PANA ACADEMY

# NEPAL ENGINEERING COUNCIL
# LICENSE EXAM PREPARATION COURSE

FOR

# CIVIL ENGINEERS

# 3. Basic Water Resources Engineering

## 3.4 Pipe flow

# Sub topics

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- Types
- governing equations
- major and minor head losses
- HGL and TEL lines
- design; pipe network problems
- unsteady flow in pipes and relief devices.

# HGL and TEL

What is Headloss

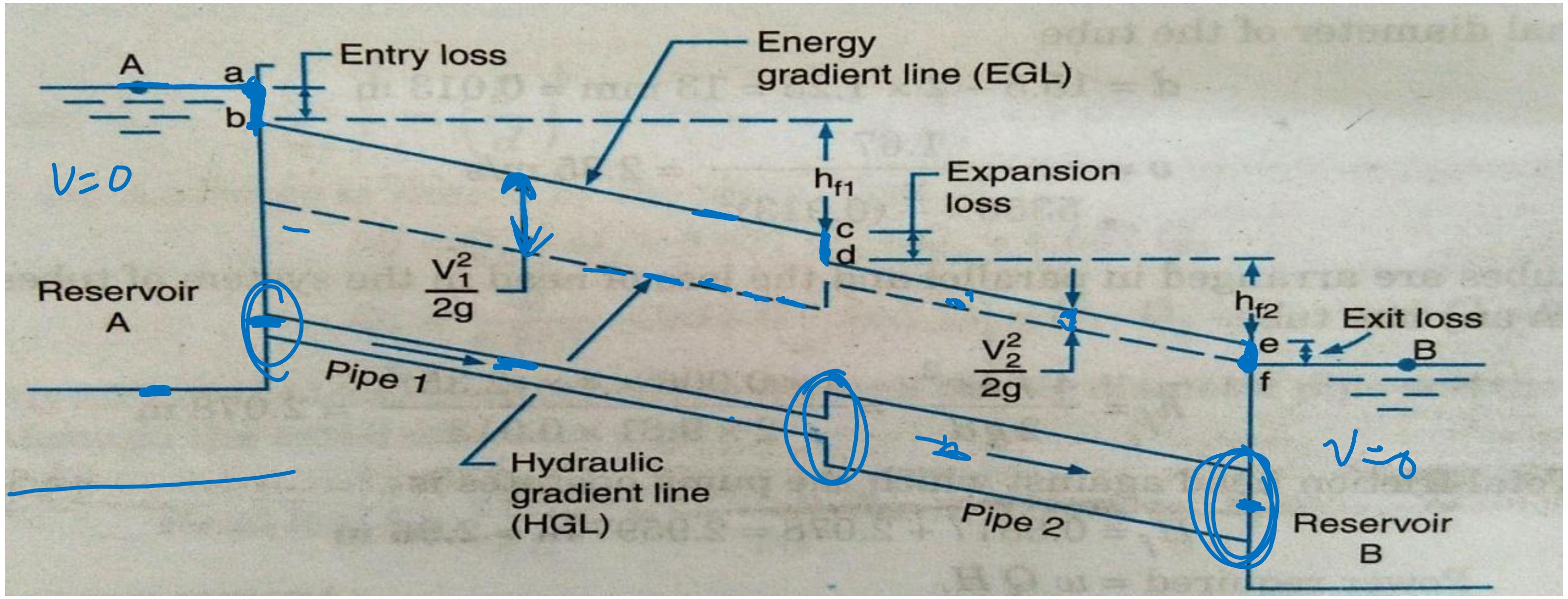
Difference between TEL at point 1 to total energy level at point 2 gives headloss in point 1 to 2

Total energy line or Energy gradient line-
Line representing the sum of pressure head, datum head, and velocity head

$$H.G.L = E.G.L - V^2/2g$$

$$\text{Headloss} = TEL_1 - TEL_2$$

Hydraulic gradient Line (H.G.L) –
Line representing the sum of pressure head and datum head.



# HGL and TEL

HGL = Hydraulic Gradient line
TEL = Total Energy line

$$E = Z + \frac{P}{\gamma} + \frac{V^2}{2g}$$

$\left(Z + \frac{P}{\gamma}\right) \rightarrow$ piezometric Head

HGL → An imaginary line connecting all piezometric Head points along the length of conduits

$$TEL = HGL + \frac{V^2}{2g}$$

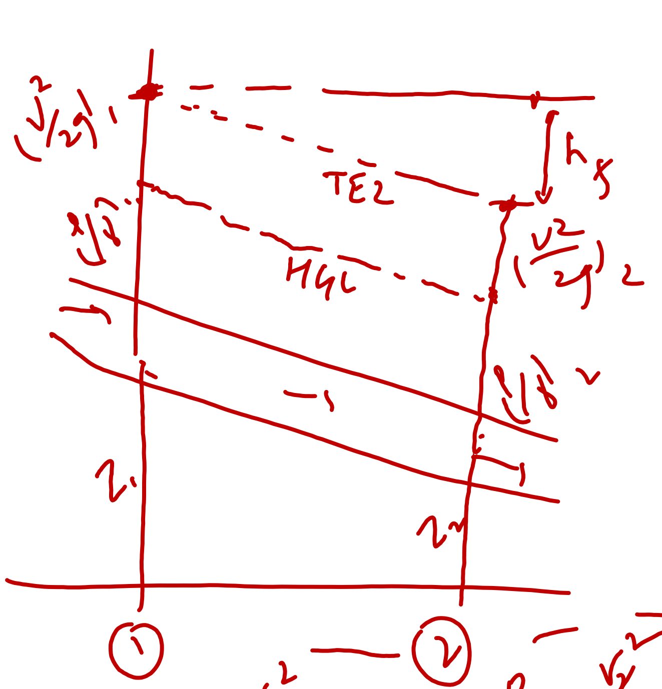
HGL in pipe flow

ⓐ will be always above center line

ⓑ will be always below center line

ⓒ coincides with center line

ⓓ can be above or below center line




$$Z_1 + \frac{P_1}{\gamma} + \frac{V_1^2}{2g} = Z_2 + \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + h_{f_{1-2}}$$

The TEL is the line that can be obtained by adding ——— in HGL

ⓐ Headloss ⓑ Velocity Head ⓒ pressure Head
ⓓ elevation Head.

# Pipe flow and types

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Reynolds number (Re) is a dimensionless quantity that helps predict fluid flow patterns.

Reynolds number =inertial force/ viscous forces

**Pipe flow:** flow at pressure
Laminar: $\boldsymbol{R_e < 2000^*}$
Transitional: $\boldsymbol{2000 < R_e < 4000}$
Turbulent: $\boldsymbol{R_e > 4000}$

$$R_e = \frac{\rho V d}{\mu} = \frac{V d}{\nu}$$

$V$ is velocity, $d$ is diameter of pipe
* $\boldsymbol{R_e < 2300}$ also used in some literature

# Pipe flow and types

The flow of fluid in pipe may be of two types:

a. Laminar flow

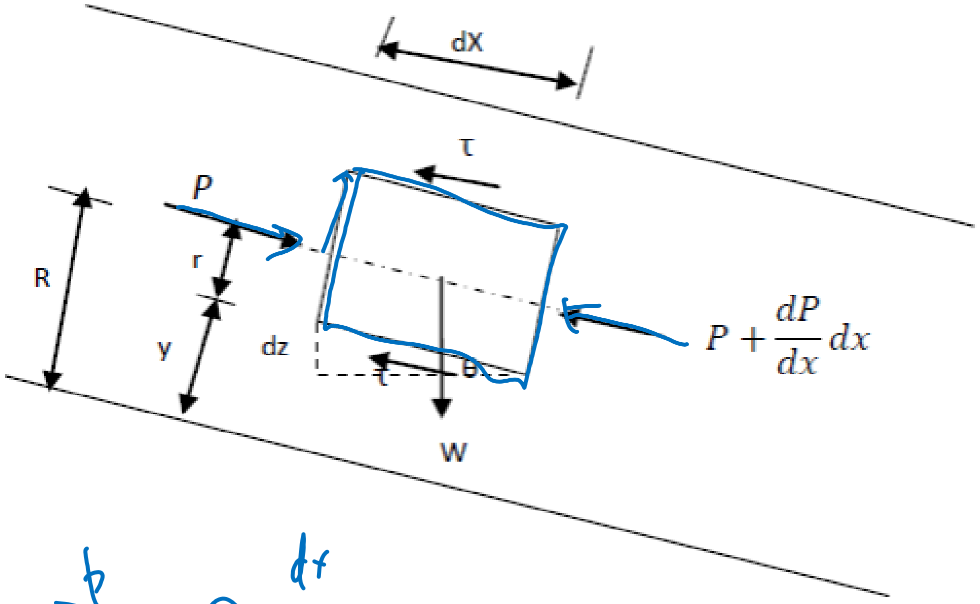
- regular, smooth and systematic
- no intermixing of fluid particles in adjacent layers
- occurs for $Re<2000$
- low velocity
- high viscosity
- e.g. flow past tiny bodies, groundwater flow, flow of blood through vessels, rise of water in plants through their roots

b. Turbulent flow

- irregular and erratic
- violent mixing of fluid particles
- occurs for $Re>4000$
- high velocity
- low viscosity
- e.g. flow through pipes, sewers, rivers

