PANA ACADEMY

NEPAL ENGINEERING COUNCIL

LICENSE EXAM PREPARATION COURSE

FOR

CIVIL ENGINEERS

3. Basic Water Resources Engineering

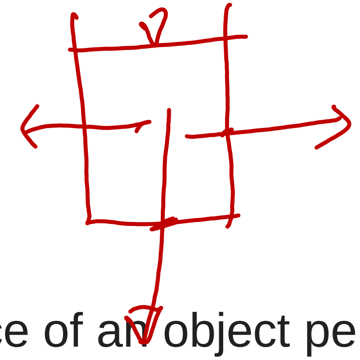
3.2 Hydrostatics Rest

Sub topics

PANA ACADEMY

- pressure and head
- Pascal’s law
- pressure-depth relationship
- Manometers
- pressure force and centre of pressure on submerged bodies (plane and curved surfaces, practical applications)
- pressure diagrams
- Buoyancy
- stability of floating/submerged bodies.

Pressure



$\gamma = \frac{W}{V}$

Pressure (P) is the force applied perpendicular to the surface of an object per unit area.

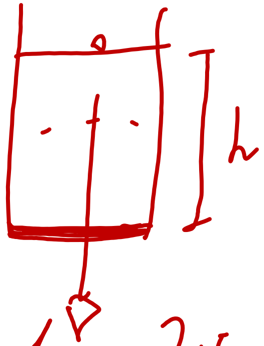
Dimensional $\frac{Kg\,m/s^2}{m^2} = \frac{kg}{m s^2}$ $N/m^2 \rightarrow Pa$

$M L^{-1} T^{-2}$

$$\boxed{P = \frac{F}{A} = \gamma h = \rho g h}$$

$p = \gamma h$

→ always vertical depth



$W = \gamma \cdot V$

$= \gamma \cdot A \cdot h$

Unit: Pa, N/m²

1 atm =101325 Pa =1.01325 bar

1 bar = 10^5 Pa

Atmospheric pressure decreases about 10% while moving 1 km higher altitude

$p = \frac{\text{force}}{\text{area}}$

$= \frac{\gamma A h}{A}$

$= \gamma h$

Pressure

Absolute Pressure: Absolute pressure is the sum of gauge pressure and atmospheric pressure.

Gauge Pressure: Gauge pressure is the pressure relative to atmospheric pressure.

Vacuum Pressure: Negative gauge Pressure

$$P_{abs} = P_g + P_{atm}$$

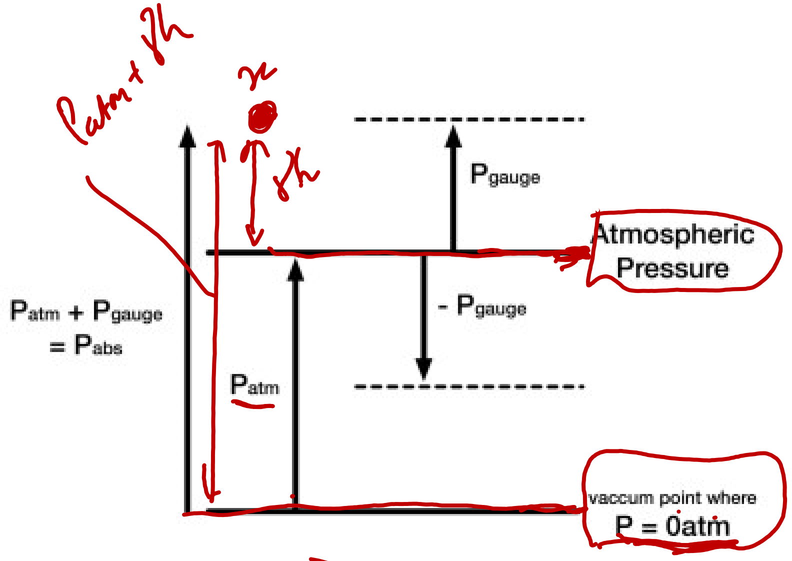
$$P_g = P_{abs} - P_{atm}, if\, P_{abs} > P_{atm}$$

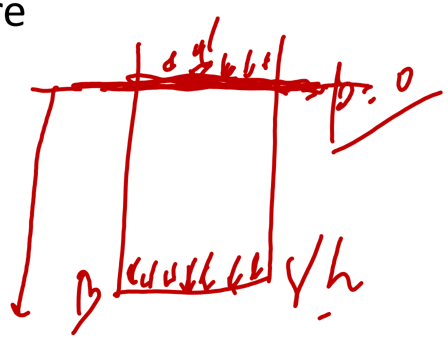
$$P_v = P_{atm} - P_{abs}\, , if\, P_{abs} < P_{atm}$$

$P \to \gamma h \to$ gauge

$\to P_{atm} + \gamma h \to$ absolute

0 K -273°C

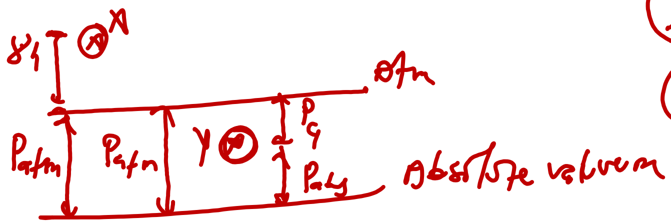




$h = 0 \to P = 0$

$P = 0$

$P = P_{atm}$



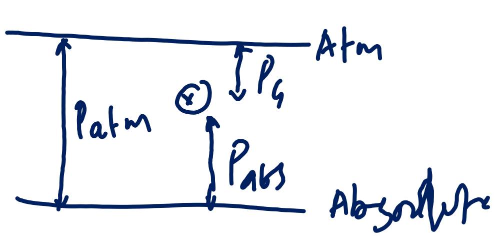
(X) $P_{abs} = P_{atm} - P_g$

(X) $P_{abs} = P_{atm} + P_g$

$\boxed{P_{abs} = P_{atm} \pm P_g}$

Pressure head

N/m^2 Pa



$P_{abs} = P_{atm} - P_g$

$P_{abs} \neq P_{abs} + P_g$

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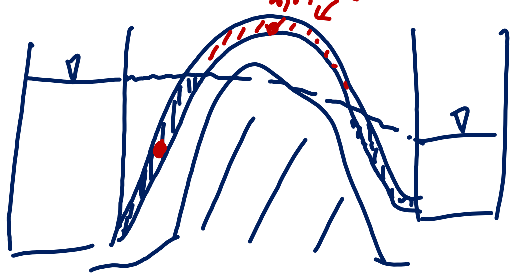
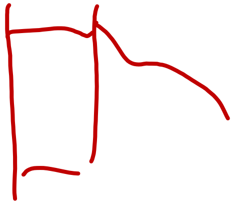
The pressure head is the height of a liquid column that corresponds to a particular pressure exerted by the liquid column on the base of its container. It may also be called **static pressure head** or simply **static head.**





$p = \gamma h$

$\frac{p}{\gamma} = h \rightarrow$ Head

$$\psi = \frac{p}{\gamma} = \frac{p}{\rho g}$$

The relationship of atmospheric, Gauge and absolute pressure P_s, (in case of vacuum) → syphon → Negative, (in case of positive pressure)

(a) $P_{abs} = P_{atm} + P_g$

(b) $P_{abs} = P_{atm} - P_g$

(c) $P_{abs} = P_{atm} \pm P_g$

(d) None

Hydrostatic Law

(?) In hydrostatic condition; there will exist

(a) Shear stress only

✓ (b) Normal stress only

(c) Both shear and normal

(d) None

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According to Hydrostatic Law, the rate of increase of pressure in a vertical direction is equal to the weight density/ specific weight of the fluid at that point when the fluid is stationary.

$$\frac{\partial P}{\partial z} = \gamma$$

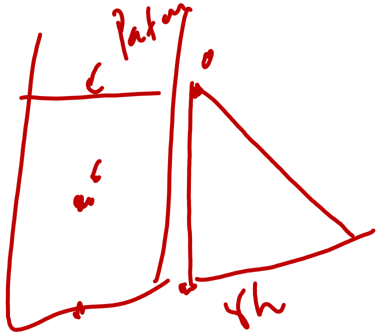
$p = \rho g h$

$p = \gamma z$

$$\boxed{\frac{dP}{dz} = \gamma}$$

The pressure at any point in a fluid at rest is obtained by the Hydrostatic Law.

