NEPAL ENGINEERING COUNCIL

Concept of Basic Electrical and Electronics Engineering

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Contents

1.2 Network theorems: concept of superposition theorem, Thevenin`s theorem, Norton`s theorem, maximum power transfer theorem. R-L, R-C, R-L-C circuits, resonance in AC series and parallel circuit, active and reactive power. (AExE0102)

Network Theorems

- Network theorems are fundamental principles in electrical engineering and circuit analysis that help simplify and analyze complex electrical networks. These theorems provide mathematical tools and techniques to determine various circuit parameters, such as current, voltage, power, and resistance.
- Super Position Theorem
- Thevenin's Theorem
- Norton's Theorem
- Maximum Power Transfer Theorem.

Concept of superposition theorem

Statement: "In any **linear and bilateral network** or circuit **having multiple independent sources**, the **response** of an element **will be equal to the algebraic sum of the responses of that element** by considering one source at a time."

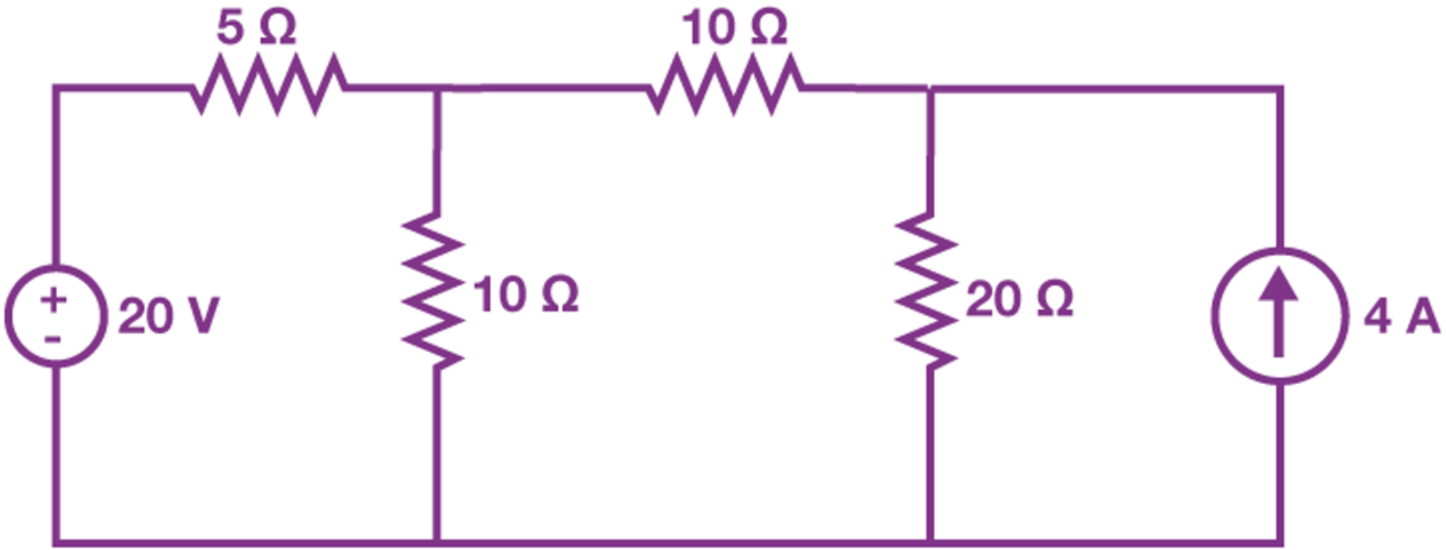
- To calculate the individual contribution of each source in a circuit, the other source must be replaced by their internal resistance.
- Replace voltage source with a short circuit. Voltage source resistance =0
- Replace current source with open circuit. Current source resistance = Infinite

Steps

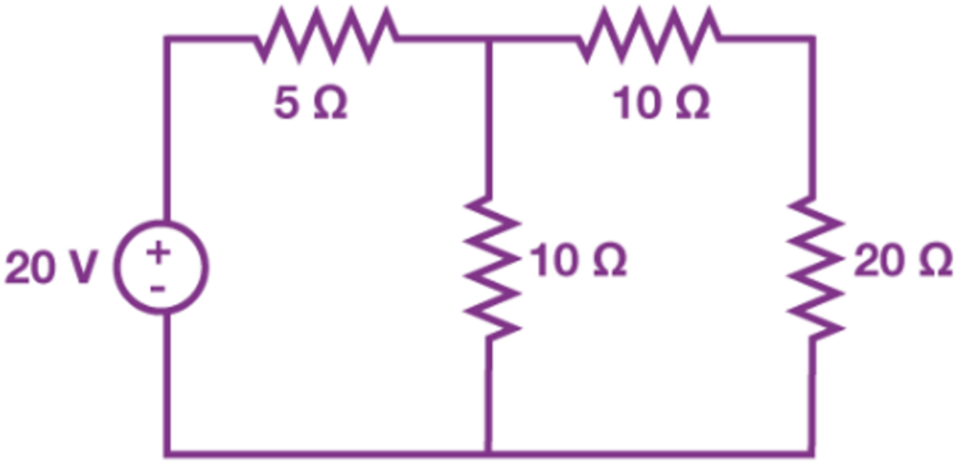
- The first step is to select one among the multiple sources present in the bilateral network. Among the various sources in the circuit, any one of the sources can be considered first.
- Except for the selected source, all the sources must be replaced by their internal impedance.
- Using a network simplification approach, evaluate the current flowing through or the voltage drop across a particular element in the network(Whatever and wherever is required).
- The same considering a single source is repeated for all the other sources in the circuit.
- Upon obtaining the respective response for individual source, perform the summation of all responses to get the overall voltage drop or current through the circuit element.