# STEADY LAMINAR FLOW IN A CIRCULAR PIPE

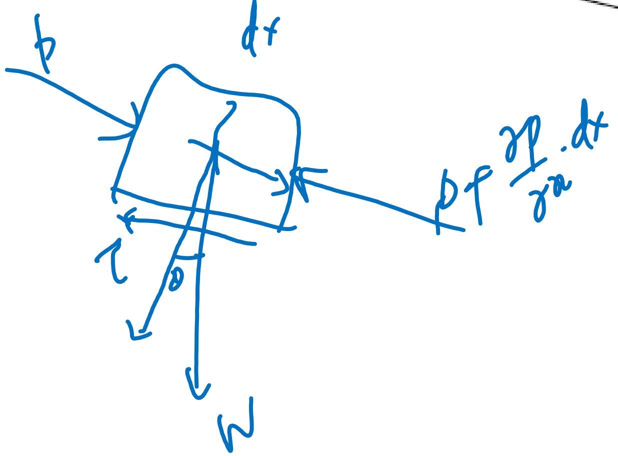
$\tau = \mu \frac{du}{dy}$




flow direction

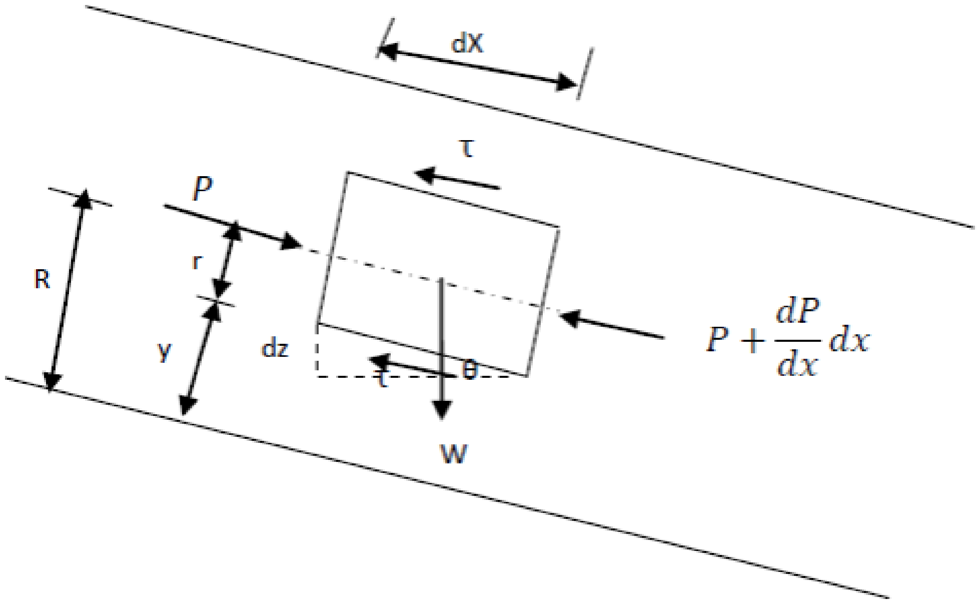
$p \cdot A - \left(p + \frac{\partial p}{\partial x} \cdot dx\right) A - \tau \cdot 2\pi r$

$+ W \sin\theta = 0$



# STEADY LAMINAR FLOW IN A CIRCULAR PIPE

Shear stress distribution




Resolving forces along the direction of motion,

$$P(\pi r^2) - \left(P + \frac{dP}{dx}dx\right)\pi r^2 - \tau(2\pi r dx) + W sin\theta = 0$$

$$P(\pi r^2) - \left(P + \frac{dP}{dx}dx\right)\pi r^2 - \tau(2\pi r dx) - \gamma\pi r^2 dx\frac{dz}{dx} = 0$$

$$\tau = -\frac{d(P+\gamma z)}{dx}.\frac{r}{2} \qquad (I)$$

(-ve sign shows that the pressure decreases in the direction of flow.)

The shear stress varies linearly with r as $-\frac{d(P+\gamma z)}{dx}$ across section is constant.

The term $\frac{d(P+\gamma z)}{dx}$ can also be written as $\frac{\gamma\, d\left(\frac{P}{\gamma}+z\right)}{dx} = \gamma\frac{d\,h}{dx}$ where h = piezometric head

# STEADY LAMINAR FLOW IN A CIRCULAR PIPE

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Velocity distribution

Assumptions: Fluid is Newtonian and there is no slip at the boundary (zero vel. at the boundary).

From Newton's law of viscosity

$$\tau = \mu \frac{dv}{dy}$$

Distance of fluid element from pipe wall (y) is

y = R – r

On differentiation

dy = -dr

$\tau = -\mu \frac{dv}{dr}$ (II)

Equating eq. (I) and (II)

$-\mu \frac{dv}{dr} = -\frac{d(P+\gamma z)}{dx} \cdot \frac{r}{2}$

$\frac{dv}{dr} = \frac{r}{2\mu} \frac{d(P+\gamma z)}{dx}$

Integrating w.r.t. r

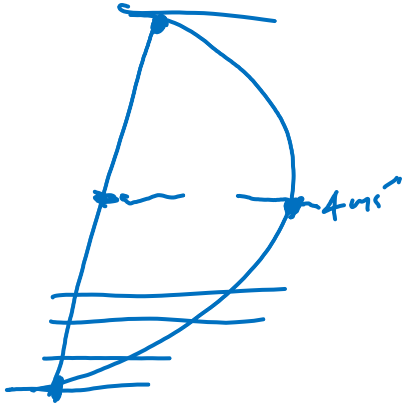
$v = \frac{r^2}{4\mu} \frac{d(P+\gamma z)}{dx} + C$

# STEADY LAMINAR FLOW IN A CIRCULAR PIPE



Velocity distribution

$$v = -\frac{1}{4\mu} \cdot \frac{d(P+\gamma z)}{dx} \cdot (R^2 - r^2) \rightarrow \text{Parabolic}$$



At r = R (At boundary), v = 0

$$0 = \frac{R^2}{4\mu}\frac{d(P+\gamma z)}{dx} + C$$

$$C = -\frac{R^2}{4\mu}\frac{d(P+\gamma z)}{dx}$$

Substituting the value of C

$$v = -\frac{1}{4\mu}\frac{d(P+\gamma z)}{dx}(R^2 - r^2)$$

Here $\mu, \frac{d(P+\gamma z)}{dx}$ and R are constant, which means u varies with r. This equation is the equation of parabola. This shows that local velocity varies parabolically along diameter.

# STEADY LAMINAR FLOW IN A CIRCULAR PIPE

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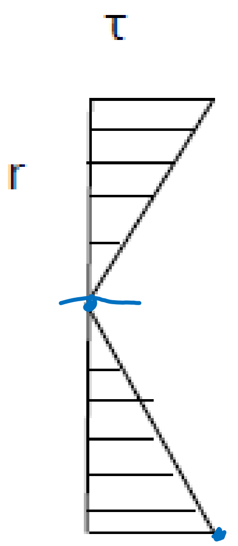
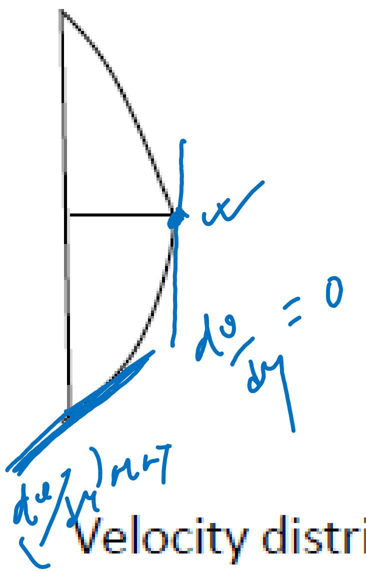
Linear




Shear stress distribution

Parabolic



Velocity distribution

$$\tau = \mu \frac{du}{dy} = 0$$

The shear stress in pipe flow at the center will be

(a) Minimum

(b) Maximum

(c) Can not be said

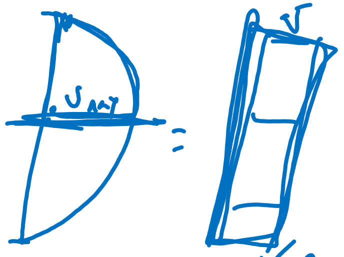
(d) depends on type of flow

# STEADY LAMINAR FLOW IN A CIRCULAR PIPE



Maximum velocity

$$v_{max} = -\frac{1}{4\mu}\frac{d(P+\gamma z)}{dx}R^2$$



Local velocity in term of Maximum velocity

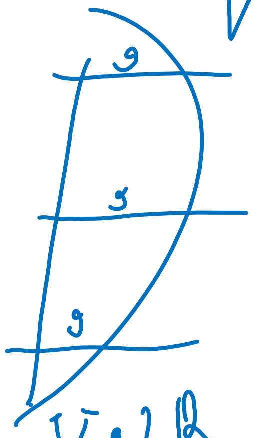
$$v = -\frac{1}{4\mu}\frac{d(P+\gamma z)}{dx}(R^2 - r^2) = -\frac{1}{4\mu}\frac{d(P+\gamma z)}{dx}R^2\left(1-\frac{r^2}{R^2}\right)$$

$$v = v_{max}\left(1-\frac{r^2}{R^2}\right)$$

$v$ - instantaneous

$v_{max}$ - Maximum

$V$ → Average



Average velocity (V)

