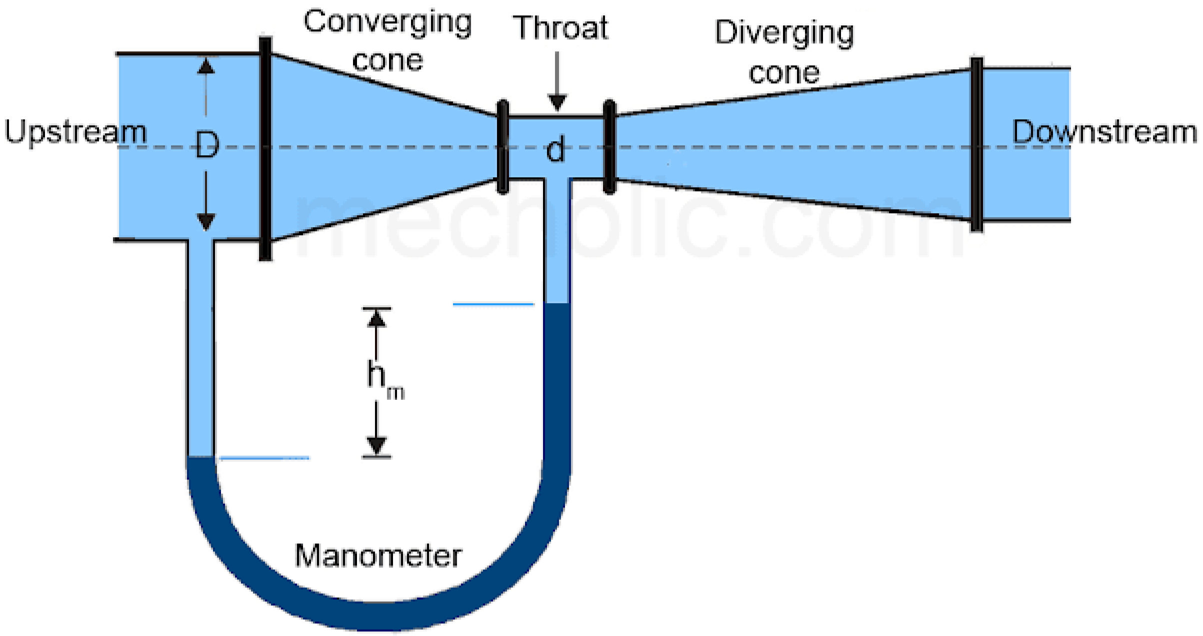
$$\frac{V_2^2}{2g}-\frac{V_1^2}{2g}=\left(\frac{P_1}{\gamma}+z_1\right)-\left(\frac{P_2}{\gamma}+z_2\right)=\mathrm{h}$$

$$\frac{1}{2g}(V_2^2-V_1^2)=\frac{1}{2g}\left(\frac{Q^2}{A_2^2}-\frac{Q^2}{A_1^2}\right)=\frac{Q^2}{2g}\left(\frac{A_1^2-A_2^2}{A_1^2A_2^2}\right)$$

$$Q=\frac{A_1A_2\sqrt{2gh}}{\sqrt{A_1^2-A_2^2}}$$

Where,

$$h=h_m\left(\frac{\gamma_m}{\gamma_L}-1\right)$$



Venturimeter

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$$\frac{P_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{P_2}{\gamma}+\frac{V_2^2}{2g}+z$$