Solved Example

Find the current flowing through 20 Ω using the superposition theorem.



- Step 1: First, let us find the current flowing through a circuit by considering only the 20 V voltage source. The current source can be open-circuited, hence, the modified circuit diagram is shown in the following figure.

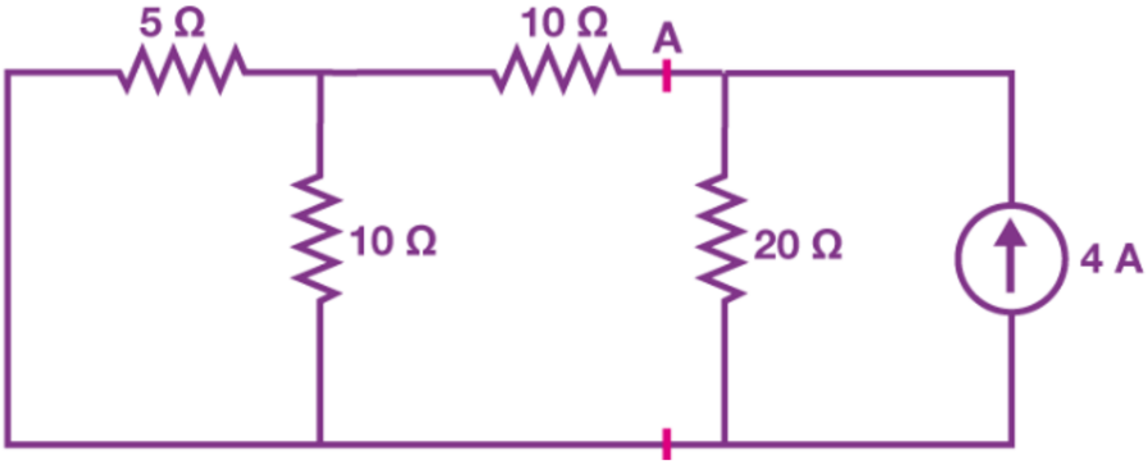


The current flowing through the 20 Ω resistor can be found using any methods:

I1= 0.4 A

Therefore, the current flowing through the 20 Ω resistor to due 20 V voltage source is 0.4 A.

Step 2: Now let us find out the current flowing through the 20 Ω resistor considering only the 4 A current source. We eliminate the 20 V voltage source by short-circuiting it. The modified circuit, therefore, is given as follows:



In the above circuit, the resistors 5 Ω and 10 Ω are parallel to each other, and this parallel combination of resistors is in series with the 10 Ω resistor. Therefore, the equivalent resistance will be: 10+50/15= 40/3 ohm

The current flowing through the 20 Ω resistor can be determined using the current division principle.

$$I_2 = I_s \frac{R_1}{R_1+R_2} \qquad I_2 = 1.6\ A$$

Step 3 The summation of currents I1 and I2 will give us the current flowing through the 20 Ω resistor. Mathematically, this is represented as follows:

$I = I1 + I2$

Substituting the values of I1 and I2 in the above equation, we get

$I = 0.4+1.6 = 2$ A

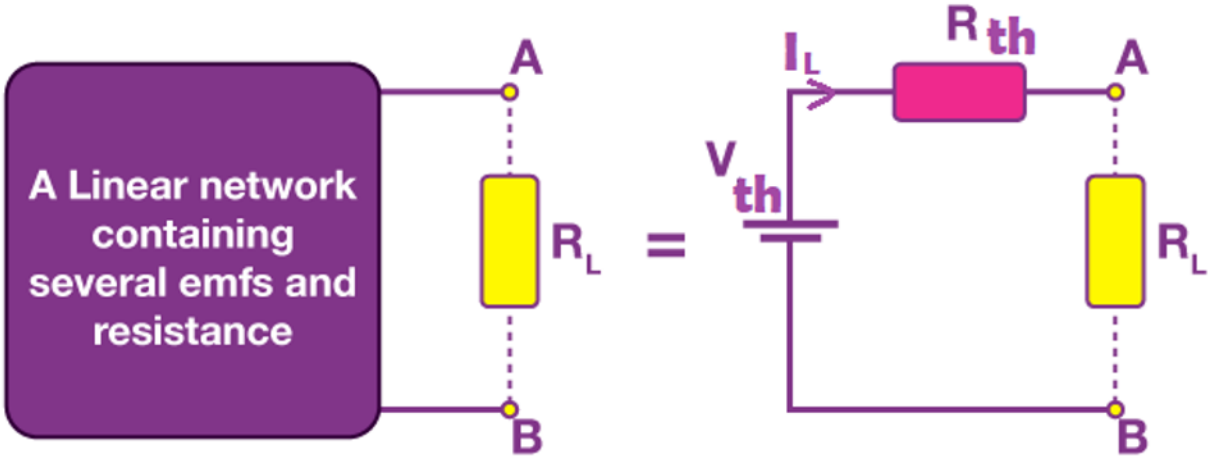
Therefore, the current flowing through the resistor is 2 A.