$$v_{max} = 2V$$

$$2 \times \int_0^R v_{max}\left(1-\frac{r^2}{R^2}\right) dr = V \cdot 2R$$

# STEADY LAMINAR FLOW IN A CIRCULAR PIPE

Local velocity in term of Maximum velocity

$$v = -\frac{1}{4\mu}\frac{d(P+\gamma z)}{dx}(R^2 - r^2) = -\frac{1}{4\mu}\frac{d(P+\gamma z)}{dx}R^2(1-\frac{r^2}{R^2})$$

$$v = v_{max}(1-\frac{r^2}{R^2})$$

Average velocity (V)

$$v_{max} = 2V$$

Point where local velocity equals mean velocity

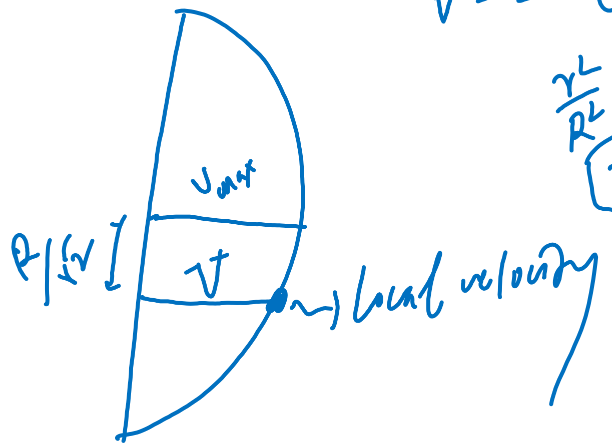
$$r = \frac{R}{\sqrt{2}}$$

$$v = v_{max}(1-\frac{r^2}{R^2})$$

$$V = 2V(1-\frac{r^2}{R^2})$$

$$\frac{r^2}{R^2} = \frac{1}{2}$$

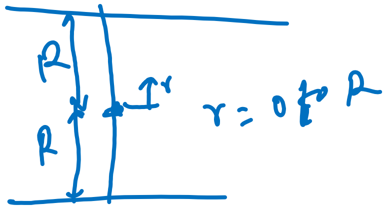
$$r = \frac{1}{\sqrt{2}} \cdot R$$



# STEADY LAMINAR FLOW IN A CIRCULAR PIPE



Pressure difference in term of average velocity: Hagen-Poiseuille Equation




Average velocity is given by

$$V = \frac{v_{max}}{2} = -\frac{1}{2} \cdot \frac{1}{4\mu} \frac{d(P+\gamma z)}{dx} R^2$$

$$-d(P + \gamma z) = \frac{8\mu V}{R^2} dx$$

Integrating

$$-\left(\int_{P_1}^{P_2} dP + \int_{Z_1}^{Z_2} \gamma z\right) = \int_{x_1}^{x_2} \frac{8\mu V}{R^2} dx$$

$$(P_1 - P_2) + \gamma(Z_1 - Z_2) = \frac{8\mu V}{R^2}(x_2 - x_1)$$

$$(P_1 - P_2) + \gamma(Z_1 - Z_2) = \frac{8\mu V}{R^2} L$$

where L = Length of the pipe

In terms of diameter, D

$$(P_1 - P_2) + \gamma(Z_1 - Z_2) = \frac{8\mu V}{(\frac{D}{2})^2} L$$

$$(P_1 - P_2) + \gamma(Z_1 - Z_2) = \frac{32\mu V L}{D^2}$$

Dividing by $\gamma$

$$\left(\frac{P_1}{\gamma} + Z_1\right) - \left(\frac{P_2}{\gamma} + Z_2\right) = \frac{32\mu V L}{\gamma D^2}$$

# STEADY LAMINAR FLOW IN A CIRCULAR PIPE



Pressure difference in term of average velocity: Hagen-Poiseuille Equation

As $V_1 = V_2$ for pipe of uniform diameter, $\left(\frac{P_1}{\gamma} + Z_1\right) - \left(\frac{P_2}{\gamma} + Z_2\right) = h_f$ (according to Bernoulli's equation)

where $h_f =$ loss of head for laminar flow

$$h_f = \frac{32\mu VL}{\gamma D^2} \text{ or } \frac{32\mu VL}{\rho g D^2}$$

This equation is known as Hagen-Poiseuille equation. This equation was derived experimentally by Hagen and Poiseuille.

Features of Hagen-Poiseuille equation

- Head loss for laminar flows varies with the first power of velocity.
- The equation does not have any empirical roughness coefficient.
- Head loss for laminar flow depends on fluid properties and pipe geometry.

# STEADY LAMINAR FLOW IN A CIRCULAR PIPE



<u>Formulae for steady, laminar flow through horizontal pipe</u>

For horizontal pipe, put Z = 0 in all expressions of inclined pipe.

The equations for shear stress and velocity distribution

$$\tau = -\frac{dP}{dx}\cdot\frac{R}{2}$$

$$v = -\frac{1}{4\mu}\frac{dP}{dx}(R^2 - r^2)$$

$$v_{max} = -\frac{1}{4\mu}\frac{dP}{dx}R^2$$

$$v = v_{max}\left(1 - \frac{r^2}{R^2}\right)$$

$V_{max} = 2V_{av}$

$$(P_1 - P_2) = \frac{32\mu VL}{D^2}$$

Power required to overcome the frictional resistance

Power ($P_w$) = force x Velocity = (pressure gradient x volume) Velocity

$$= -\frac{\partial P}{\partial x}ALV = \frac{P_1 - P_2}{L}\,ALV = Q(P_1 - P_2) = Q\,\frac{(P_1 - P_2)}{\gamma}\,\gamma$$

Power $= \gamma Q h_f$

$$h_f = \frac{fLv^2}{2gD}$$

$$f = \frac{64}{Re} = \frac{64\mu}{\rho v D}$$

$$h_f = \frac{32\mu VL}{\gamma D^2}$$

$$h_f = \frac{128\mu QL}{\pi\gamma D^4}$$

# TURBULENCE FLOW IN A CIRCULAR PIPE

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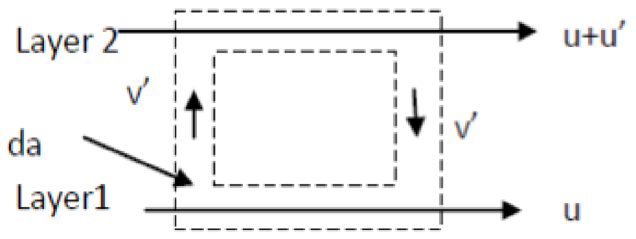
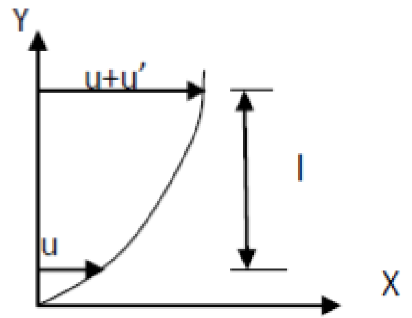
## Prandtl's mixing length theory

According to Prandtl, lumps of fluid particles move from one layer of some velocity to another layer of different velocity. Momentum exchange between two layers occurs due to the displacement of fluid particles. During this process, the fluid particles traveling from a layer lose its own momentum and acquire the momentum of the new layer. The distance between the two layers is the mixing length.

## Prandtl's mixing length equation

$$\tau_t = \rho l^2 \left(\frac{dv}{dy}\right)^2$$



# TURBULENCE FLOW IN A CIRCULAR PIPE

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Prandtl assumed mixing length, $l = ky$
K=Karman universal constant =0.4

Prandtl's mixing length equation

$$\tau_t = \rho k^2 y^2 \left(\frac{dv}{dy}\right)^2$$

$$\frac{dv}{dy} = \frac{1}{ky}\sqrt{\frac{\tau}{\rho}}$$

*For small y near pipe wall*, $\tau = \tau_o$

$\sqrt{\frac{\tau_o}{\rho}}$ has the dimension of velocity and it is called shear velocity $V_*$

$$V^* = \sqrt{\frac{\tau_o}{\rho}} = \sqrt{\frac{kg\,m/s^2 / m^2}{kg/m^3}} = \sqrt{m^2/s^2} = m/s$$

# TURBULENCE FLOW IN A CIRCULAR PIPE

$$\frac{dv}{dy} = \frac{V_*}{ky}$$

The equation gives

$$v = v_{max} + 2.5V_* \ln\left(\frac{y}{R}\right)$$

$$\frac{v_{max} - v}{V_*} = -2.5 \ln\left(\frac{y}{R}\right) = 2.5 \ln\left(\frac{R}{y}\right) = 5.75 \log\left(\frac{R}{y}\right)$$

$v_{max} - v$ is known as velocity deficit

$\frac{v_{max} - v}{V_*}$ is seen to be function of y/R which means no consideration of boundary smoothness. So this equation usually holds good near center of pipe away from boundary