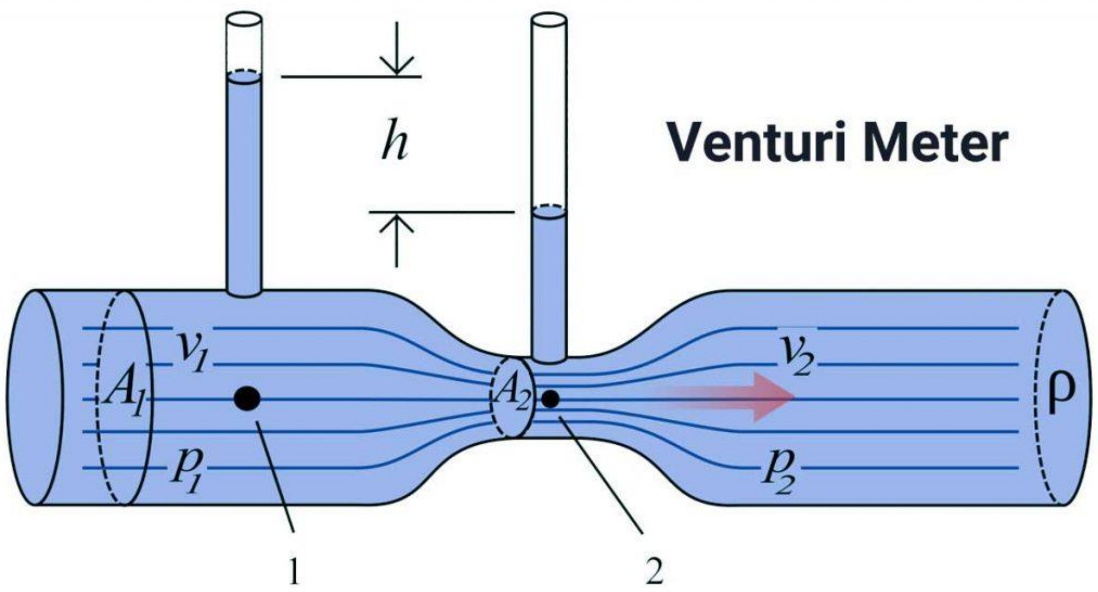
$$\frac{V_2^2}{2g}-\frac{V_1^2}{2g}=\left(\frac{P_1}{\gamma}+z_1\right)-\left(\frac{P_2}{\gamma}+z_2\right)=\mathrm{h}$$

$$\frac{1}{2g}(V_2^2-V_1^2)=\frac{1}{2g}\left(\frac{Q^2}{A_2^2}-\frac{Q^2}{A_1^2}\right)=\frac{Q^2}{2g}\left(\frac{A_1^2-A_2^2}{A_1^2A_2^2}\right)$$

$$Q=\frac{A_1A_2\sqrt{2gh}}{\sqrt{A_1^2-A_2^2}}$$

Where,

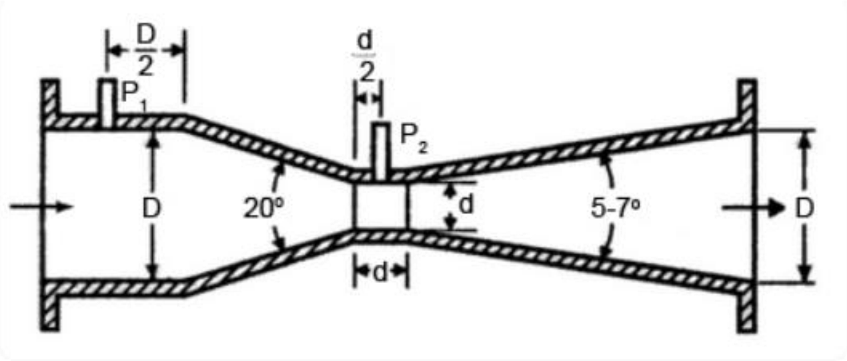
$$h=difference\ between\ piezometric\ head$$