$v = 0$

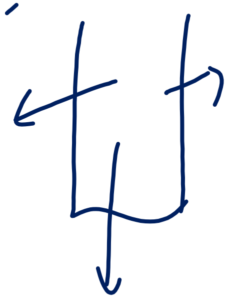


$$P_{abs} = P_{atm} + \gamma h$$

P_a

$p = \gamma z$

$\tau = \mu \frac{du}{dy} = 0$



Hydrostatic Law

$$\boxed{\begin{aligned}\frac{\partial P}{\partial z} &= -\rho(a_z + g)\\ \frac{\partial P}{\partial x} &= -\rho(a_x)\\ \frac{\partial P}{\partial y} &= -\rho(a_y)\end{aligned}}$$

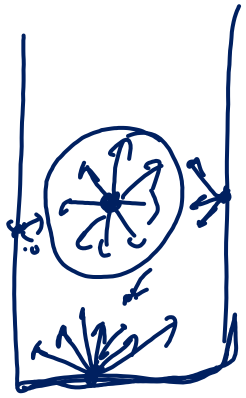
$a_z = 0 \quad \frac{\partial p}{\partial z} = \gamma$

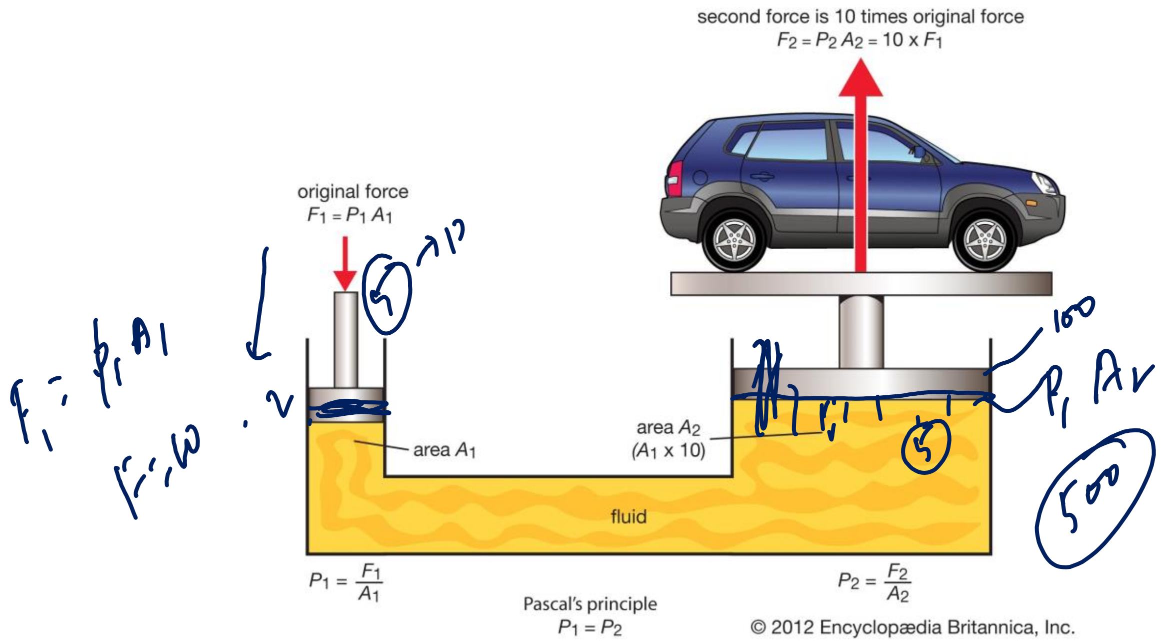
$a_x = 0 \quad \frac{\partial p}{\partial x} = 0$

$a_y = 0 \quad \frac{\partial p}{\partial y} = 0$

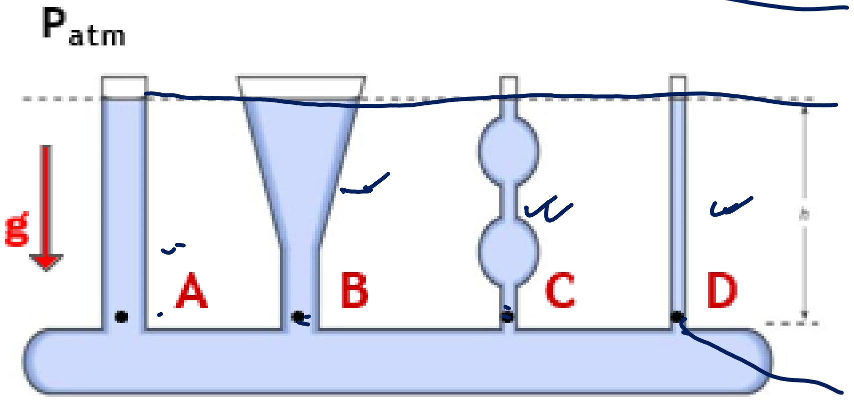
Pascal’s Law

The external static pressure applied on a confined liquid is distributed or transmitted evenly throughout the liquid in all directions





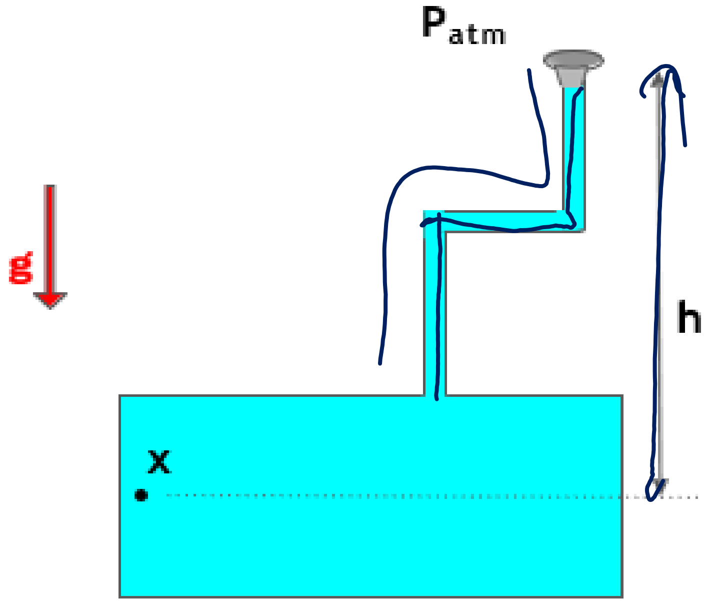
Fluid Pressure does NOT depend on shape of Vessel