# TURBULENCE FLOW IN A CIRCULAR PIPE

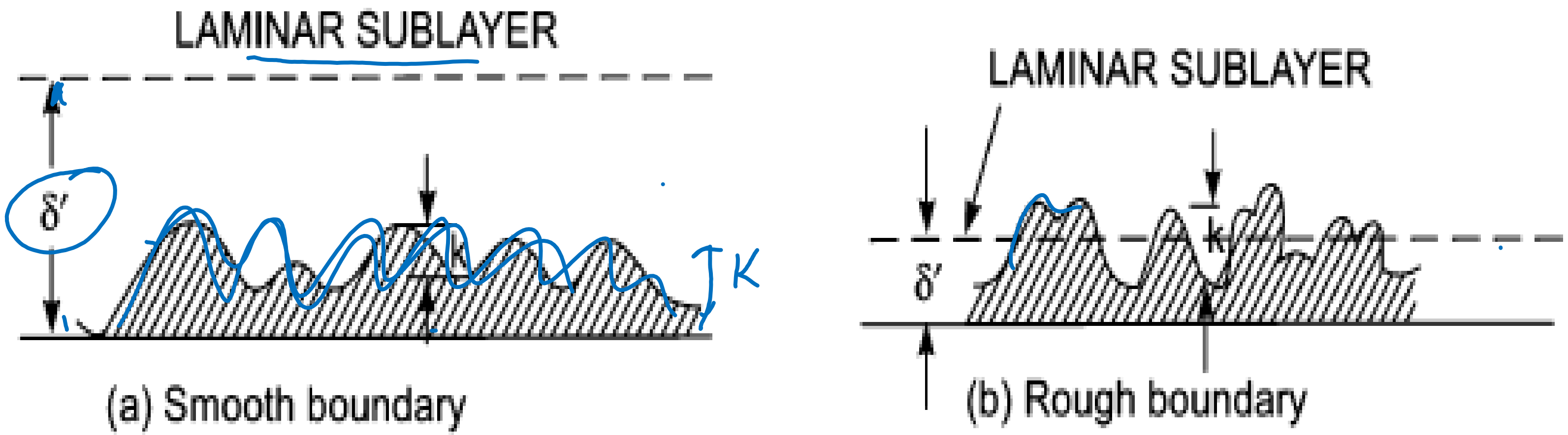


Fig. 10.4 *Smooth and rough boundaries.*

# TURBULENCE FLOW IN A CIRCULAR PIPE

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From Nikuradse's experiment:

$\frac{k}{\delta'}<0.25$: Smooth boundary

$\frac{k}{\delta'}>6$: rough boundary

$0.25<\frac{k}{\delta'}<6$: transition

$\delta' = \frac{11.6\nu}{V^*}$  $V^* = \sqrt{\frac{\tau_0}{\rho}}$

$\frac{V^* k}{\nu} = Re_{Roughness}$

In terms of roughness, Reynold number (Rn) = $\frac{V_* K}{\upsilon}$ where $V_*$ = shear velocity, k = roughness height

Rn<4: smooth

Rn>70: rough

4<Rn<70: transition

**Dimensionless Roughness parameter** $(R_n)$:

# TURBULENCE FLOW IN A CIRCULAR PIPE

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**KARMAN-PRANDTL VELOCITY**

$$v = v_{max} + 2.5V_* \ln\left(\frac{y}{R}\right)$$

At some distance $y'$ near boundary v=0

$$0 = v_{max} + 2.5V_* \ln\left(\frac{y'}{R}\right)$$

Subtracting two

$$v = 2.5V_* \ln\left(\frac{y}{y'}\right) = 5.75 \log\left(\frac{y}{y'}\right)$$

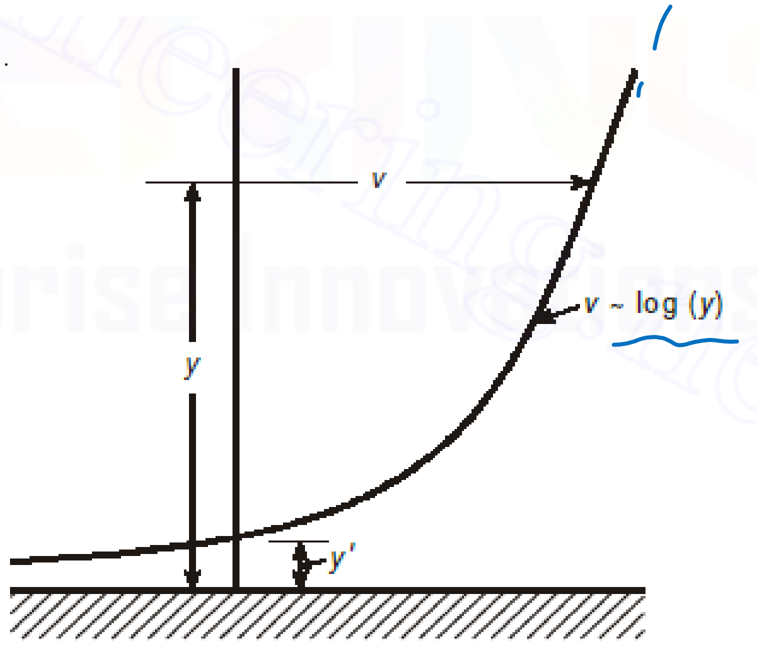


**Figure 14.5.** *Logarithmic velocity distribution curve for turbulent flow*

# TURBULENCE FLOW IN A CIRCULAR PIPE

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If flow occur with minimum energy loss it is smooth

a. velocity distribution for smooth pipe in turbulent flow

From Nikuradse's experiment for smooth boundary

$y' = \frac{\delta'}{107}$ where $\delta'$ = thickness of laminar sublayer.

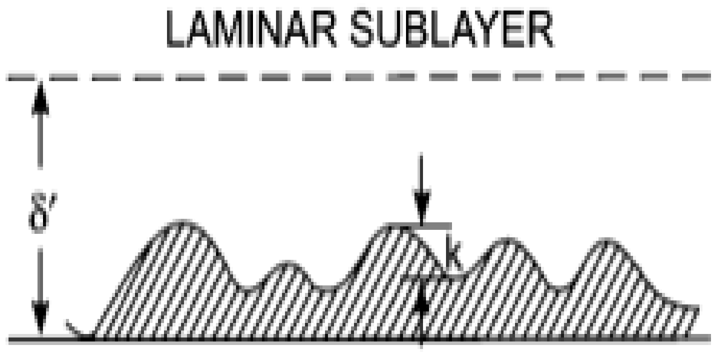
$\delta' = \frac{11.6\upsilon}{V_*}$

$y' = \frac{\delta'}{107} = \frac{11.6\upsilon}{107V_*} == \frac{0.108\upsilon}{V_*}$

Substituting y' in eq. (I) and simplifying

$$v = 5.75V_* log_{10}\left(\frac{y}{\frac{0.108\upsilon}{V_*}}\right)$$

$$\frac{v}{V_*} = 5.75 log_{10}\left(\frac{V_* y}{\upsilon}\right) + 5.5$$




(a) Smooth boundary

# TURBULENCE FLOW IN A CIRCULAR PIPE

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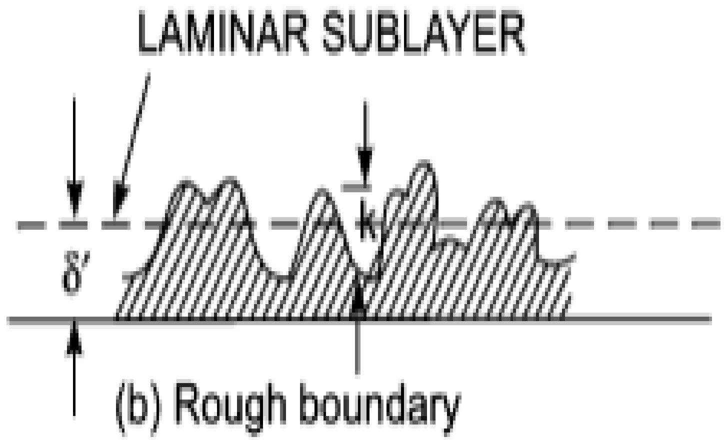
b. velocity distribution for rough pipes in turbulent flow

From Nikuradse's experiment for rough boundary

$y' = \frac{k}{30}$ where K = roughness height

Substituting y' in eq. (I) and simplifying

$\frac{v}{V_*} = 5.75 log_{10}(y/k) + 8.5$



# TURBULENCE FLOW IN A CIRCULAR PIPE

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Velocity distribution in terms of mean velocity

Average velocity for smooth pipes

$$\frac{v}{V_*} = 5.75 log_{10}\left(\frac{V_* y}{\upsilon}\right) + 5.5$$

Mean velocity $(\bar{V}) = \frac{Q}{A} = \frac{\int v dA}{A} = \frac{\int_0^R v(2\pi r)dr}{\pi R^2}$

After integration (see appendix for integration)

$$\frac{\bar{V}}{V_*} = 5.75 log_{10}\left(\frac{V_* R}{\upsilon}\right) + 1.75$$

Average velocity for rough pipes

$$\frac{v}{V_*} = 5.75 log_{10}(y/k) + 8.5$$

$$\bar{V} = \frac{Q}{A} = \frac{\int v dA}{A} = \frac{\int_0^R v(2\pi r)dr}{\pi R^2}$$

After integration (see appendix for integration)

$$\frac{\bar{V}}{V_*} = 5.75 log_{10}\left(\frac{R}{K}\right) + 4.75$$

# TURBULENCE FLOW IN A CIRCULAR PIPE

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Relationship between mean velocity, shear velocity and local velocity

For smooth pipes

$\frac{v}{V_*} = 5.75 log_{10}\left(\frac{V_* y}{\upsilon}\right) + 5.5$ (a1)

$\frac{\bar{V}}{V_*} = 5.75 log_{10}\left(\frac{V_* R}{\upsilon}\right) + 1.75$ (a2)

For rough pipes

$\frac{v}{V_*} = 5.75 log_{10}(y/k) + 8.5$ (b1)

$\frac{\bar{V}}{V_*} = 5.75 log_{10}\left(\frac{R}{K}\right) + 4.75$ (b2)

Subtracting a2 from a1 or b2 from b1

$\frac{v-\bar{V}}{V*} = 5.75 log_{10}\left(\frac{y}{R}\right) + 3.75$

With reference to the mean velocity of flow, Karman-Prandtl expressions for the velocity distribution in both rough and smooth pipe become identical.