Venturimeter

$$Q_{act} = C_d Q_{th}$$

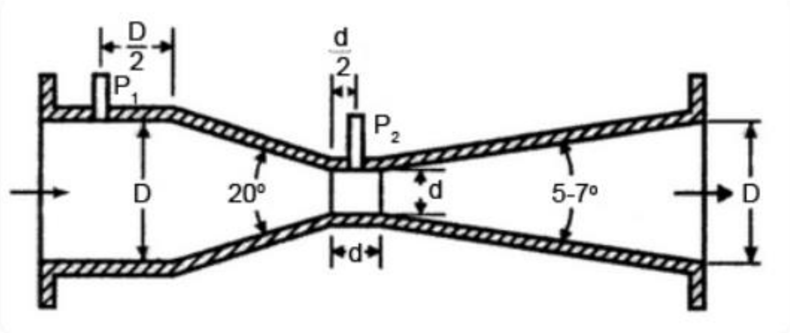
$$C_d \; = coefficient\ of\ discharge$$

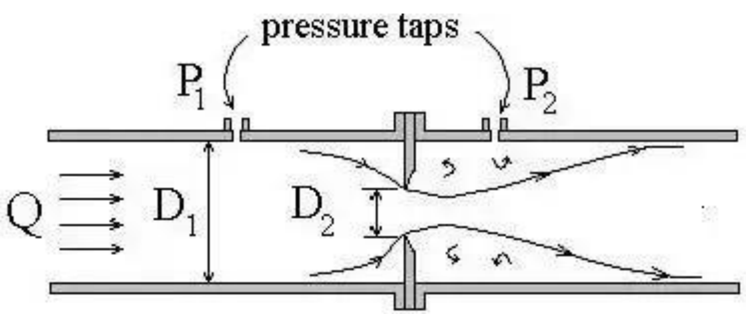


Venturimeter

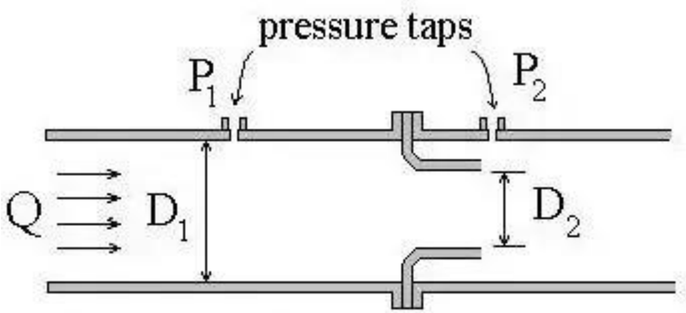
$$Q_{act} = C_d Q_{th}$$

C_d $= coefficient\ of\ discharge$
.jbh,





Orifice Meter Parameters



Flow Nozzle Meter Parameters

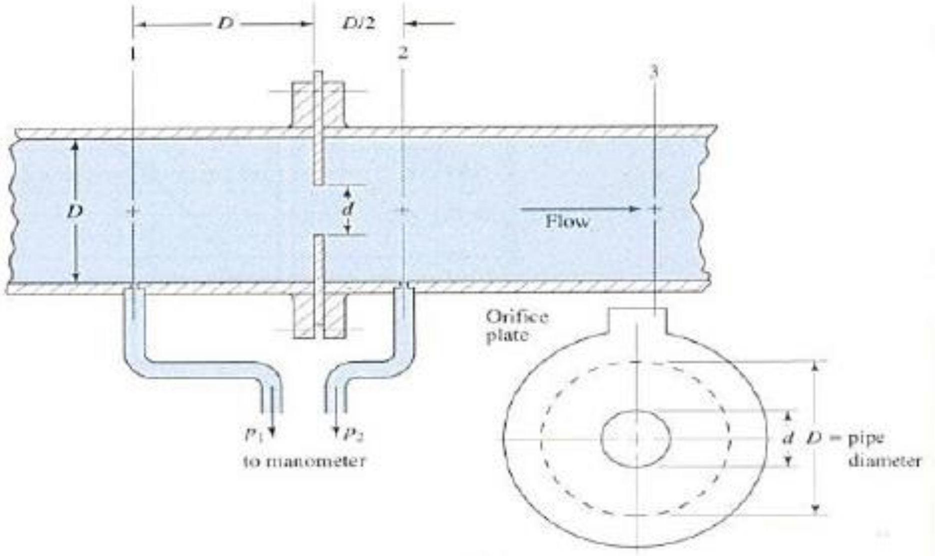


Figure 8 Orificemeter

$$Q = C_d \frac{A_0 A_1 \sqrt{2gh}}{\sqrt{A_1^2 - A_0^2}}$$

$$C_d = C_c \frac{\sqrt{1 - \frac{A_0^2}{A_1^2}}}{\sqrt{1 - \frac{A_0^2}{A_1^2} C_c^2}}$$

$$\Rightarrow C_c = C_d \frac{\sqrt{1 - \frac{A_0^2}{A_1^2} C_c^2}}{\sqrt{1 - \frac{A_0^2}{A_1^2}}}$$

A_1 is Area of section 1
A_0 is Area of Opening
C_c is Coefficient of Contraction
C_d is Coefficient of discharge
C_v is Coefficient of velocity
$C_d = C_c * C_v$

C_d, C_v, C_c

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Coefficient of velocity: $C_v = \frac{V_{th}}{V_{act}}$

Its value ranges from 0.95 to 0.99

For sharp edged orifices it is about 0.97

Coefficient of contraction: $C_c = \frac{A_{vc}}{A_{ori}} = \frac{Area\ at\ vena\ contracta}{Area\ of\ orifice\ opening}$