Limitation of Superposition Theorem

- The theorem does not apply to non-linear circuits. The requisite of linearity indicates that the superposition theorem is only applicable to determine voltage and current but not power.

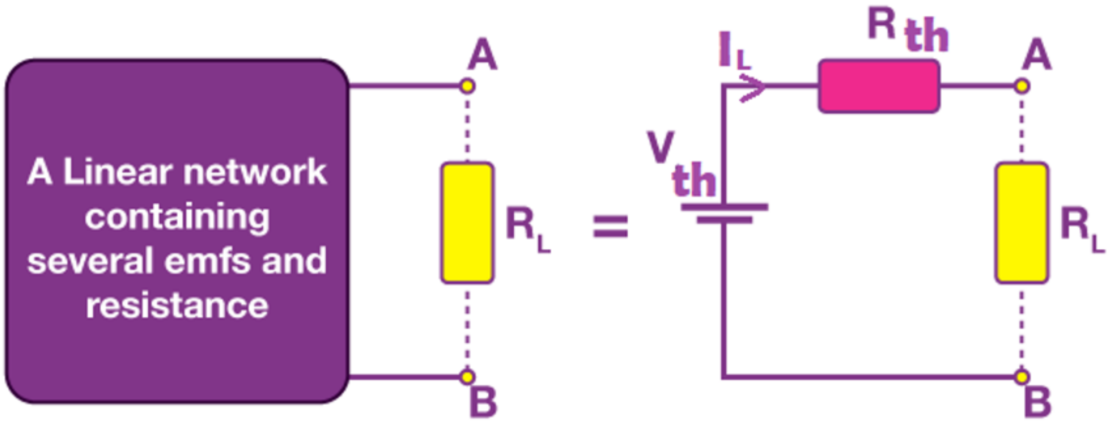
Thevenin`s theorem

Thevenin's theorem states that any **linear circuit** containing several sources and resistors can be simplified to a **equivalent circuit** (known as Thevenin-Equivalent Circuit)as with a **single voltage source and resistance** connected in series with a load.



Rth is single equivalent resistance (Thevenin resistance)

Vth is equivalent voltage source (Thevenin Voltage) instead of multiple energy sources

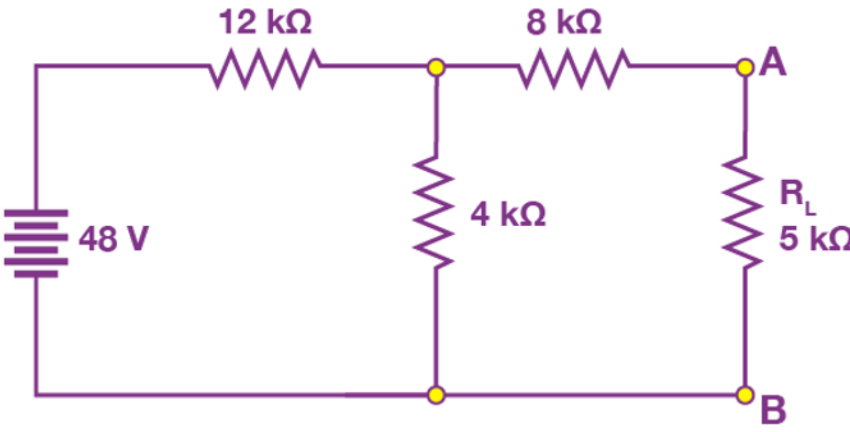


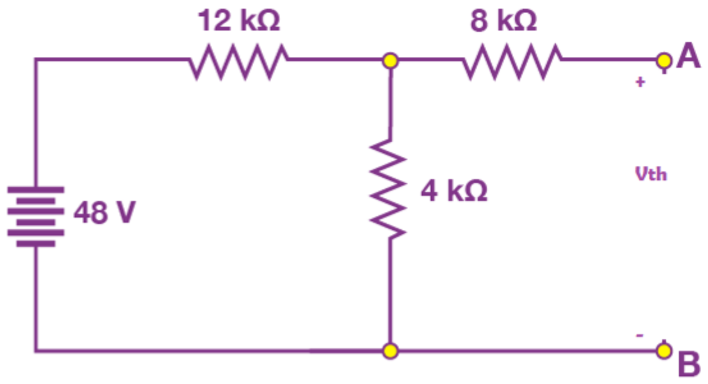
- Thevenin voltage V_{th} is found by **removing load R_L** from the original circuit and calculating the **open circuit potential difference** from one load connection point to the other (A to B)
- Thevenin resistance R_{th} is found by removing load R_L from the original circuit and calculating the total equivalent resistance between the two load connection points A B
- The load current is given by $$I_L = \frac{V_{\text{th}}}{R_{\text{th}} + R_L},$$

Solved Example

Find V_{th}, R_{th} and the load current I_L flowing through R_L and load voltage across the load resistor in the circuit below using Thevenin's Theorem.