The point where the point velocity = mean velocity

For $v = \bar{V}$, above equation reduces to

$0 = 5.75 log_{10}\left(\frac{y}{R}\right) + 3.75$

y= 0.223R

# TURBULENCE FLOW IN A CIRCULAR PIPE

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## Relationship between maximum velocity and average velocity

At y=R, v= $v_{max}$

$$\frac{v_{max}-\bar{V}}{V_*} = 3.75$$

$$\frac{V_{max} - \bar{V}}{V^*} = 3.75$$

$$V_{max} = 2\bar{V}$$

# Major and minor loss

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## Major Loss / Friction Loss In Pipe Flow

The friction loss in a pipe of length $(l)$ and diameter (d) is generally calculated using ***Darcy-Weibach Equation*** as below:

$$h_f = \frac{flv^2}{2gd}$$

$f = \frac{64}{Re}$ → Hagen equation → Laminar

Turbulent: $f = \phi(Re, \frac{K}{D})$ → Moodys diagram

where,

f = friction factor

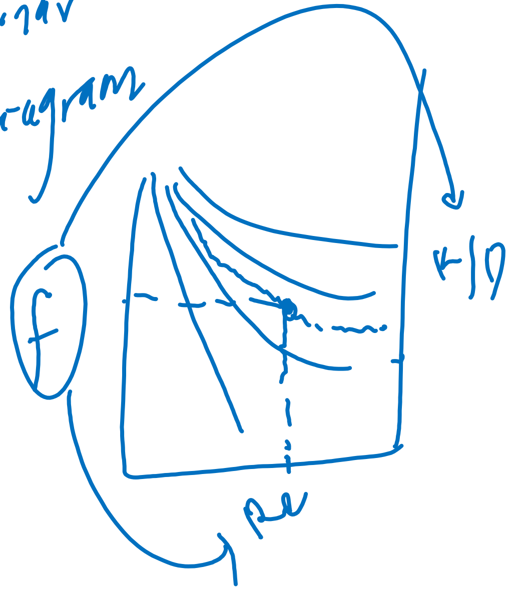
l = length of pipe

v = velocity of flow in the pipe

g = acceleration due to gravity

= $9.81 m/s^2$

d = diameter of pipe



# Major and minor loss

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## Major Loss / Friction Loss In Pipe Flow

The friction loss in a pipe of length $(l)$ and diameter (d) is generally calculated using ***Darcy-Weibach Equation*** as below:

$$h_f = \frac{flv^2}{2gd} \rightarrow h_f = \frac{8flQ^2}{\pi^2 g D^5}$$

in terms of discharge,

$$h_f = \frac{fLV^2}{2gD} = \frac{fL}{2gD}\frac{Q^2}{A^2} = \frac{fL}{2gD}\frac{Q^2}{\left(\frac{\pi}{4}D^2\right)^2} = \frac{8fL}{\pi^2 gD^5}Q^2 = rQ^2$$

Where r = $\frac{8fL}{\pi^2 gD^5}$ or $\frac{fL}{12.1D^5}$

Coefficient r is called resistance coefficient.

# Major and minor loss

## For laminar flow

The friction factor is evaluated depending upon the regime of flow (laminar or turbulent) and the type of surface boundary (smooth or rough). The friction factor for pipe flow is determined using ***Moody diagram*** for which Reynold's Number, pipe roughness, and diameter of pipe is required to be known. Moody diagram can be used for laminar as well as turbulent flow regime. Alternatively, for laminar flow, friction factor can be formulated as below:

$$f = \frac{64}{Re}$$

Here, the Renyold's Number (Re) can be calculated as below:

$$Re = \frac{v\rho d}{\mu}$$

$$Re = \frac{vd}{\nu}$$

where, kinematic viscosity $(\nu)$ for water is approximately equal to $10^{-6} m^2/s$ and is formulated as below:

$$\nu = \frac{\mu}{\rho}$$

# Major and minor loss

For turbulent flow in Smooth pipe

In case of turbulent flow, friction factor is determined based on whether the pipe is hydrodynamically smooth or rough. For turbulent flow in hydrodynamically smooth pipe, friction factor is given as:

$$f = \frac{0.3164}{Re^{1/4}} \; for \; 4 * 10^3 < Re < 4 * 10^5$$

For higher Reynold's number from Nikuradse experiment

$$\frac{1}{\sqrt{f}} = 2.0 \log\left(Re\sqrt{f}\right) - 0.8$$

$$f = 0.0032 + \frac{0.221}{(Re)^{0.237}}$$

# Major and minor loss

PANA ACADEMY

For turbulent flow in Rough pipe

$$\frac{1}{\sqrt{f}} = 2.0 \log\left(\frac{R}{k}\right) + 1.74$$

as long as the parameter $\left(\frac{\text{Re}\sqrt{f}}{R/k}\right)$ is less than 17 the pipe will behave as hydrodynamically smooth pipe and when $\left(\frac{\text{Re}\sqrt{f}}{R/k}\right)$ becomes greater than 400, then the pipe will behave as hydrodynamically rough pipe. For $17 < \left(\frac{\text{Re}\sqrt{f}}{Rk}\right) < 400$ the pipe boundary will behave as in transition.

# Major and minor loss

The friction factor for turbulent flow in commercial pipes can be calculated using ***Colebrook-White Equation*** as below:

$$\frac{1}{\sqrt{f}} = -2log\left(\frac{e}{3.70d} + \frac{2.51}{Re\sqrt{f}}\right)$$

Moody's Diagram

Here, the roughness height (e) for different pipe material is given below:

| Pipe Material | Roughness Height, e (mm) |
|---|---|
| Concrete | 0.3 - 3 |
| Wood | 0.3 |
| Cast Iron | 0.26 |
| Galvanized Iron | 0.15 |
| Wrought Iron | 0.046 |

# Minor loss



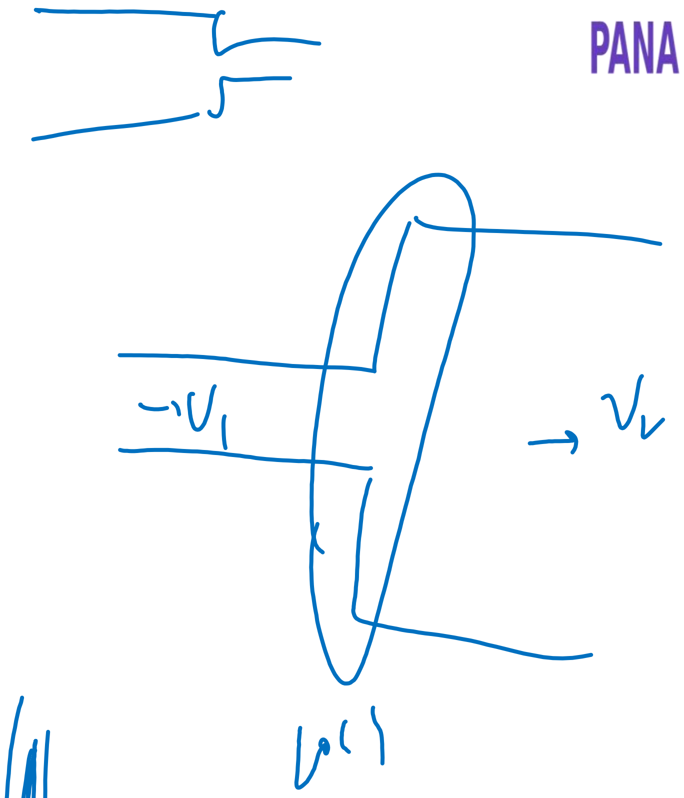
1. **Head loss due to sudden enlargement**

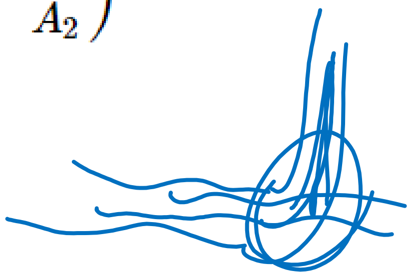
$$h_m = \frac{(v_1 - v_2)^2}{2g}$$

$$= k\frac{v_1^2}{2g}$$

where,

$$k = \left(1 - \frac{A_1}{A_2}\right)^2$$





# Minor loss

PANA ACADEMY

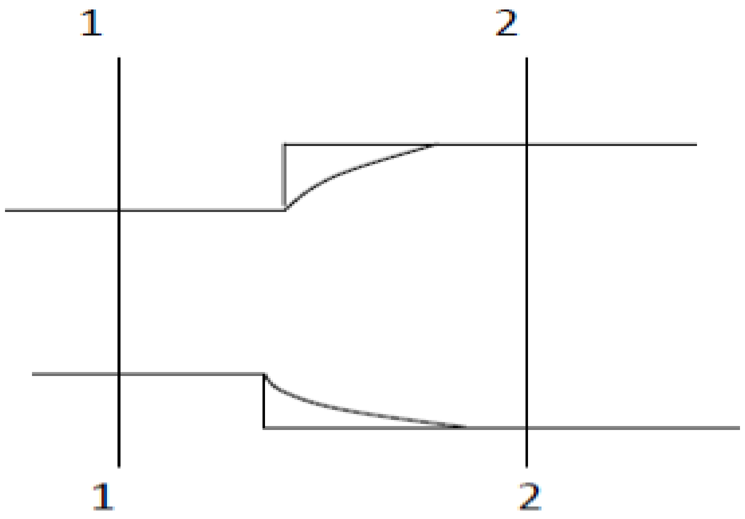



Fig. 1.11: Head loss due to sudden enlargement

Applying Bernoulli's equation between section 1 and 2

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2 + h_e$$

$Z_1$=$Z_2$ (horizontal)

$$h_e = \left(\frac{P_1}{\gamma} - \frac{P_2}{\gamma}\right) + \left(\frac{V_1^2}{2g} - \frac{V_2^2}{2g}\right) \quad \text{(a)}$$