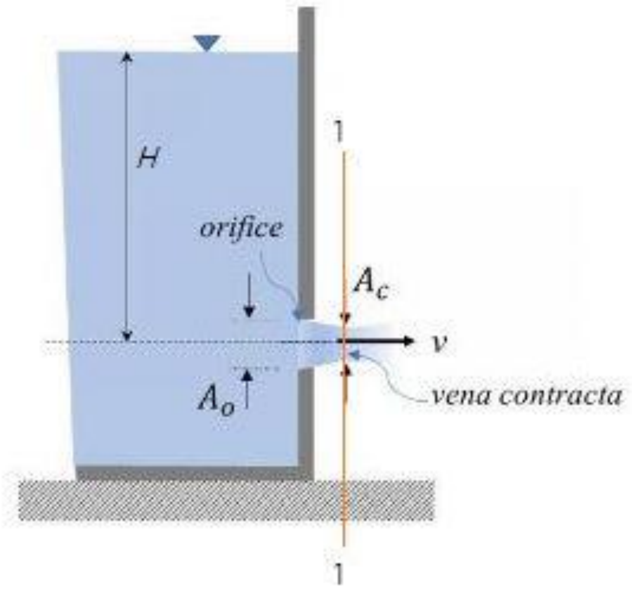
Its value ranges from 0.61 to 0.69

For sharp edged orifices it is about 0.65

Coefficient of discharge: $C_d = \frac{Q_{th}}{Q_{act}}$

$$C_d = C_c \times C_v$$

Its value ranges from 0.61 to 0.65



Pitot Tube

$$\frac{P}{\gamma} + \frac{V^2}{2g} + z = \mathrm{H}_1$$

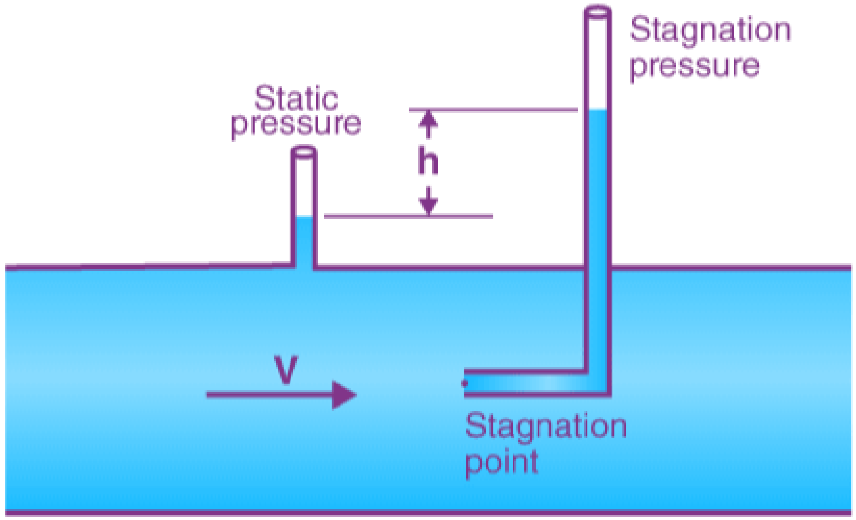
$$\frac{P}{\gamma} + z = \mathrm{H}_2$$

$$\mathrm{H}_2 + \frac{V_1^2}{2g} = \mathrm{H}_1$$

$$\frac{V_1^2}{2g} = \mathrm{H}_1 - \mathrm{H}_2 = h$$

$$V = \sqrt{2gh}$$

$$V_{act} = C_v V_{th}$$



2.14. Hydraulic gradient line (H.G.L):

It is defined as the line which gives the sum of pressure head $(\frac{p}{\gamma})$ and datum head (z) of a flowing fluid in pipe with respect to some reference or it is line which is obtained by joining the top of all vertical ordinates, showing the pressure head $(\frac{p}{\gamma})$ of a flowing fluid in a pipe from the centre of the pipe. The line so obtain is called the H.G.L.