$$P_A = P_B = P_C = P_D = P_{atm} + h\rho g$$

$F = p \cdot A$

$p = \rho g h$



$$P_x = P_{atm} + h\rho g$$

$p = \frac{F}{A}$

$F = p \cdot A$

Pressure Measurement

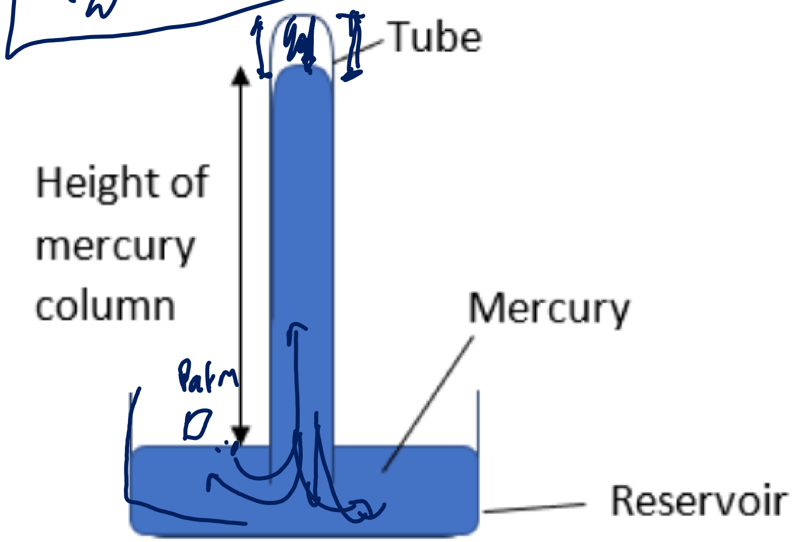
Barometer

$$P_{atm} = P_v + \gamma h$$

$S_m = 13600$

$h_m = \frac{P}{\gamma}$

$\boxed{h_w = 13.6 \times h_m}$



Mercury is suitable as it has very low vapour pressure such that

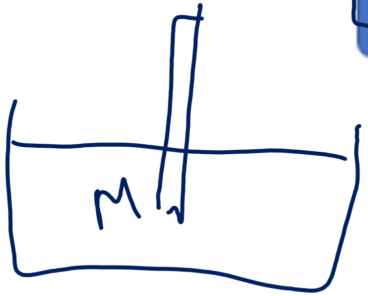
$$\boxed{P_{atm} = \gamma h}$$

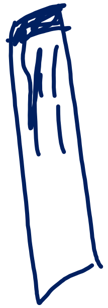
$\rightarrow P_{atm} = h \gamma_m$

It cannot measure negative gauge pressure

Pressure measured in mm of Hg

$$P_{atm} = 760\ mm\ of\ Hg = ?\ m\ of\ H_2O$$





Pressure Measurement

760 mm of Hg

76 cm of Hg

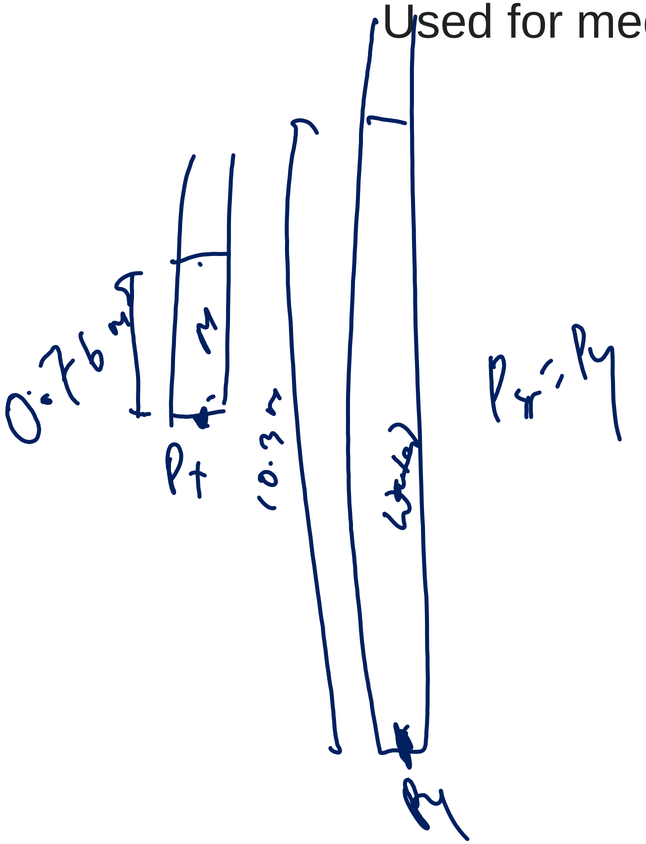
13.6×76 cm of water

10.3 m of water

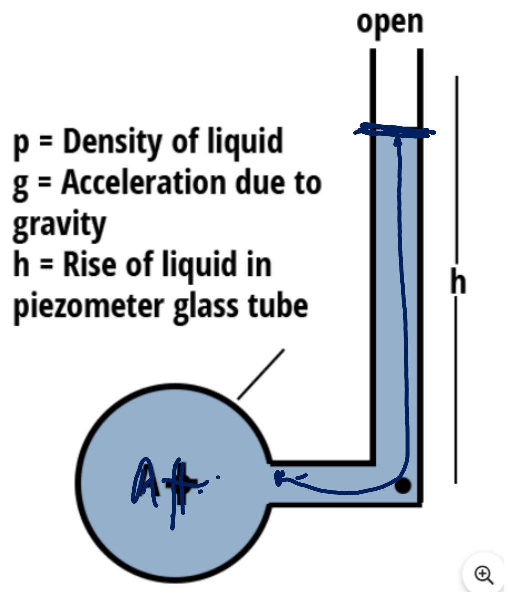
Manometer

Mercury generally used as it is higher density liquid

Used for medium pressure liquid



$$P_A = P_{atm} + \gamma_m h$$



A Schematic Diagram of the Piezometer

$P_A = \gamma_L \cdot h$

Pressure Measurement

Manometer

Mercury generally used as it is higher density liquid

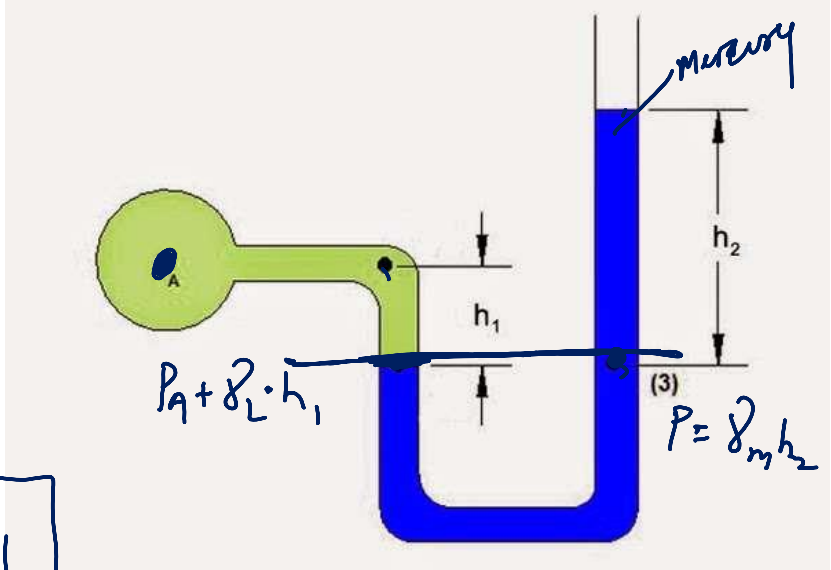
$$P_3 = P_{atm} + \gamma_m h_2$$
$$P_3 = P_A + \gamma_l h_1$$

$$P_{atm} + \gamma_m h_2 = P_A + \gamma_l h_1$$
$$P_A = P_{atm} + \gamma_m h_2 - \gamma_l h_1$$

$P_A + \gamma_l h_1 = \gamma_m h_2$

G — $\boxed{P_A = \gamma_m h_2 - \gamma_l h_1}$

$P_A = P_{atm} + \gamma_m h_2 - \gamma_l h_1$



U - Tube Manometer

Pressure Measurement

$\gamma_m = 13600 \times 9.81$

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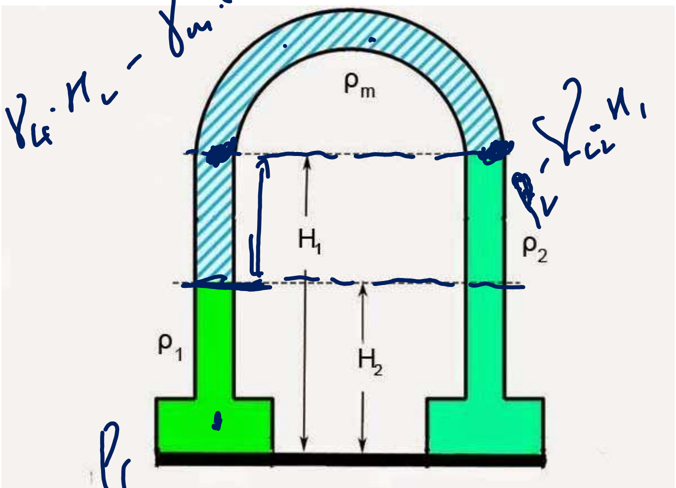
Inverted U tube Manometer

$$P_A = P_2 + \rho_2 g(H_1)$$
$$P_B = P_2 + \rho_m g(H_1 - H_2) + \rho_1 g(H_2)$$

$$P_A - P_B = \rho_2 g(H_1) - \rho_m g(H_1 - H_2) - \rho_1 g(H_2)$$

Density of manometric fluid should be less than other liquid

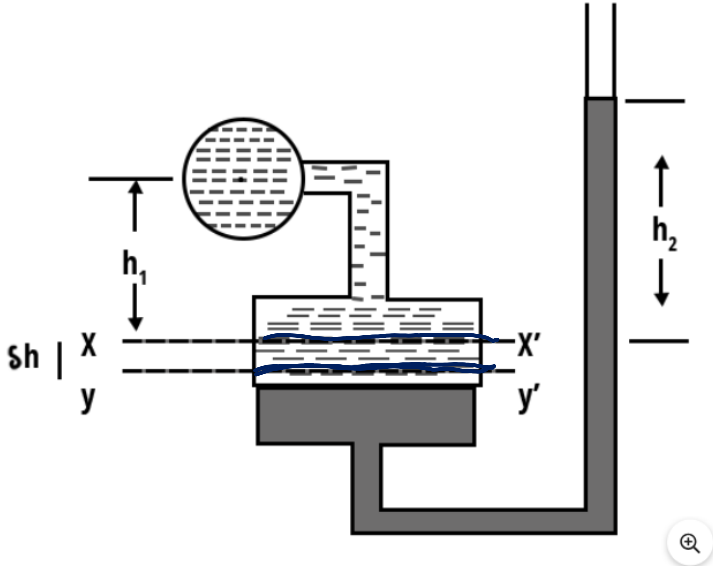


Inverted U tube Manometer

Pressure Measurement

Vertical single column Manometer

$$P_3 = \rho_m g(h_2 + \Delta h)$$
$$P_3 = P_A + \rho_l g(h_1 + \Delta h)$$
$$P_A = \rho_m g(h_2 + \Delta h) - \rho_l g(h_1 + \Delta h)$$
$$P_A \approx \rho_m g(h_2) - \rho_l g(h_1)$$



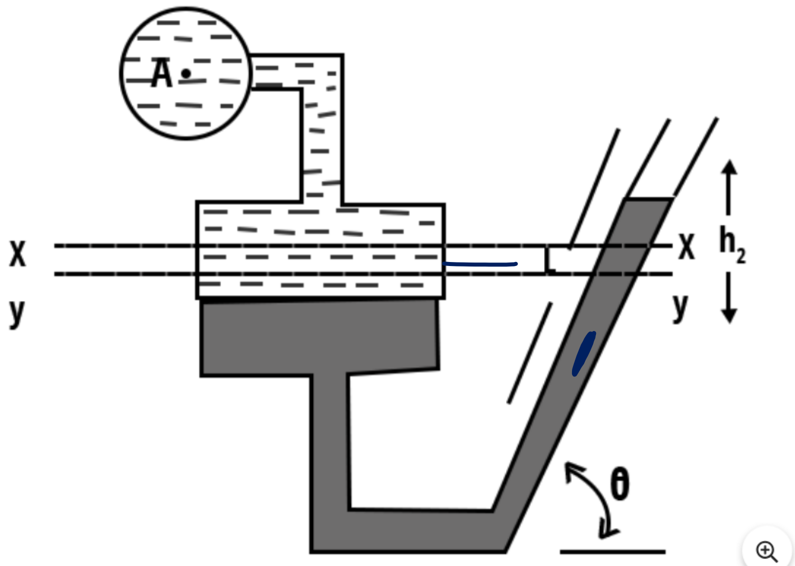
A Schematic Representation of a Vertical Single Column Manometer