# Minor loss

PANA ACADEMY

According to momentum principle,

$$\sum F_x = \rho Q\,(V_2 - V_1)$$

$$P_1A_1 + P_x(A_2 - A_1) - P_2A_2 = \rho Q(V_2 - V_1)$$

But experimentally it is found that $P_x = P_1$

$$P_1A_1 + P_1(A_2 - A_1) - P_2A_2 = \rho Q(V_2 - V_1)$$

$$(P_1 - P_2)A_2 = \rho Q(V_2 - V_1)$$

$$(P_1 - P_2)A_2 = \rho A_2V_2(V_2 - V_1)$$

$$\frac{(P_1 - P_2)}{\rho g} = \frac{1}{g}(V_2^2 - V_1V_2)$$

$$\frac{(P_1 - P_2)}{\gamma} = \frac{1}{g}(V_2^2 - V_1V_2) \qquad \text{(b)}$$

From eq. a and b

$$h_e = \frac{1}{g}(V_2^2 - V_1V_2) + \left(\frac{V_1^2}{2g} - \frac{V_2^2}{2g}\right)$$

$$h_e = \frac{(V_1 - V_2)^2}{2g}$$

$$h_e = \frac{V_1^2}{2g}\left(1 - \frac{V_2}{V_1}\right)^2 = \frac{V_1^2}{2g}\left(1 - \frac{Q/A_2}{Q/A_1}\right)^2 = \frac{V_1^2}{2g}\left(1 - \frac{A_1}{A_2}\right)^2 = K\frac{V_1^2}{2g}$$

This shows that minor loss varies as the square of velocity.

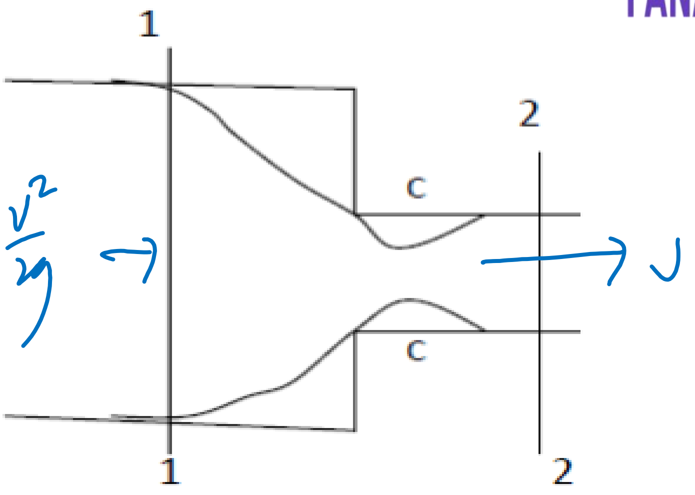
# Minor loss

**2. Head loss due to sudden contraction**

$$h_m = k\frac{v_2^2}{2g}$$

where,

$$k = \left(\frac{1}{C_c} - 1\right)^2$$




$C_c$ is coefficient of contraction. The value of $C_c$ or k is not constant but depends on the ratio $\left(\frac{A_2}{A_1}\right)$. The value of 'k' is generally taken as 0.50.

# Minor loss

**3. Head loss at the entrance to a pipe**

$$h_m = 0.5\frac{v^2}{2g}$$

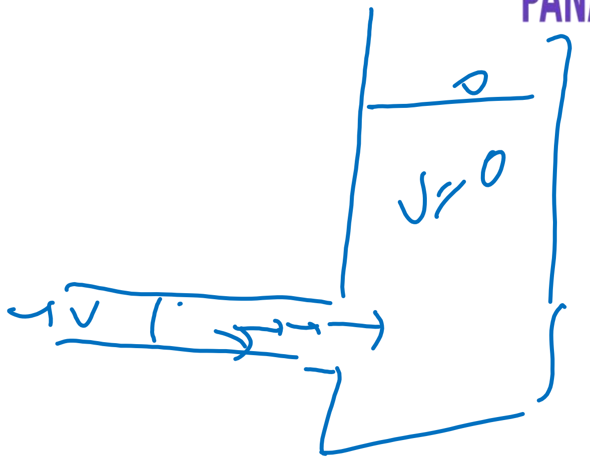
| Type of entrance | K |
|---|---|
| Sharp cornered | 0.5 (common case) |
| Rounded | 0.2 |
| Bell mouthed | 0.05 |

# Minor loss

PANA ACADEMY

**4. Head loss at exit from a pipe**

$$h_m = \frac{v^2}{2g}$$




sudden expansion → $\frac{(v_1 - v_2)^2}{2g}$

sudden contraction = $0.5\,\frac{v^2}{2g}$

entry to pipe = $0.5\, v^2/2g$

exit from pipe = $v^2/2g$

$\left(k\,\frac{v^2}{2g}\right)$

# Minor loss

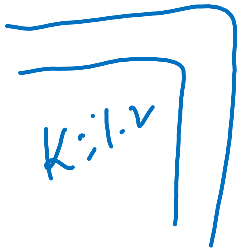
PANA ACADEMY

**5. Head loss due to bend**

$$h_m = k\frac{v^2}{2g}$$

The value of 'k' depends on total angle of bend and relative radius of curvature R/d, where, R is radius of curvature of pipe axis and d is diameter of pipe.

Sharp $90^0$ bend, K = 1.2
Sharp $180^0$ bend, K = 2.2




# Minor loss

PANA ACADEMY

**6. Head loss in various pipe fittings**

$$h_m = k\frac{v^2}{2g}$$

The value of coefficient 'k' actually depends on the type of pipe fittings.

| Fittings | K |
|---|---|
| Standard T (branch flow) | 1.8 |
| Standard T (line flow) | 0.4 |
| $90^0$ elbow (long radius) | 0.6 |
| $90^0$ elbow (short radius) | 1.5 |
| Gate valve (fully open) | 0.2 |

# Minor loss

PANA ACADEMY

| Situation | Head loss $= h_L$ | Remarks/Explanation |
|---|---|---|
| 1. Sudden expansion | $\frac{(V_1 - V_2)^2}{2g}$ | Expansion from section 1 to 2 |
| 2. Sudden contraction | $\left(\frac{1}{C_c} - 1\right)^2 \frac{V_2^2}{2g}$ | $C_c$ = coefficient of contraction<br>$V_2$ = velocity in contracted section |
| 3. Square edged entrance to a pipe | $0.5\ \frac{V^2}{2g}$ | Square edged entrance from a reservoir |
| 4. Exit from a pipe | $\frac{V^2}{2g}$ | |
| 5. Conical expansion | $k\ \frac{(V_1 - V_2)^2}{2g}$ | $k = \text{fn}\,(\theta, D_2/D_1)$ |
| 6. Bends | $k\ \frac{V^2}{2g}$ | $k = \text{fn}\,(\theta, R/D)$ |
| 7. Pipe fittings | $k\ V_1^2/2g$ | Values of $k$ given in Table 11.2. |
| 8. Nozzle | $\left(\frac{1}{C_v^2} - 1\right) V^2/2g$ | $C_v$ = coeff. of velocity of the nozzle |

# Minor loss

Equivalent length of pipe representing minor loss ($L_e$)

$$\frac{kV^2}{2g} = \frac{fL_eV^2}{2gD}$$

$$L_e = \frac{KD}{f}$$

For more than one type of losses, K represents the sum of several losses.