2.15. Total energy loss (TEL or EGL)

It is known that the total head (which is also total energy per unit weight) with respect to any arbitrary datum, is the sum of the elevation (potential) head, pressure head and velocity head.

$$Total\ Energy = \frac{p}{\gamma} + \frac{v^2}{2g} + z$$

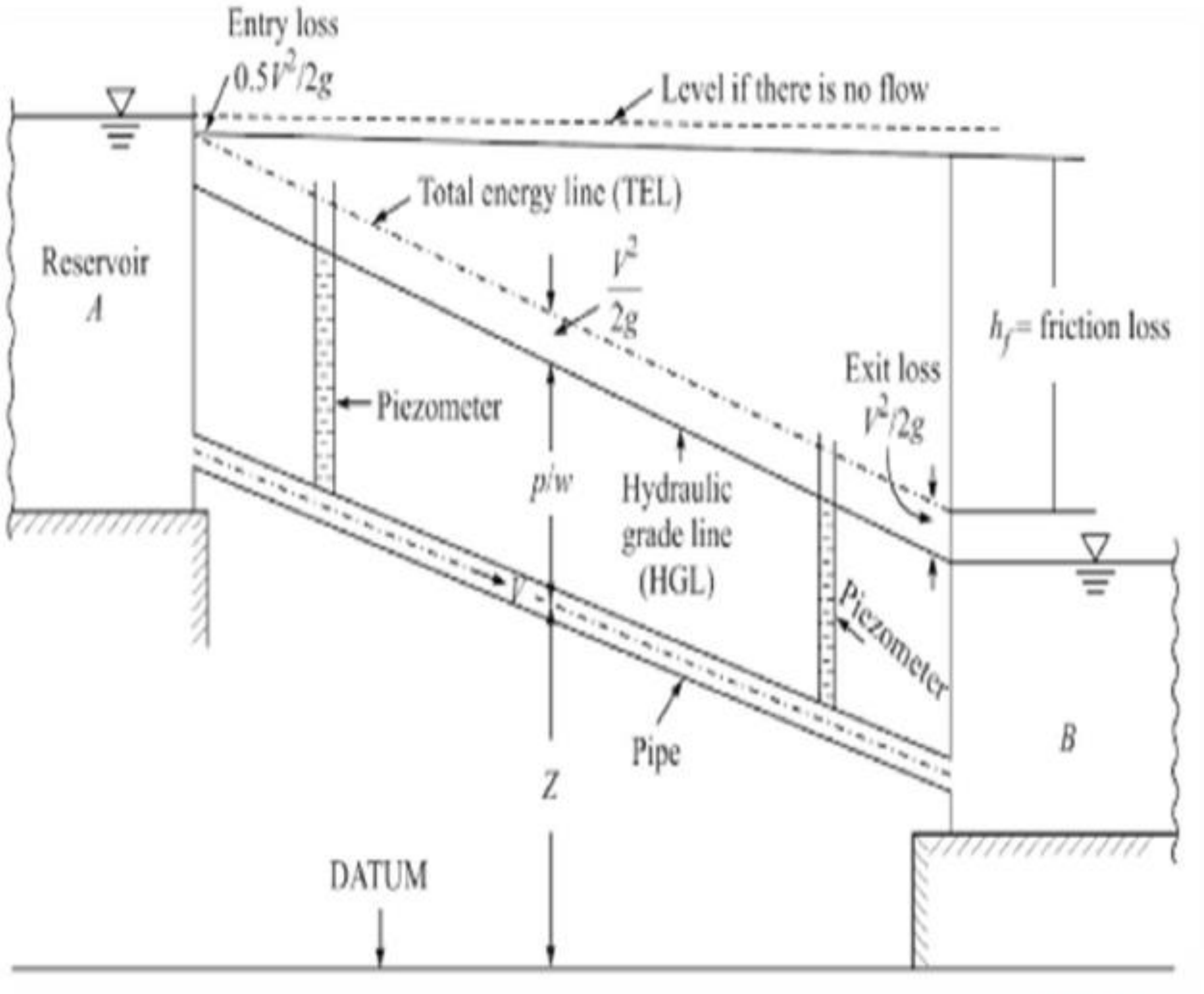


Figure 9 TEL and HGL

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3.1. Weir and Notches:

3.2. Weir :

A concrete or masonry structure built across rivers in order to raise the level of water on the u/s side to allow the excess water to flow over its entire length to d/s side.

Similar to small dam constructed across river but in dam excess water flows d/s through small portion called spillway, in weir water flows in entire length.

Nappe is sheet of water flowing through weir or notch.