Pressure Measurement

Inclined single column Manometer

$$P_3 = \rho_m g sin\theta(h_2+\Delta h)$$
$$P_3 = P_A + \rho_l g(h_1+\Delta h)$$
$$P_A = \rho_m g sin\theta(h_2+\Delta h) - \rho_l g(h_1+\Delta h)$$
$$P_A \approx \rho_m g h_2 sin\theta - \rho_l g h_1$$

Error decreases and sensitivity increases by factor of $1/\sin\theta$



A Schematic Representation of Inclined Single Column Manometer

Pressure Measurement

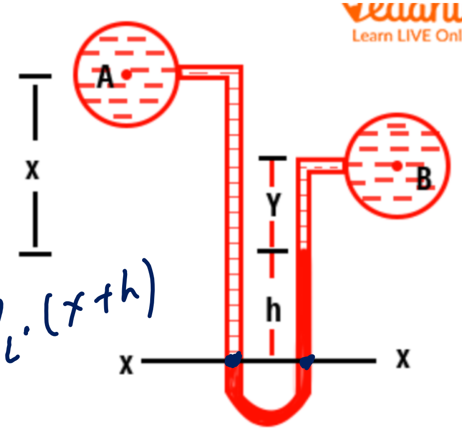
U tube differential Manometer

$$P_A - P_B = hg(\rho_g - \rho_1) + \rho_2 g y - \rho_1 g x$$

Balancing liquid column

$P_A + \gamma_L \cdot (x+h)$

$P_B + \gamma_L \cdot Y + \gamma_m h$



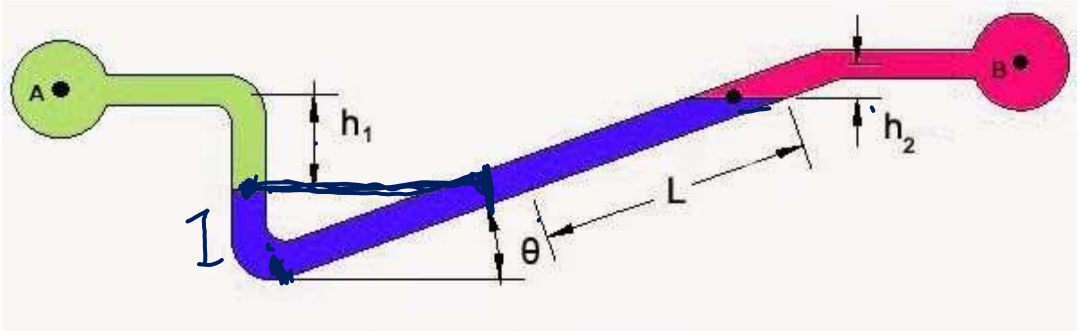
(a) Two pipes at different levels

A Schematic Representation of a Differential U-Tube Manometer

Pressure Measurement

Inclined U tube Manometer

$$P_B + \gamma_L h_2 + \gamma_m \cdot L \sin\theta = P_A + h_1 \gamma_L$$



Inclined Manometer

Pressure Measurement

Double U tube Manometer

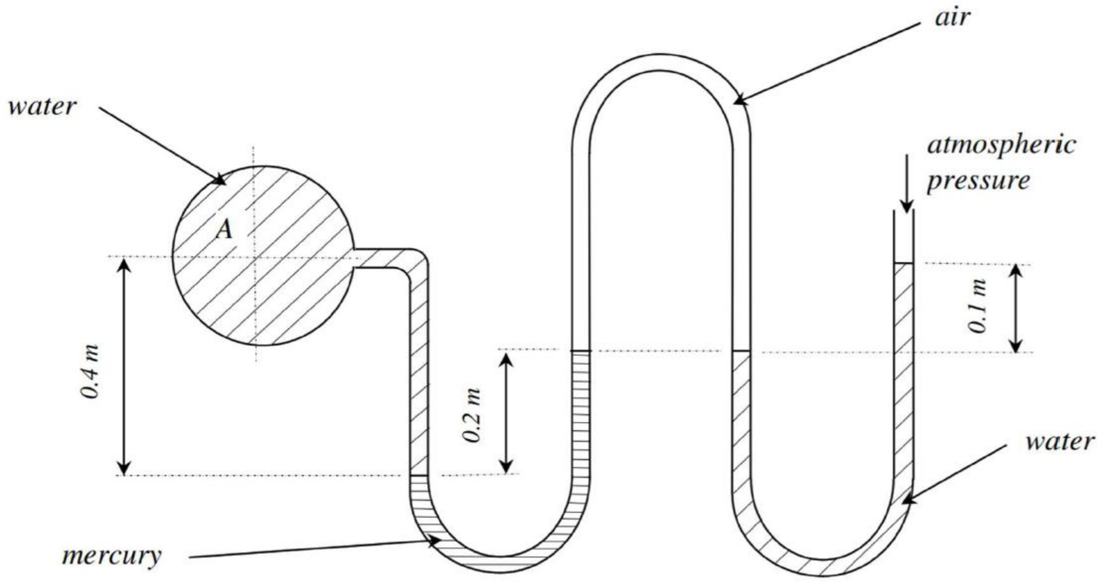


Figure Q1: Double U-tube manometer

Pressure Distribution

Pressure is compressive and acts normal to surface.

Force = $p \times$ Area

γh ; $p = 0$

Pressure at top = 0

Pressure at bottom = γh

Average pressure = $\gamma h/2$

Force on wall per unit width = $\frac{1}{2}\gamma h^2$

Resultant force acts at $\frac{2}{3}h$ from free surface

Δ area

$= \frac{1}{2}\gamma h \cdot h$

$= \frac{1}{2}\gamma h^2$

Force

pressure diagram area

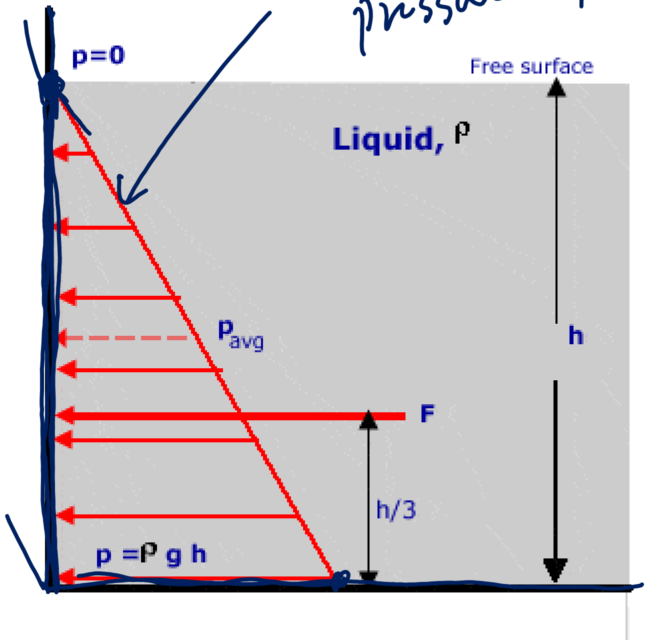
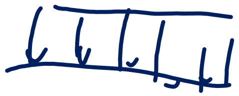


Figure: Pressure and Resultant Force on a Vertical Plane Surface



$F = \left(\frac{0+\gamma h}{2}\right)\cdot(h\cdot 1) = \frac{1}{2}\gamma h^2$

Pressure Distribution

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$$\bar{h} = \bar{x} + \frac{I_G \sin^2\theta}{A\bar{x}}$$

$$= \frac{h}{2} + \frac{1\cdot h^3/12}{h\cdot 1\cdot \frac{h}{2}}$$

$$= h/2 + \frac{h}{6} = \frac{2h}{3}$$

Pressure is compressive and acts normal to surface.