Step 1: Remove the 5 kΩ from the circuit.
Step 2: Compute the open-circuit voltage. This will give you the Thevenin's voltage (V_{TH}).
Step 3: Compute Thevenin resistance(Rth) across AB by replacing all source with their internal impedance.
Step4: Draw Thevenin equivalent circuit and Compute I_L=Vth/(RL+Rth)





Step 1: Remove the 5 kΩ from the circuit as showi

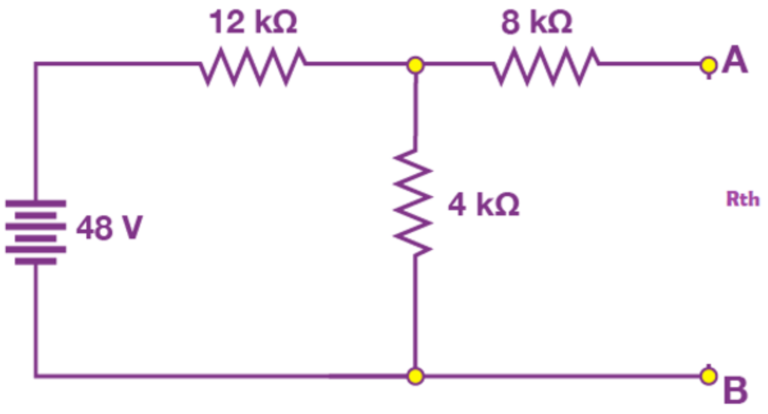
Step 2: We calculate Thevenin's voltage by determining the current that flows through 12 kΩ and 4 kΩ resistors.

As both the resistors are in series, the current that flows across them can be calculated as follows:

I = 48 V /(12 kΩ + 4 kΩ) = 3 mA

As there is no current flowing through the 8 kΩ resistor, so there is no voltage drop across it and hence the voltage across the terminals AB is same as the voltage across the 4 kΩ resistor. Therefore, 12 V will appear across the AB terminals. Hence, the Thevenin's voltage, V_{TH} = 12 V

Step 3: For Rth, Short the voltage sources as shown in the figure.



By measuring the open circuit resistance, we can measure Thevenin's resistance.

We notice that the 8 kΩ resistor is in series with the parallel connection of 12 kΩ and 4 kΩ resistors. Therefore, the equivalent resistance or the Thevenin's resistance is calculated as follows:

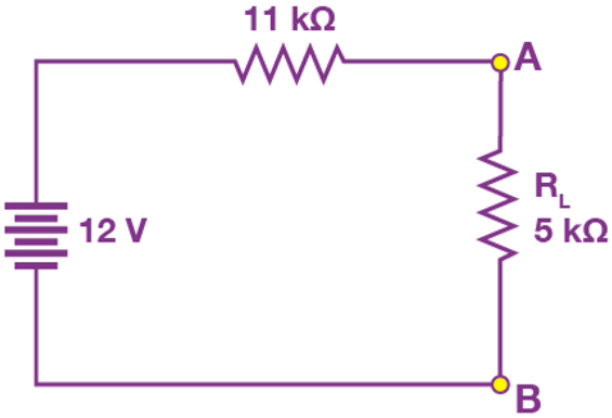
$R_{TH} = 8k\Omega + (4k\ \Omega \,||\, 12k\Omega)$

$R_{TH} = 8\ k\Omega + [(4\ k\Omega \times 12\ k\Omega) / (4\ k\Omega + 12\ k\Omega)]$

$R_{TH} = 8\ k\Omega + 3\ k\Omega$

$R_{TH} = 11\ k\Omega$

Step 4: Now, connect the R_{TH} in series with Voltage Source V_{TH} and the load resistor R_L as shown in the figure.



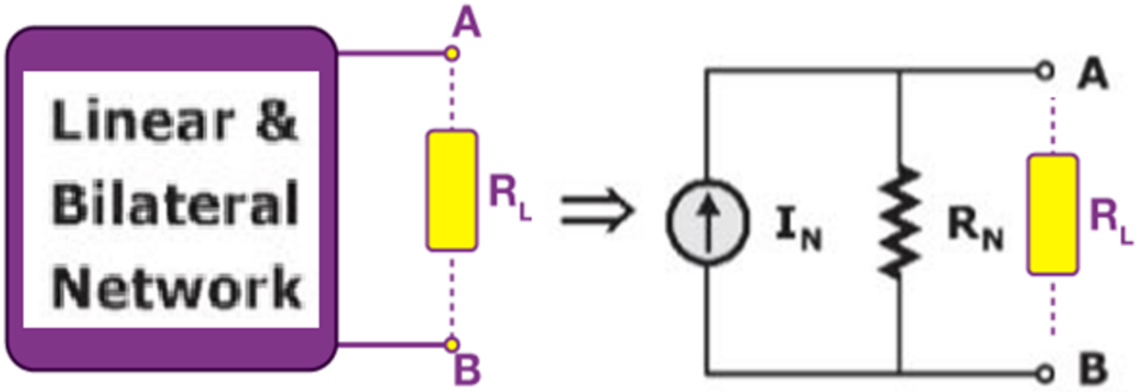
$I_L = V_{TH} / (R_{TH} + R_L)$

$I_L = 12\ V / (11\ k\Omega + 5\ k\Omega) = 12\ V/16\ k\Omega = 0.75\ mA$

Norton`s theorem

Norton's theorem states that any 2-terminal linear and bilateral network having multiple sources can be represented in a simplified equivalent circuit known as Norton's equivalent circuit. Norton's equivalent circuit consists of Norton's current source, I_N in parallel with Norton's resistance, R_N.