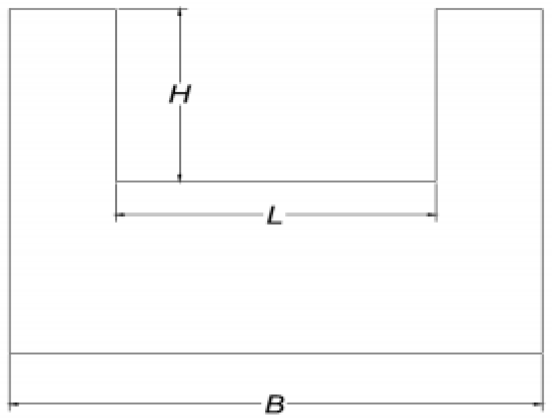
3.3. Notches:

Opening provided in side of tank (or vessel) such that the liquid surface in the tank is below the top edge of the opening.

Notches are used to measure rate of flow of liquid from a tank or in channel.

Notches	Weir	Weir According to shape of Crest	Weir According to discharging Behavior
Rectangular	Rectangular	Sharp edge weir	Freely Discharging
Triangular	Triangular	Narrow Crested	Submerged
Trapezoidal	Trapezoidal	Broad Crested	
Parabolic		Ogee Shaped	
Stepped			

3.4. Rectangular Weir:



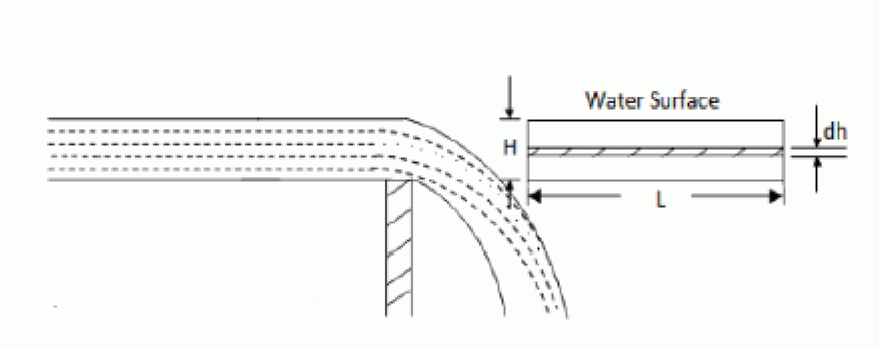


Figure 10 Rectangular Notch and Weir

Discharge Formula:

$Q = \frac{2}{3} C_d \sqrt{2g}\ L H^{3/2}$ – C_d is coefficient of discharge, L is length & H is head over crest

Considering End Contraction:

$Q = \frac{2}{3} C_d \sqrt{2g} * (L - 0.1nH) H^{3/2}$ – Where n is number of end contraction.

3.5. V notch Notch:

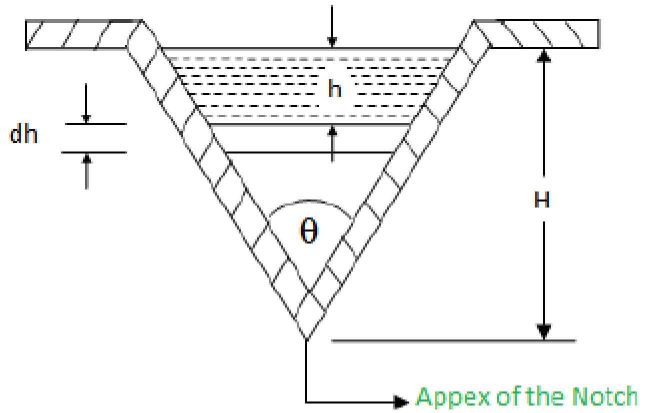


Fig : Triangular Notch

Figure 11 Triangular Notch

Discharge Formula

$Q = \frac{8}{15} C_d \sqrt{2g}\, tan \frac{\theta}{2} H^{5/2}$ – C_d is coefficient of discharge

Considering Approach Velocity

$Q = \frac{8}{15} C_d \sqrt{2g}\, tan \frac{\theta}{2} [(H + h_a)^{\frac{5}{2}} - h_a^{\frac{5}{2}}]$ – Where h_a is velocity head = $\frac{v_a^2}{2g}$