Pressure at top = 0

Pressure at bottom = γh

Average pressure = $\gamma h/2$

Force on wall = $\gamma A\bar{x}$

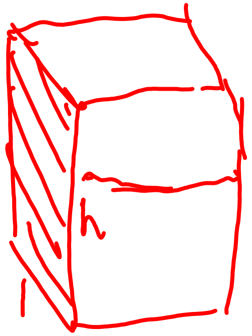
$\frac{1}{2}\gamma h^2$

center of pressure

$$\frac{\gamma A \bar{x}}{\gamma\cdot(h\cdot 1)\cdot\frac{h}{2}} \qquad \frac{\gamma h^2}{2}$$

Resultant force acts at $\bar{h}$ from free surface

$$\bar{h} = \bar{x} + \frac{I_{GG}\sin^2\theta}{A\bar{x}}$$



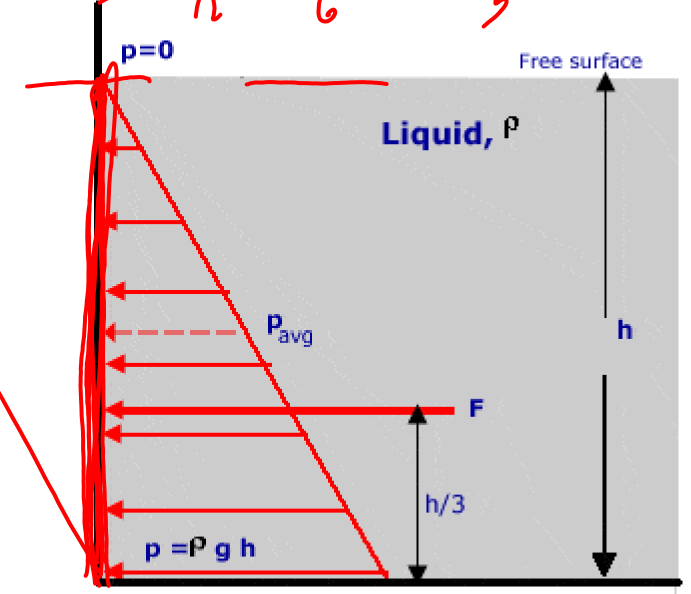


Figure: Pressure and Resultant Force on a Vertical Plane Surface

Pressure Distribution

Pressure is compressive and acts normal to surface.