This $L_e$ can be added to L for computing head loss using Darcy- Weisbach equation.

$$h_f = \frac{f(L+L_e)V^2}{2gD}$$

# Pipe in series

PANA ACADEMY

Example

$$h_f = \frac{8fLQ^2}{\pi^2 g D^5}$$

✓ $h_f = h_{f1} + h_{f2} + h_{f3}$

✓ $Q = Q_1 = Q_2 = Q_3$

Headloss
↳ add

Discharge
↳ same

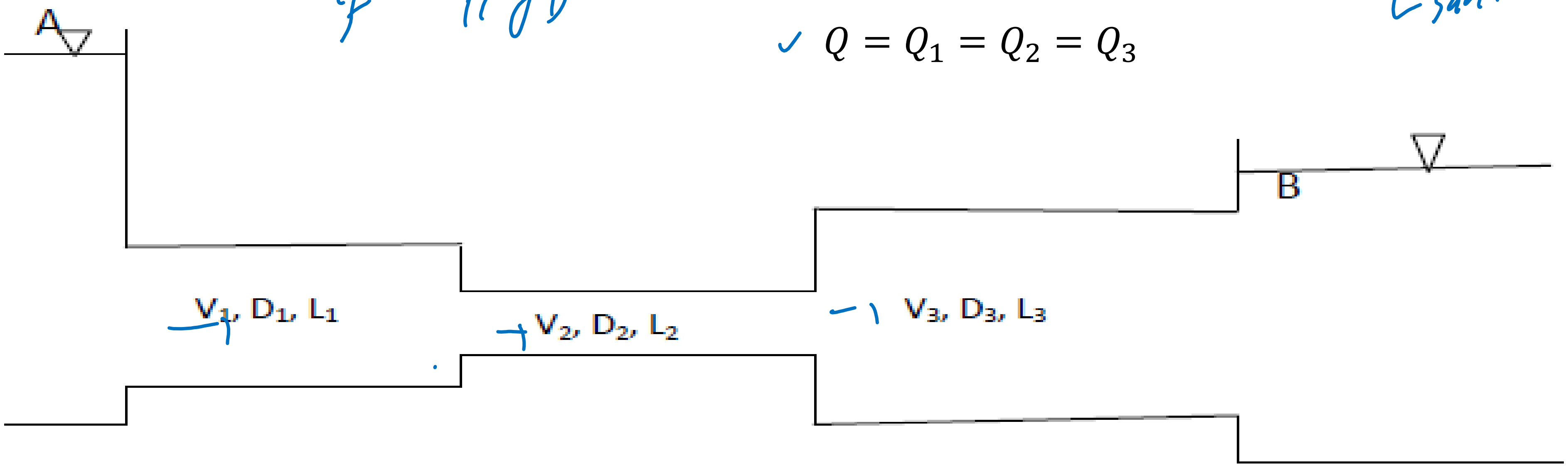



Fig. 2.1: pipes in series

# Pipe in series

PANA ACADEMY

Bernoulli's equation at A and B

$$\frac{P_A}{\gamma} + \frac{V_A^2}{2g} + Z_A = \frac{P_B}{\gamma} + \frac{V_B^2}{2g} + Z_B + total\ head\ loss$$

$$0 + 0 + Z_A = 0 + 0 + Z_B + 0.5\frac{V_1^2}{2g} + \frac{f_1 L_1 V_1^2}{2gD_1} + 0.5\frac{V_2^2}{2g} + \frac{f_2 L_2 V_2^2}{2gD_2} + \frac{(V_2 - V_3)^2}{2g} + \frac{f_3 L_3 V_3^2}{2gD_3} + \frac{V_3^2}{2g}$$

$$H = 0.5\frac{V_1^2}{2g} + \frac{f_1 L_1 V_1^2}{2gD_1} + 0.5\frac{V_2^2}{2g} + \frac{f_2 L_2 V_2^2}{2gD_2} + \frac{(V_2 - V_3)^2}{2g} + \frac{f_3 L_3 V_3^2}{2gD_3} + \frac{V_3^2}{2g}$$

where H = difference in level between A and B

Continuity equation: $Q = Q_1 = Q_2 = Q_3$

Neglecting minor losses

$$H = \frac{f_1 L_1 V_1^2}{2gD_1} + \frac{f_2 L_2 V_2^2}{2gD_2} + \frac{f_3 L_3 V_3^2}{2gD_3}$$

In terms of Q

$H = r_1 Q_1^2 + r_2 Q_2^2 + r_3 Q_3^2$ where $r_i = \frac{8 f_i L_i}{\pi^2 g D_i^5}$ or $\frac{f_i L_i}{12.1 D_i^5}$

$$H = Q^2 \sum r_i$$

# Pipe in series

PANA ACADEMY

When a compound pipe is replaced by a single pipe of uniform diameter having head loss and discharge equal to the loss of head and discharge of a compound pipe consisting of several pipes of different lengths and diameter, then the single pipe is called equivalent pipe. The uniform diameter of the equivalent pipe is known as equivalent diameter of the compound pipe.

# Pipe in series

if D and L be the diameter and length respectively of the equivalent pipe carry same discharge Q and head loss in the equivalent pipe is same that as that in the compound pipe.

Loss of head in equivalent pipe is $h_L = r\,Q^2$

Equating two head loss

$r\,Q^2 = (r_1 + r_2 + r_3 + \cdots)Q^2$

$r = (r_1 + r_2 + r_3 + \cdots)$

$$\frac{fL}{12.D^5} = \left[\frac{f_1 L_1}{12.1D_1^5} + \frac{f_2 L_2}{12.1D_2^5} + \frac{f_3 L_3}{12.1D_3^5} + \cdots\right]$$

$$\frac{fL}{D^5} = \left[\frac{f_1 L_1}{D_1^5} + \frac{f_2 L_2}{D_2^5} + \frac{f_3 L_3}{D_3^5} + \cdots\right]$$

This equation is called Dupuit's equation, which may be used to determine the size of the equivalent pipe.

If $f = f_1 = f_2 = f_3 = \ldots = f$

$$\frac{L}{D^5} = \left[\frac{L_1}{D_1^5} + \frac{L_2}{D_2^5} + \frac{L_3}{D_3^5} + \cdots\right]$$

# Pipe in parallel

PANA ACADEMY

$$h_f = h_{f1} = h_{f2}$$

$$Q = Q_1 + Q_2$$

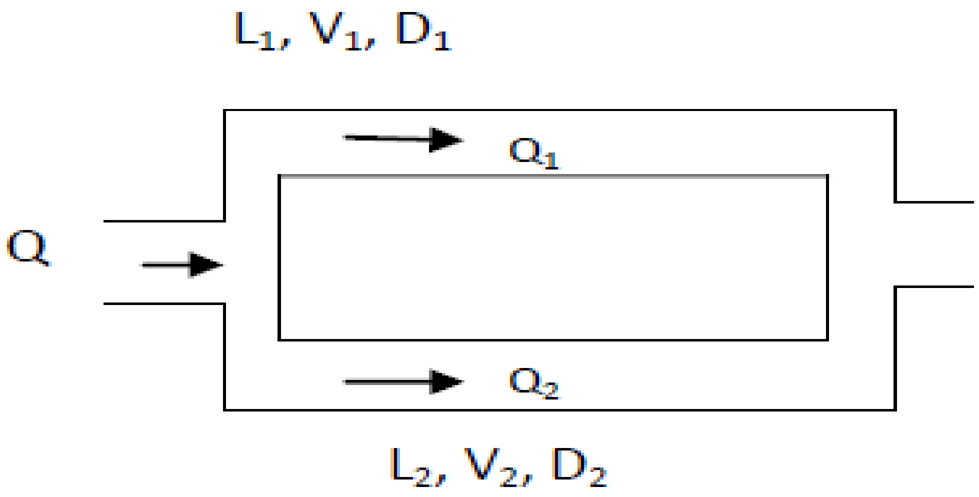



Fig. 2.2: Pipes in parallel

# Pipe in parallel

$$\frac{f_1 L_1 V_1^2}{2gD_1} = \frac{f_2 L_2 V_2^2}{2gD_2}$$

In terms of Q

$$r_1 Q_1^2 = r_2 Q_2^2$$

# Pipe in parallel

PANA ACADEMY

Equivalent pipe for pipes in parallel

We know,

$h_f = rQ^2$

Total discharge through the main is

$Q = \sqrt{\frac{h_f}{r}}$

$h_f$ is same for all pipes in case of parallel connection.

Discharge through each pipe

$Q_1 = \sqrt{\frac{h_f}{r_1}}, Q_2 = \sqrt{\frac{h_f}{r_2}}$ ........

# Pipe in parallel

PANA ACADEMY

$Q = Q_1 + Q_2 + \ldots$

$$\sqrt{\frac{h_f}{r}} = \sqrt{\frac{h_f}{r_1}} + \sqrt{\frac{h_f}{r_2}} + \cdots$$