3.6. Trapezoidal Notch:

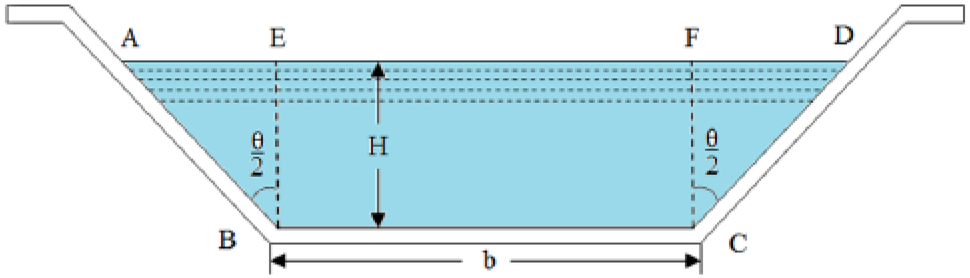


Fig : Trapezoidal Notch

Figure 12 Trapazoidal Notch

Discharge Formula

$$Q = \frac{2}{3} C_{d1}\sqrt{2g}\; L H^{3/2} + \frac{8}{15} C_{d2}\sqrt{2g}\; tan\frac{\theta}{2} H^{5/2}$$

If $C_{d1} = C_{d2} = C_d$

$$Q = C_d\sqrt{2g}\; H^{\frac{3}{2}} (\frac{2}{3}L + \frac{8}{15}\; H\; tan\frac{\theta}{2})$$

3.7. Cippoletti Notch

$$\frac{\theta}{2} = 14$$

Special condition of trapezoidal notch where side slope is kept, 1H:4V

$$tan\frac{\theta}{2} = \frac{1}{4}$$

$$Q_1 = \frac{2}{3}C_d\sqrt{2g} * (L - 0.1nH)H^{3/2}$$
$$Q_2 = \frac{8}{15}C_d\sqrt{2g}\, tan\frac{\theta}{2}H^{5/2}$$
$$Q = Q_1 + Q_2$$
$$Q = \frac{2}{3}C_d\sqrt{2g}\, LH^{3/2}, C_d = 0.632$$
$$Q = 1.86\, LH^{3/2}$$

3.8. Submerged Weir

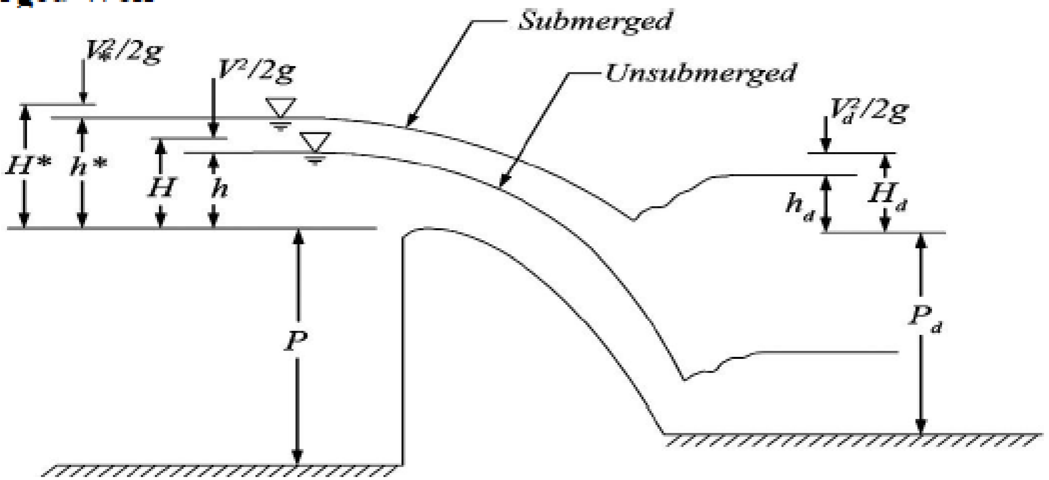
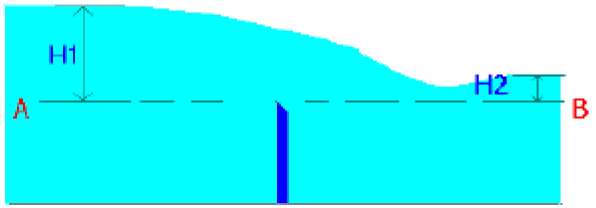


Figure 13 Submerged and Unsubmerged Weir

If d/s water table is below crest level weir is unsubmerged or free flowing weir
If d/s water table is above crest level weir is submerged of Drowned weir



$$Q_1 = \frac{2}{3} C_{d1} \sqrt{2g}\, L (H_1 - H_2)^{3/2}$$

$$Q_2 = C_{d2} \sqrt{2g(H_1 - H_2)}\, L H_2$$

$$Q = Q_1 + Q_2$$