The Norton current I_N is the short-circuit current from the terminals, and the Norton resistance R_N is calculated similarly as the Thevenin resistance (Open circuit equivalent resistance across the terminals.

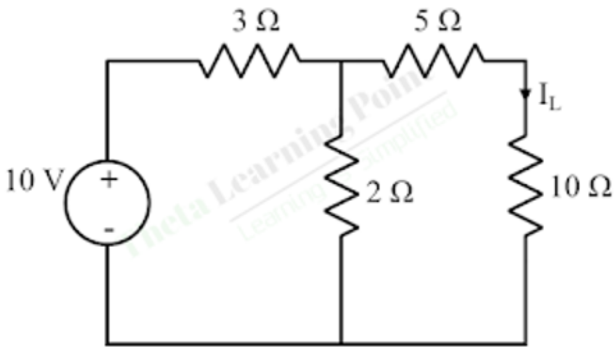


Steps

- Step 1: Remove the element, where we are supposed to find the response from the given circuit. After the removal of the element, the terminals will be open.
- Step 2: Find the current flowing through the terminals of the circuit obtained in Step 1 after shorting them. This current is known as short circuit current or Norton's equivalent current or Norton's current, IN in short.
- Step 3: Replace all the independent sources with their internal resistances in the circuit obtained in Step 1.
- Step 4: Find the equivalent resistance across the open-circuited terminals of the circuit obtained in Step 3 indirect methods. This equivalent resistance is known as Norton's equivalent resistance or Norton's resistance, RN in short.
- Step 5: Calculate the load current I_L using current divider rule $I_l = \frac{I_N * RN}{RN+RL}$

Solved Example

Determine the electric current through the 10 Ω resistor in the following circuit by using Norton's theorem.



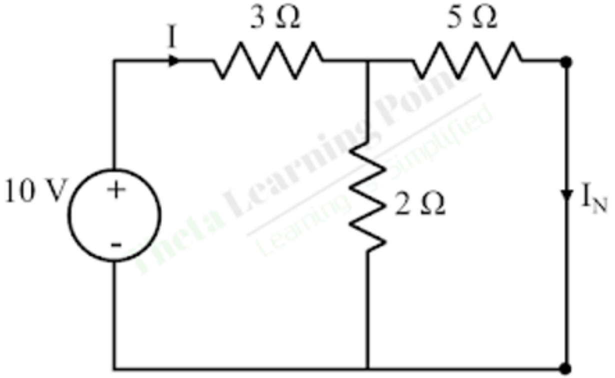
Solution

1. Find I_N (Norton Current):

$$R_{eq} = \left(\frac{5 \times 2}{5 + 2}\right) + 3$$

$$\Rightarrow R_{eq} = \left(\frac{10}{7}\right) + 3$$

$$R_{eq} = 4.43\Omega$$



Therefore, the total current in the circuit from the voltage source is,

$$I = \frac{V}{R_{eq}} = \frac{10}{4.43}$$

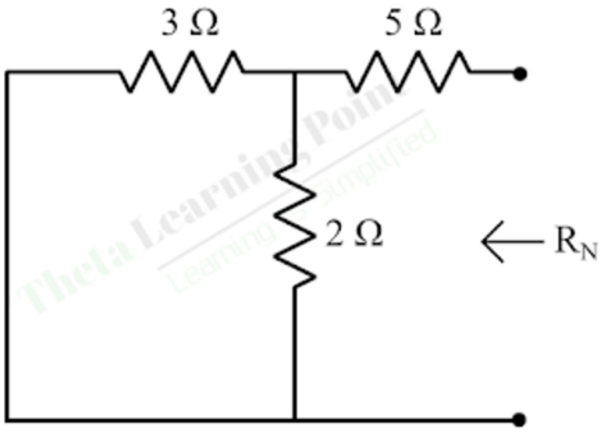
$$I = 2.26A$$

Now, using the current division rule, the Norton current will be

$$I_N = 2.26 \times \left(\frac{2}{2 + 5}\right)$$

$$\therefore I_N = 0.645A$$

Find RN (Norton Resistance):