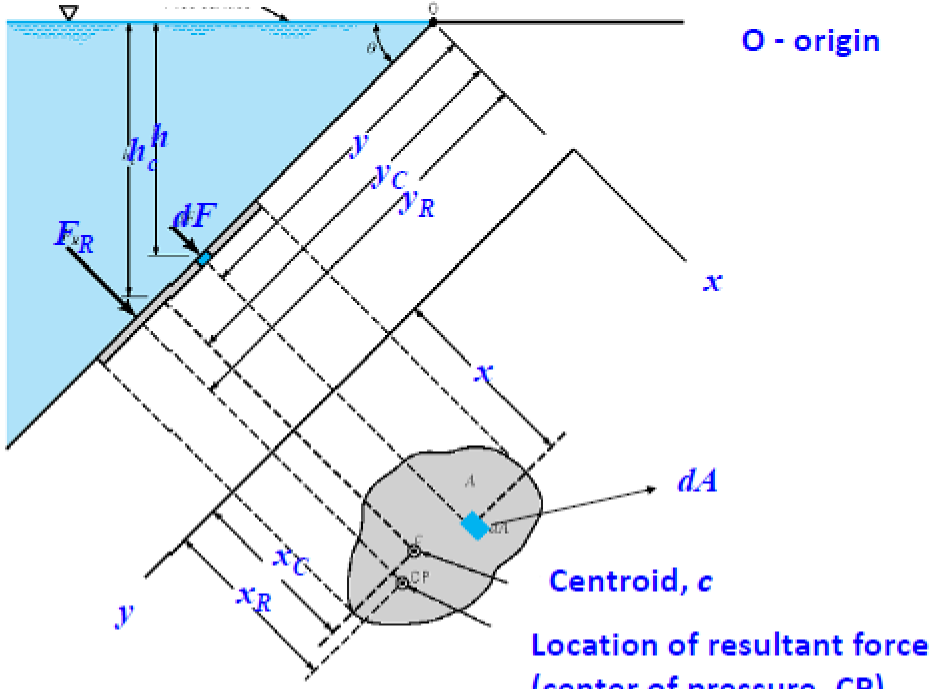
Pressure at top = γh_1

Pressure at bottom = γh_2

Force = $\gamma A\bar{x}$

Resultant force acts at $\bar{h}$ from free surface

$$\bar{h} = \bar{x} + \frac{I_{GG}\sin^2\theta}{A\bar{x}}$$



Pressure Distribution

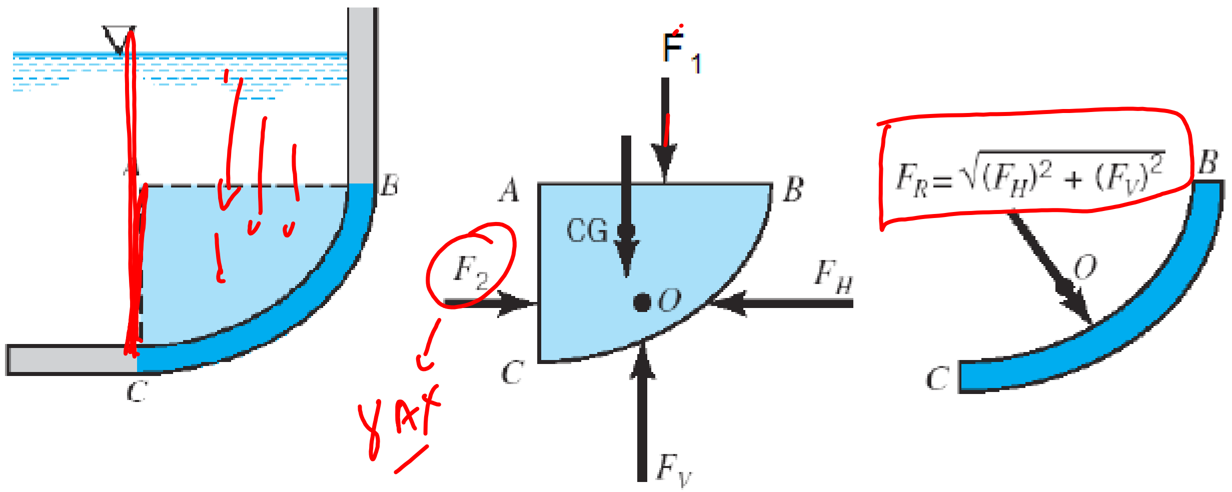
PANA ACADEMY

Horizontal force component

Force = $\gamma A \bar{x}$

Resultant force acts at $\bar{h}$ from free surface

$$\bar{h} = \bar{x} + \frac{I_{GG}}{A\bar{x}}$$



Vertical force component

Force = wt. of liquid above the curved surface

Downward direction depends on wetting surface being inside area considered

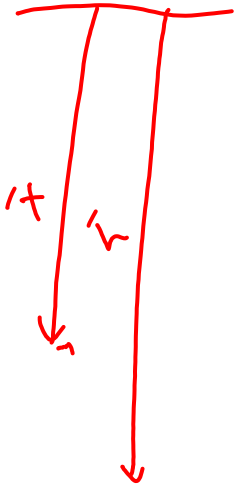
Pressure Distribution

Horizontal force component

Force = $\gamma A\bar{x}$

Resultant force acts at $\bar{h}$ from free surface

$$\boxed{\bar{h} = \bar{x} + \frac{I_{GG}}{A\bar{x}}}$$



cg - $\bar{x}$

cp - $\bar{h}$

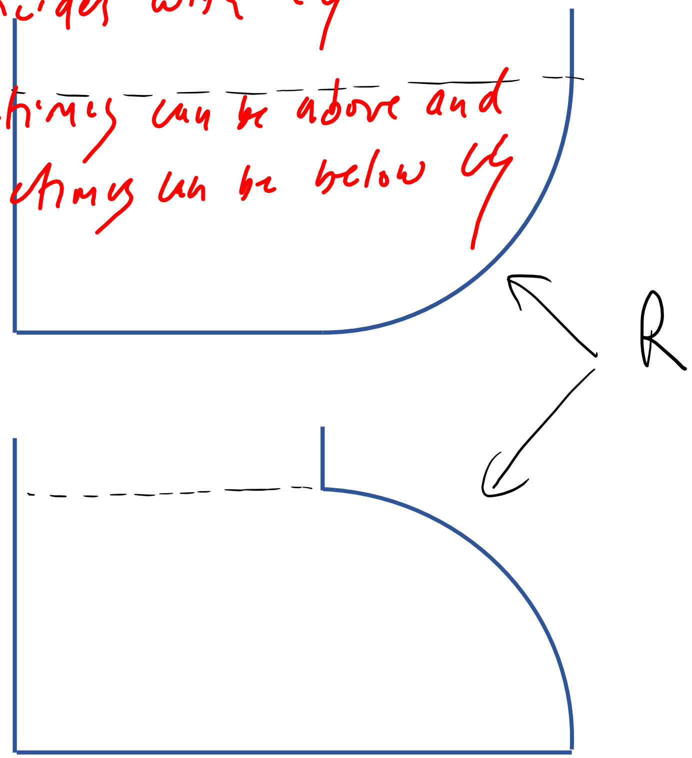
(7) The center of pressure will

(a) always be lower than cg

(b) always be above cg

(c) coincides with cg

(d) Sometimes can be above and sometimes can be below cg

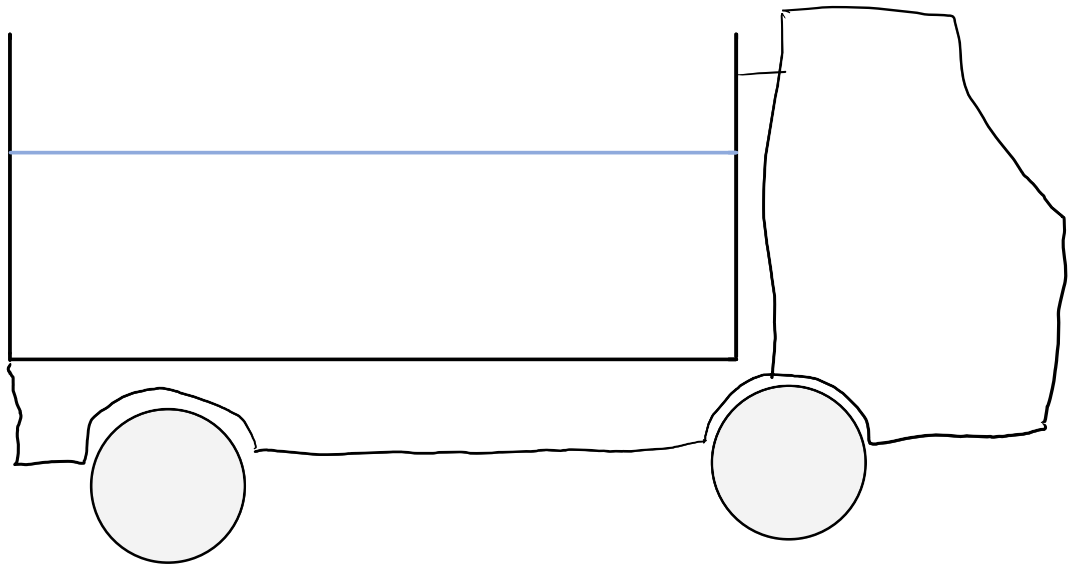


Pressure Distribution

$$\frac{\partial P}{\partial z} = -\rho(a_z + g), \frac{\partial P}{\partial x} = -\rho(a_x), \frac{\partial P}{\partial y} = -\rho(a_y)$$

$Ph \Rightarrow \rho g h \rightarrow \rho(g + a_z)h$

$$tan\theta = \frac{dz}{dx} = \frac{a_x}{a_z + g}$$

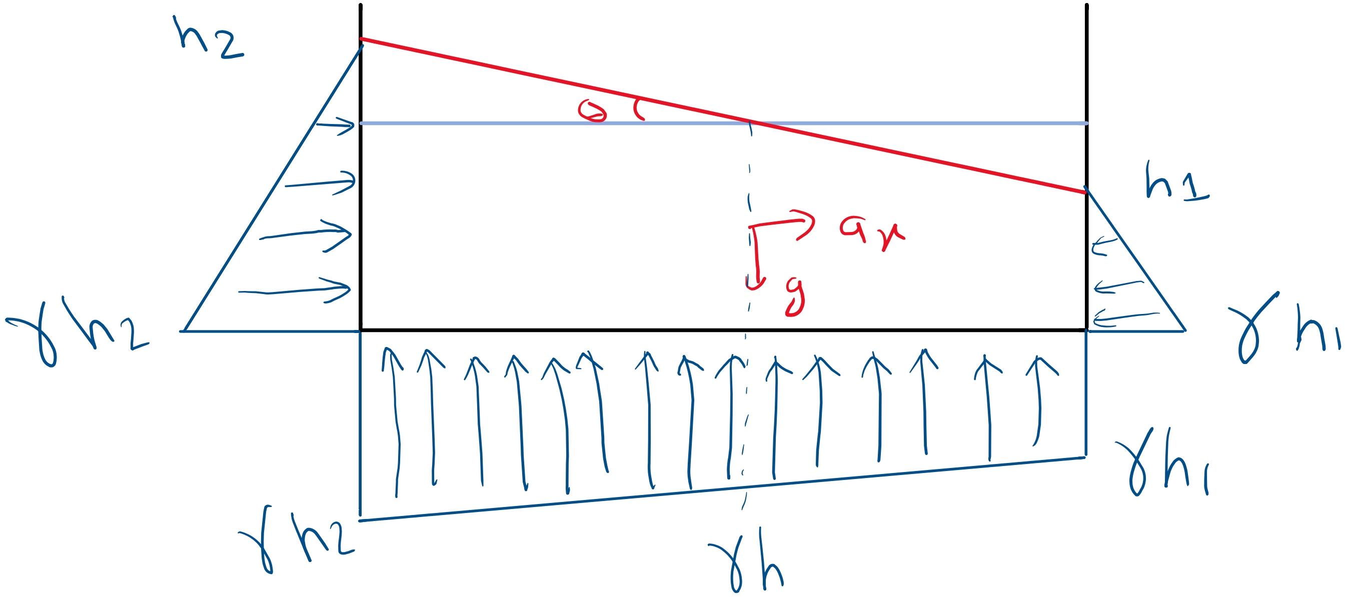


Pressure Distribution

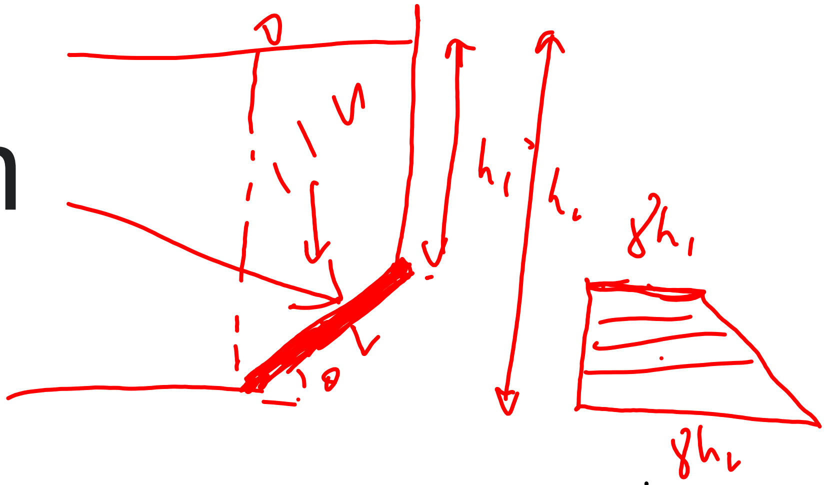
$$h_2 = h + \frac{L}{2} sin\theta$$

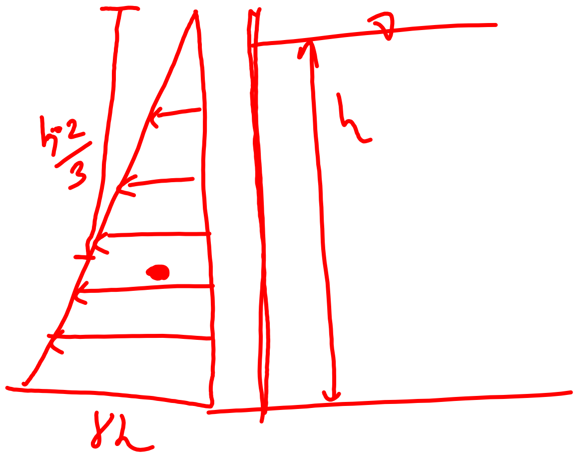
$$h_1 = h - \frac{L}{2} sin\theta$$

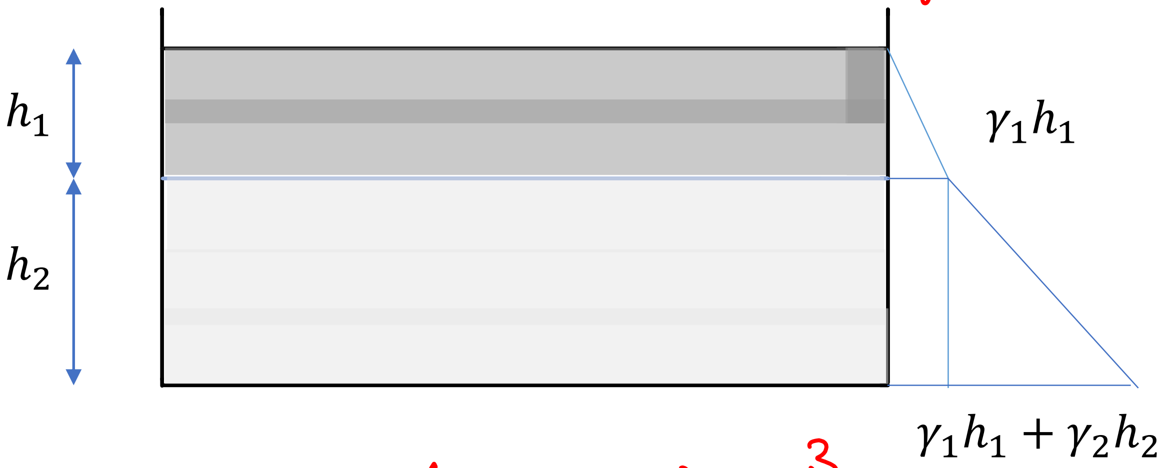
$$tan\theta = \frac{dz}{dx} = \frac{a_x}{g}$$