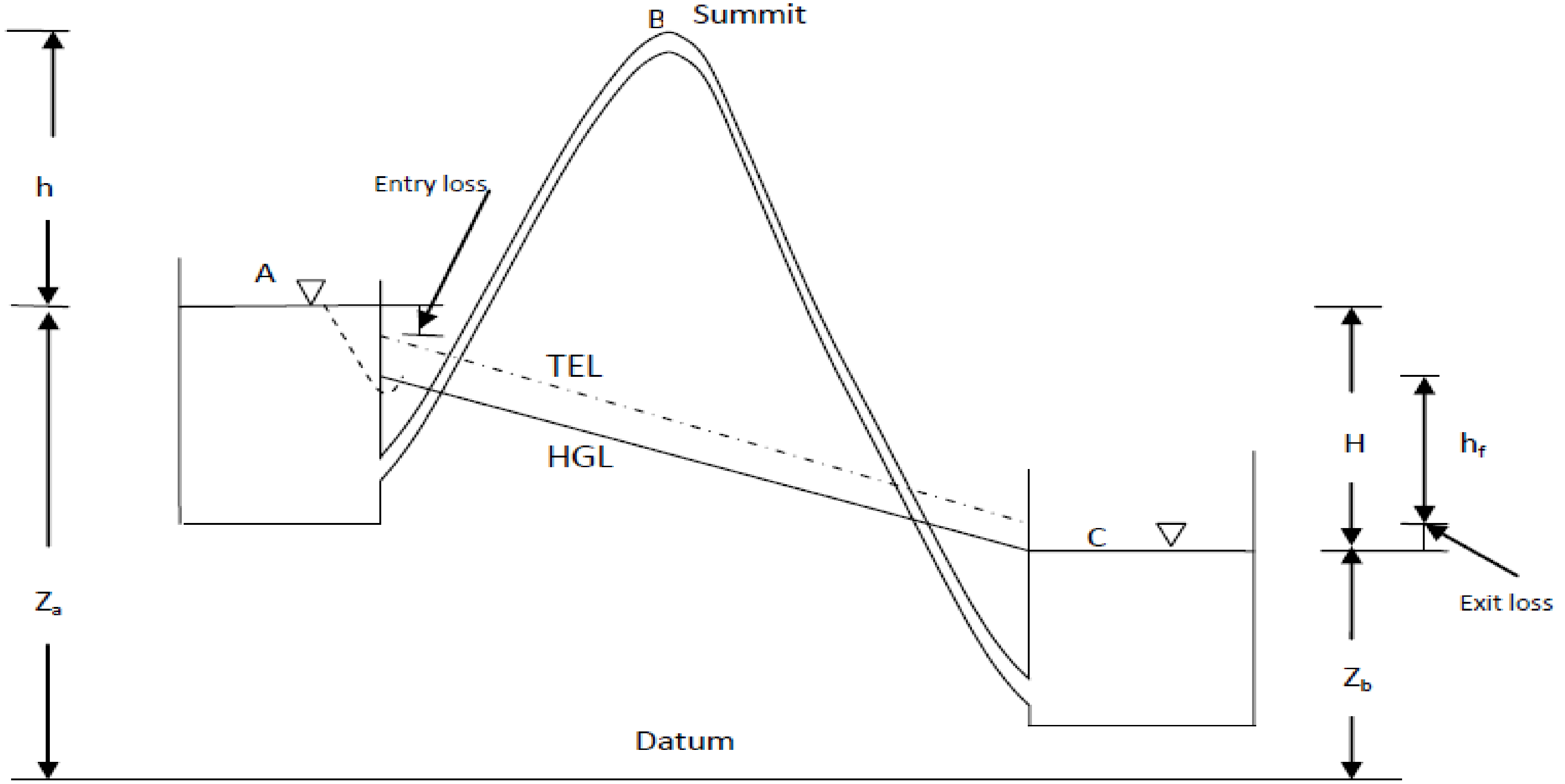
$$\frac{1}{\sqrt{r}} = \frac{1}{\sqrt{r_1}} + \frac{1}{\sqrt{r_2}} + \cdots$$

r value of equivalent single pipe replacing parallel pipes with discharge Q can be found out t

expression. In terms of f,

$$\frac{1}{\sqrt{\frac{fL}{D^5}}} = \frac{1}{\sqrt{\frac{f_1 L_1}{D_1^5}}} + \frac{1}{\sqrt{\frac{f_2 L_2}{D_2^5}}} + \cdots$$

# Siphon



# Siphon

PANA ACADEMY

a. Determination of head loss between two tanks

Assuming the siphon to run full, then applying Bernoulli's equation between two points A and C, we get

$$\frac{P_A}{\gamma}+\frac{V_A^2}{2g}+Z_A=\frac{P_C}{\gamma}+\frac{V_C^2}{2g}+Z_C+Losses$$

(Working in terms of absolute pressure)

$$10.3+0+Z_A=10.3+0+Z_C+0.5\frac{V^2}{2g}+\frac{V^2}{2g}+\frac{fLV^2}{2gD}$$

V= Velocity of flow, L = Length of siphon, D = Diameter of siphon

Entry loss = $0.5V^2/2g$, exit loss = $V^2/2g$, loss due to friction (Major loss) = $\frac{fLV^2}{2gD}$

$$Z_A-Z_C=\frac{V^2}{2g}\left(1.5+\frac{fL}{D}\right)$$

$$H=\frac{V^2}{2g}\left(1.5+\frac{fL}{D}\right)$$

b. Determination of maximum height of summit

Applying Bernoulli's equation is applied between the point A and summit B. (Working in terms of absolute pressure given the absolute pressure at summit)

$$\frac{P_A}{\gamma} + \frac{V_A^2}{2g} + Z_A = \frac{P_B}{\gamma} + \frac{V_B^2}{2g} + Z_B + Losses$$

(Working in terms of absolute pressure)

$$\frac{P_A}{\gamma} + 0 + Z_A = \frac{P_B}{\gamma} + \frac{V^2}{2g} + (Z_A + h) + 0.5\frac{V^2}{2g} + \frac{flV^2}{2gD}$$

$$h = \frac{P_A}{\gamma} - \frac{P_B}{\gamma} - \frac{V^2}{2g}\left(1.5 + \frac{fl}{D}\right)$$

$P_A$ = Atmospheric pressure = 10.3 m of water
$P_B$ = Absolute pressure at summit
l = length of inlet leg of siphon

If absolute pressure at summit is given, above expression is used to compute h. If gauge pressure is given, then take $P_a/\gamma = 0$.

# Siphon

(Working in terms of absolute pressure)

$$\frac{P_A}{\gamma} + 0 + Z_A = \frac{P_B}{\gamma} + \frac{V^2}{2g} + (Z_A + h) + 0.5\frac{V^2}{2g} + \frac{flV^2}{2gD}$$

$$h = \frac{P_A}{\gamma} - \frac{P_B}{\gamma} - \frac{V^2}{2g}\left(1.5 + \frac{fl}{D}\right)$$

$P_A$ = Atmospheric pressure = 10.3 m of water
$P_B$ = Absolute pressure at summit
l = length of inlet leg of siphon

If absolute pressure at summit is given, above expression is used to compute h. If gauge pressure is given, then take $P_a/\gamma = 0$.

Abs. pressure at B can be 0atm theoretically but practically abs. pressure going below 2.5m of water, dissolved gasses would come out and form air plug stopping flow

# Siphon

c. Determination of pressure at summit

Applying Bernoulli's equation is applied between the point A and summit B. (Working in terms of absolute pressure given the absolute pressure at summit)

$$\frac{P_A}{\gamma} + \frac{V_A^2}{2g} + Z_A = \frac{P_B}{\gamma} + \frac{V_B^2}{2g} + Z_B + Losses$$

(Working in terms of absolute pressure)

$$\frac{P_A}{\gamma} + 0 + Z_A = \frac{P_B}{\gamma} + \frac{V^2}{2g} + (Z_A + h) + 0.5\frac{V^2}{2g} + \frac{flV^2}{2gD}$$

$$\frac{P_B}{\gamma} = \frac{P_A}{\gamma} - h - \frac{V^2}{2g}\left(1.5 + \frac{fl}{D}\right)$$

(Note: If the end C is discharging to atmosphere, then take velocity at C = Velocity of flow through pipe in Bernoulli's equation.)

# Water hammer

PANA ACADEMY

When a liquid flowing through a long pipe is suddenly brought to rest by closing the valve at the end of a pipe, then a pressure wave of high intensity is produced behind the valve. This pressure wave of high intensity has the effect of hammering action on the walls of the pipe. This phenomenon is known as water hammer or hammer blow.

# Water hammer

PANA ACADEMY

The **pressure rise due to water hammer depends upon** the following factors:

- The **velocity of flow of water in the pipe**.
- The length of the pipe.
- Time is taken to close the valve or the closure of the valve.
- **Elastic properties of the material of the pipe.**
- **The density of fluid.**

# Water hammer

**Sudden and Gradual closure of valve:**

- If T be the time required by the pressure wave to travel once up and down the pipe and c is the celerity then:
- T = $\frac{2L}{c}$
- Let ‘t’ be the actual time taken for a closer of the valve

**If “t >T” then such closure is referred to as the gradual closure of the valve.**

- When valve closure is gradual, fluid is considered incompressible, and pipe material is considered rigid.

**If “t < T” then such closure is referred to the as instantaneous or sudden closure of the valve.**

- When valve closure is instantaneous or sudden fluid is considered to be compressible and pipe material is considered both as rigid and elastic.

# Water hammer

PANA ACADEMY

**1. Gradual closer of the valve**: when the valve closing time is more than the wave propagation time then this is known as the gradual closing of the valve.

**In gradual closer pressure rise is given by**

$$P = \frac{\rho V L}{t}$$

Where P Rise in pressure, ρ is the density of the fluid, V is the initial velocity of flow, L is the length of the pipe, t is the time of closure of the valve

# Water hammer

**2. Sudden closer of the valve**: when valve closing time is less than the wave propagation time then is known as the sudden closure of the valve.

- **The sudden closure of the valve when the pipe is assumed rigid**:

The pressure rise is given by:

$$P = \rho V C$$

Where P is a rise in pressure, ρ is the density of the fluid, V is the initial velocity of flow, and C is the pressure wave velocity = $\sqrt{\frac{K}{\rho}}$

$$\therefore P = V\sqrt{K\rho}$$

# Water hammer

- **The sudden closure of the valve when the pipe is assumed to be elastic:**

The pressure rise is given by:

$$P = \rho V C'$$

$$C' = \frac{C}{\sqrt{1+\frac{DK}{Et}}} \text{ and } C = \sqrt{\frac{K}{\rho}}$$

$$\therefore P = \sqrt{\rho}\, V \frac{\sqrt{K}}{\sqrt{1+\frac{DK}{Et}}}$$

Where P is the rise in pressure, ρ is the density of the fluid, V is the initial velocity of flow, and C is pressure wave velocity, D is the diameter of the pipe, K is the bulk modulus of fluid, E is the modulus of elasticity, t is the thickness of the pipe.