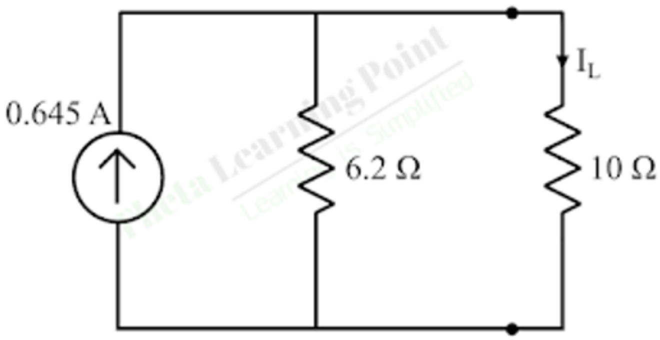


$$R_N = \left(\frac{2 \times 3}{2+3}\right) + 5$$

$$R_N = \left(\frac{6}{5}\right) + 5$$

$$\therefore R_N = 6.2\Omega$$

Equivalent Norton Circuit and the load current is,

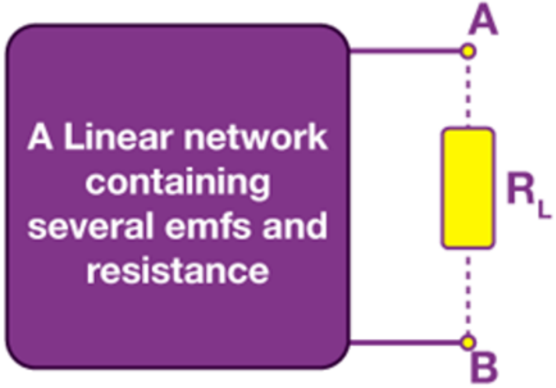


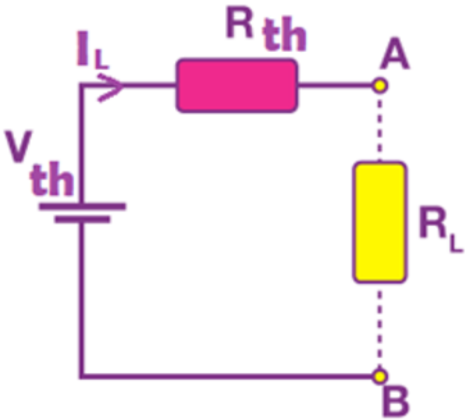
$$I_L = 0.645 \times \left(\frac{6.2}{6.2+10}\right)$$

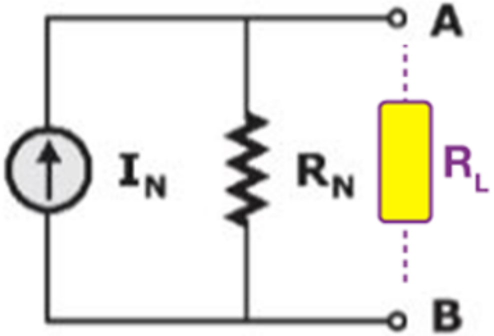
$$\therefore I_L = 0.246A$$

Thevenin's and Norton's Theorem

- Norton's theorem is similar to Thevenin's theorem in that it also allows us to simplify any linear circuit to an equivalent circuit. However, instead of using a voltage source and a series resistance, the Norton equivalent circuit consists of a current source with a parallel resistance.
- $Rth = R_N$
- $Vth = I_N * R_N$ (Source conversion techniques)

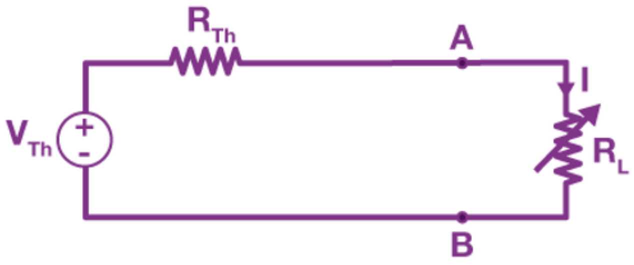






Maximum power transfer theorem

The MPTT states that, to obtain maximum power from a power source with internal resistance R_{th}, the resistance of the load R_L must equal the resistance of the source $R_{th.}$



Maximum Power transfer takes place if $R_L = R_{th}$

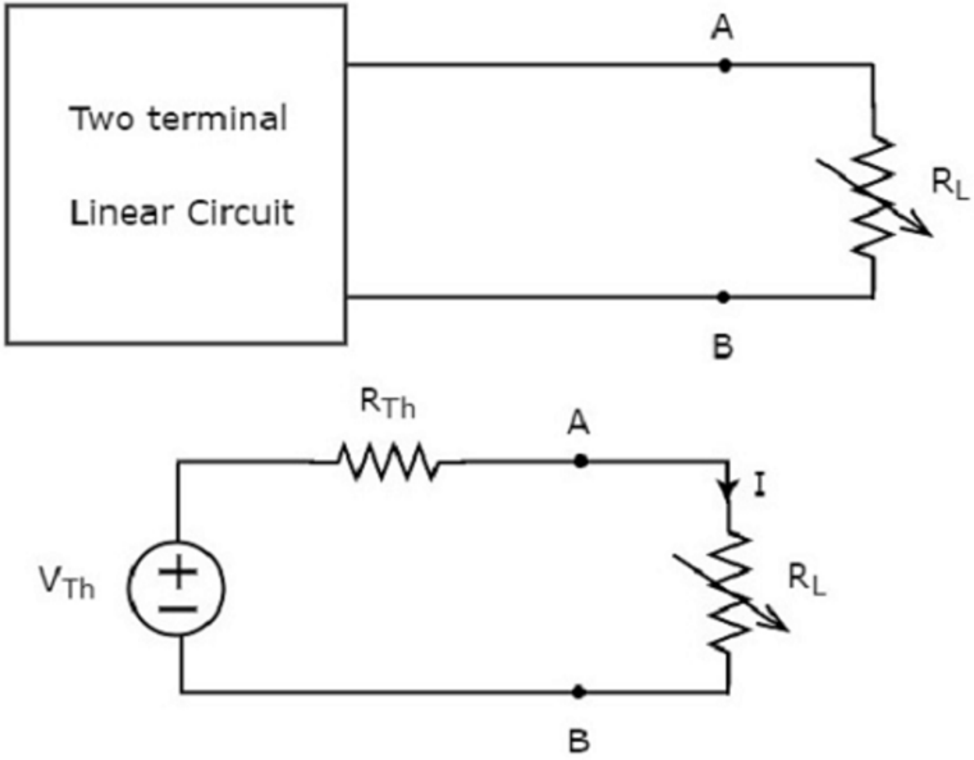
The value of maximum power is

$$P_{L,Max} = \frac{{V_{Th}}^2}{4R_L}$$

MPTT and Thevenin's Theorm

Replace any two terminal linear network or circuit with a variable load resistor having resistance of R_L ohms with a Thevenin's equivalent circuit.