Pressure Distribution







force $= \gamma A \bar{x}$ = Area of pressure diagram

cp $\bar{h} = \bar{x} + \frac{I_G \sin^2\theta}{A\bar{x}}$ = centroid of pressure diagram.

$$\bar{h} = \left\{h_1 + \left(\frac{h_2 - h_1}{2}\right)\right\} + \frac{3/12 \cdot \sin^2\theta}{(L \cdot 1) \cdot \bar{x}}$$

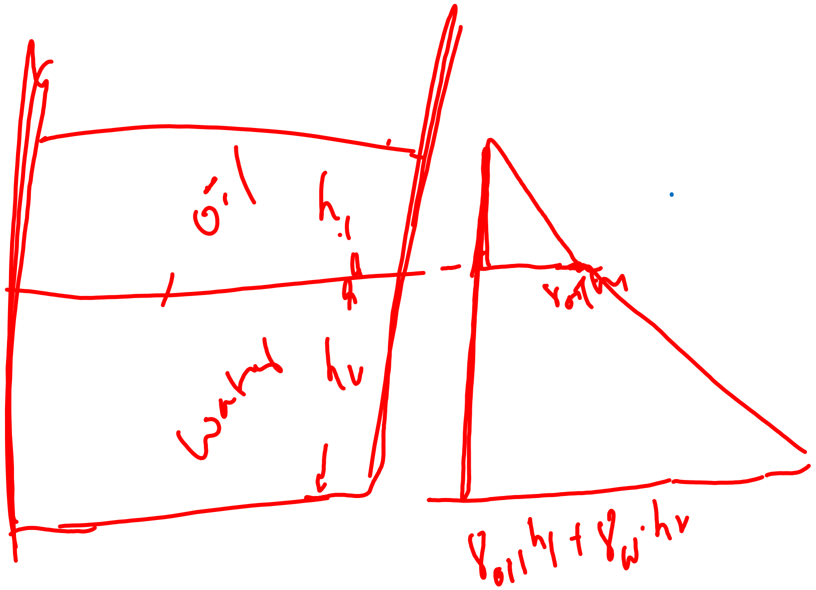
$$\bar{x} = \left(h_1 + \frac{(h_2 - h_1)}{2}\right)$$

Buoyancy

Buoyancy or upthrust, is an upward force exerted by a fluid that opposes the weight of a partially or fully immersed object.

Buoyancy

If B < W, Body sinks



Buoyancy

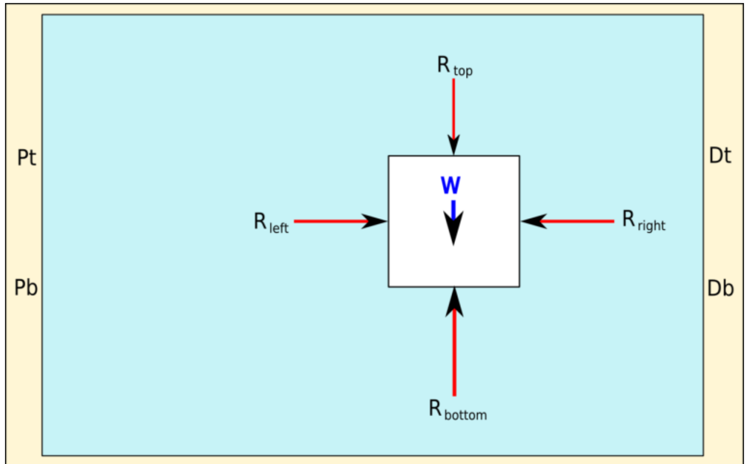
Buoyancy or upthrust, is an upward force exerted by a fluid that opposes the weight of a partially or fully immersed object.

If B < W, Body sinks

If B = W, Body floats, partially or fully immersed

Buoyancy = Weight of liquid displaced

And it acts at center of volume displaced



Forces on an immersed cube

Buoyancy

Buoyancy = Weight of liquid displaced

If V_{imm} be the volume of body submersed

Buoyancy = $\gamma_w V_{imm}$

For a floating body,

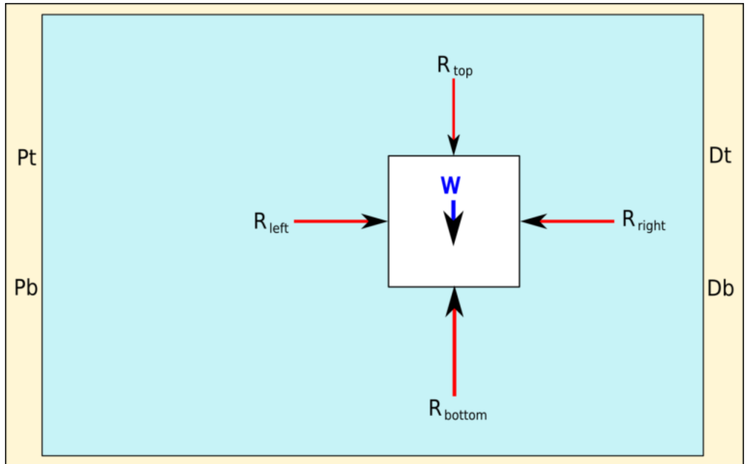
Buoyancy = Weight

$$\gamma_w V_{imm} = \gamma_m V$$

$$\frac{V_{imm}}{V} = \frac{\gamma_m}{\gamma_w}$$

Part of body submersed = $\frac{\gamma_m}{\gamma_w}$

Part of body outside water = $1 - \frac{\gamma_m}{\gamma_w}$



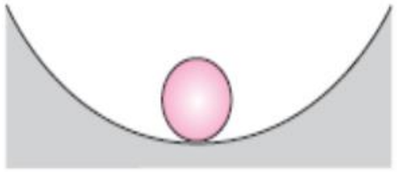
Forces on an immersed cube

Stability

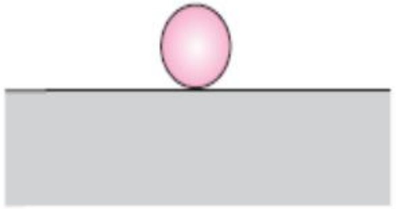
STABLE since any small disturbance (someone moves the ball to the right or left) generates a restoring force (due to gravity) that returns it to its initial position.

NEUTRALLY STABLE because if someone moves the ball to the right or left, it would stay put at its new location. It has no tendency to move back to its original location, nor does it continue to move away.

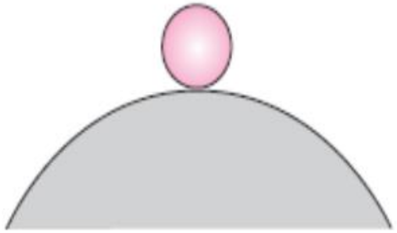
UNSTABLE since any disturbance, even an infinitesimal one, causes the ball to roll off the hill – it does not return to its original position; rather it diverges from it.



(*a*) Stable



(*b*) Neutrally stable



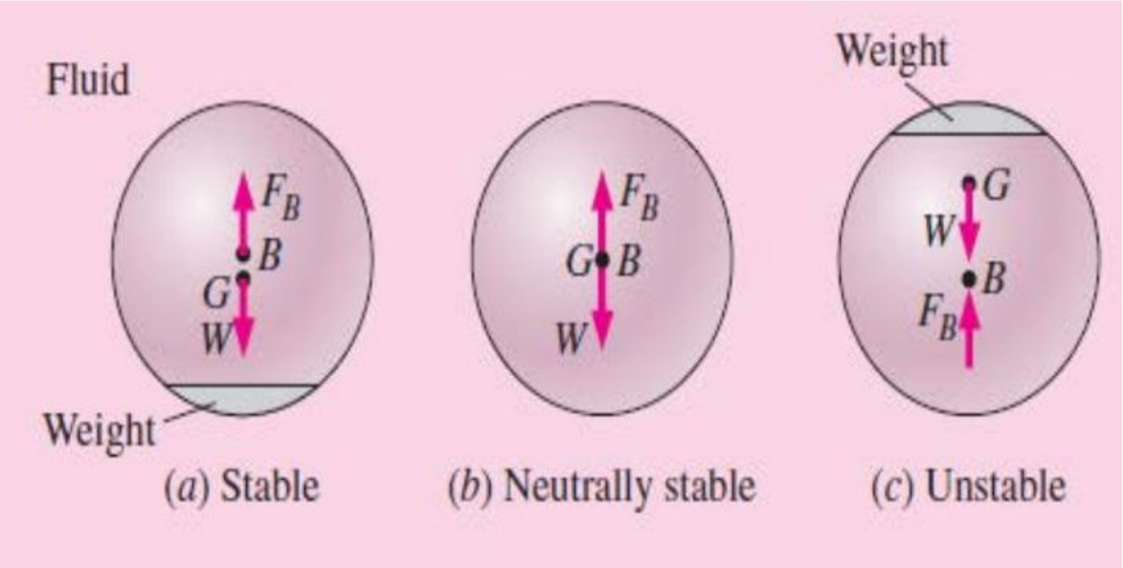
(*c*) Unstable

Stability of Immersed Bodies

STABLE: Center of buoyancy higher than CG of object

UNSTABLE: Center of buoyancy higher than CG of object.