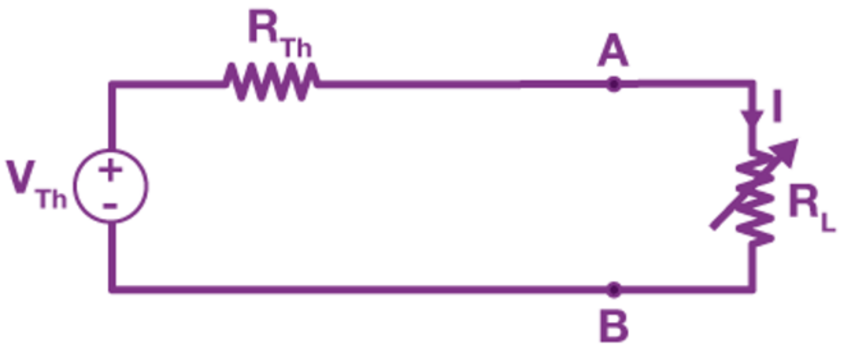
We know that Thevenin's equivalent circuit resembles a practical voltage source.



$$P_{L,Max} = \frac{{V_{Th}}^2}{4R_L}$$

At $R_L = R_{th}$

Proof



The amount of power dissipated across the load resistor is

$$P_L = I^2 R_L$$

Substitute $I = \frac{V_{Th}}{R_{Th}+R_L}$ in the above equation.

$$P_L = \left(\frac{V_{Th}}{(R_{Th}+R_L)}\right)^2 R_L$$

$$\Rightarrow P_L = {V_{Th}}^2 \left\{\frac{R_L}{(R_{Th}+R_L)^2}\right\}$$ **Equation 1**

For maximum or minimum, first derivative will be zero. So, differentiate Equation 1 with respect to R_L and make it equal to zero.

$$\frac{dP_L}{dR_L} = {V_{Th}}^2 \left\{\frac{(R_{Th}+R_L)^2 \times 1 - R_L \times 2(R_{Th}+R_L)}{(R_{Th}+R_L)^4}\right\} = 0$$

$$\Rightarrow (R_{Th}+R_L)^2 - 2R_L(R_{Th}+R_L) = 0$$

$$\Rightarrow (R_{Th}+R_L)(R_{Th}+R_L-2R_L) = 0$$

$$\Rightarrow (R_{Th}-R_L) = 0$$

$$\Rightarrow R_{Th} = R_L \text{ or } R_L = R_{Th}$$

Therefore, the condition for maximum power dissipation across the load is $R_L = R_{Th}$. That means, if the value of load resistance is equal to the value of source resistance i.e., Thevenin's resistance, then the power dissipated across the load will be of maximum value.

Proof Contd.

The value of Maximum Power Transfer

Substitute $R_L = R_{Th}$ & $P_L = P_{L,Max}$ in Equation 1.

$$P_{L,Max} = {V_{Th}}^2\{\frac{R_{Th}}{(R_{Th}+R_{Th})^2}\}$$

$$P_{L,Max} = {V_{Th}}^2\{\frac{R_{Th}}{4{R_{Th}}^2}\}$$

$$\Rightarrow P_{L,Max} = \frac{{V_{Th}}^2}{4R_{Th}}$$

$$\Rightarrow P_{L,Max} = \frac{{V_{Th}}^2}{4R_L},\ since\ R_L = R_{Th}$$

Is maximum amount of power transferred to the load

Efficiency at Maximum Power Transfer

We can calculate the efficiency of maximum power transfer, η_{Max} using following formula.

$$\eta_{Max} = \frac{P_{L,Max}}{P_S} \qquad \textbf{Equation 2}$$

Where,

- $P_{L,Max}$ is the maximum amount of power transferred to the load.
- P_S is the amount of power generated by the source.

The **amount of power generated** by the source is

$$P_S = I^2 R_{Th} + I^2 R_L$$

$$\Rightarrow P_S = 2I^2 R_{Th},\ since\ R_L = R_{Th}$$

Substitute $I = \frac{V_{Th}}{2R_{Th}}$ in the above equation.

$$P_S = 2\left(\frac{V_{Th}}{2R_{Th}}\right)^2 R_{Th}$$

$$\Rightarrow P_S = 2\left(\frac{{V_{Th}}^2}{4{R_{Th}}^2}\right) R_{Th} = \frac{{V_{Th}}^2}{2R_{Th}}$$