NEUTRALLY STABLE: Center of buoyancy higher than CG of object.



Stability of Immersed Bodies

STABLE: Center of buoyancy higher than CG of object

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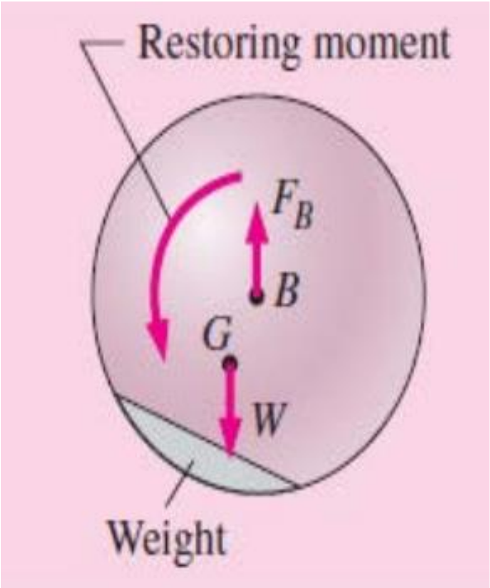
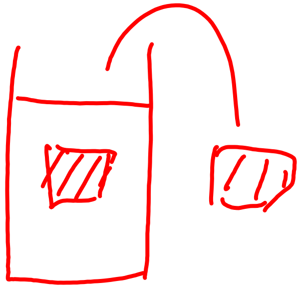
NEUTRALLY STABLE: Center of buoyancy higher than CG of object.



Upthrust / Buoyant force

Buoyancy

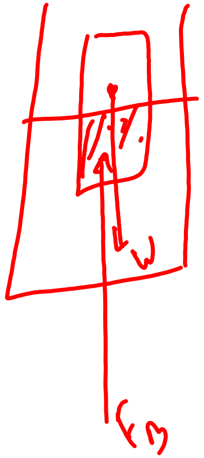
$F_B = \gamma_L \cdot V_0$



$F_B = \gamma_L \cdot$ volume of liquid displaced

F_B = Weight of liquid displaced

Partially submerged



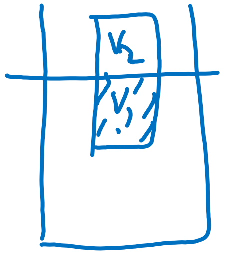
F_B = weight of body

w = weight of liquid displaced

$$\frac{V_{submerged}}{V} = \frac{G_{object}}{G_{liquid}}$$

A body of specific gravity 3.40 is partially submerged in mercury; what % of its volume will be above free surface?

(a) 12.5% (b) 25% (c) 50% (d) 75%



$$\gamma_0 (V_1 + V_2) = \gamma_m \cdot V_1$$

$$\frac{\gamma_0}{\gamma_m} = \frac{V_1}{V} = \frac{G_0 \cdot \gamma_w}{G_m \cdot \gamma_w} = \frac{G_0}{G_m} = \frac{3.40}{13.60} = \frac{1}{4}$$

Stability of floating Bodies

A measure of stability is the metacentric height GM, the distance between G and the metacenter M

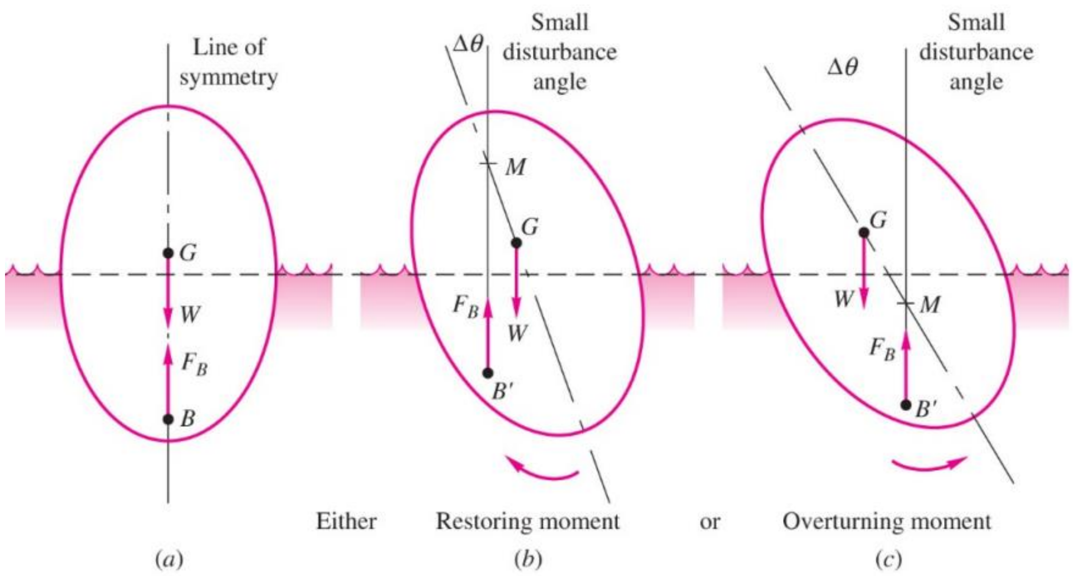
GM = BM – BG

$$BM = \frac{I}{V_{dis}}$$

STABLE: if M is above G (GM > 0)

UNSTABLE: if M is below G (GM < 0

NEUTRALLY STABLE: if M coincides G (GM = 0)



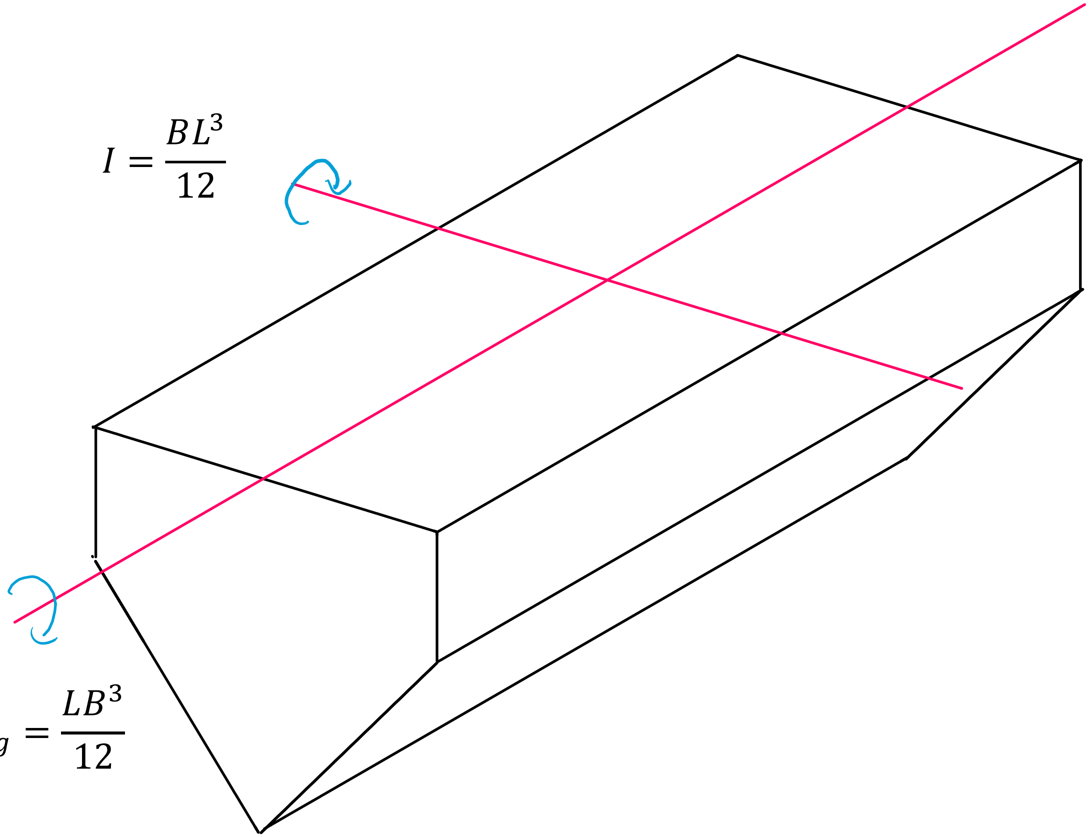
Stability of floating Bodies

A measure of stability is the metacentric height GM, the distance between G and the metacenter M

GM = BM – BG

$$BM = \frac{I}{V_{dis}}$$

$$I = \frac{BL^3}{12}$$



$$I_{rolling} = \frac{LB^3}{12}$$

PANA ACADEMY

Thank YOU !!!