Substitute the values of $P_{L,Max}$ and P_S in Equation 2

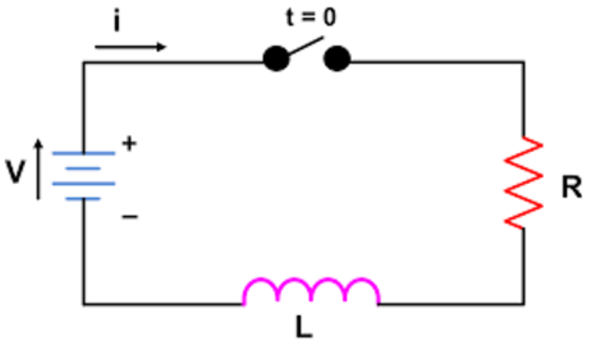
$$\eta_{Max} = \frac{\left(\frac{{V_{Th}}^2}{4R_{Th}}\right)}{\left(\frac{{V_{Th}}^2}{2R_{Th}}\right)}$$

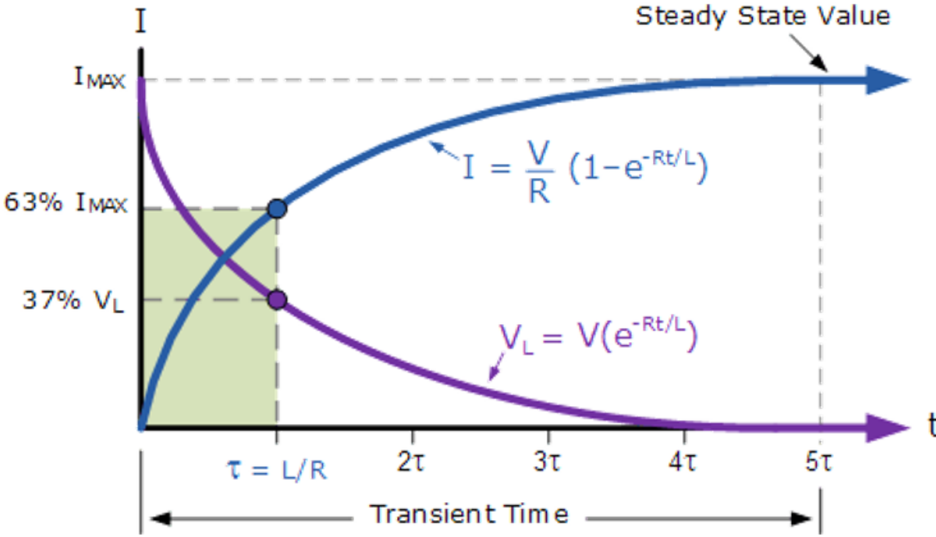
$$\Rightarrow \eta_{Max} = \frac{1}{2}$$

$$\%\eta_{Max} = 50\%$$

Therefore, the efficiency of maximum power transfer is **50 %**.

RL Circuit



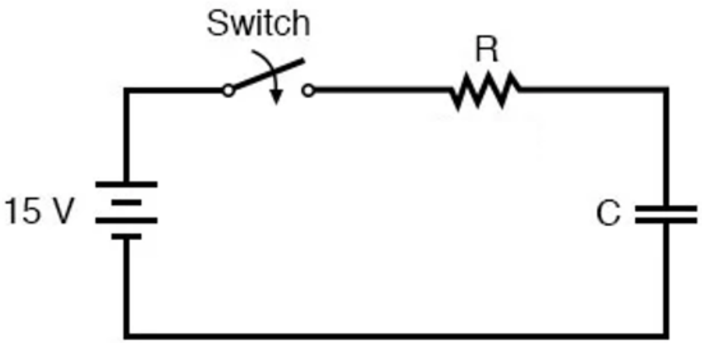


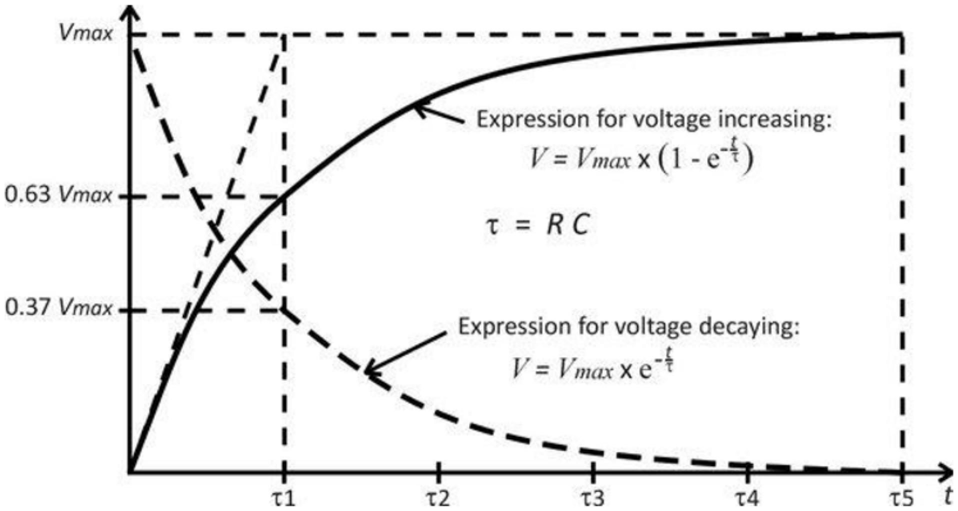
Remember:

- Time Constant= L/R

Time constant is the time required to reach 63.7% of its steady state value.

RC circuit





Remember:

- Time Constant= RC

Time constant is the time required to reach 63.7% of its steady state value.